MATHEMATICS

Grade 9

Book 2

CAPS

Learner Book

sasol inzalo foundation

UKUQONDA institute

Developed and funded as an ongoing project by the Sasol Inzalo Foundation in partnership with the Ukuqonda Institute.

Published by The Ukuqonda Institute
9 Neale Street, Rietondale 0084
Registered as a Title 21 company, registration number 2006/026363/08
Public Benefit Organisation, PBO Nr. 930035134
Website: http://www.ukuqonda.org.za

First published in 2014

ISBN: 978-1-920705-29-9

This book was developed with the participation of the Department of Basic Education of South Africa with funding from the Sasol Inzalo Foundation.

Contributors:
Piet Human, Erna Lampen, Marthinus de Jager, Louise Keegan, Paul van Koersveld, Nathi Makae, Enoch Masemola, Therine van Niekerk, Alwyn Olivier, Cerenus Pfeiffer, Renate Röhrs, Dirk Wessels, Herholdt Bezuidenhout

Acknowledgements:
For the chapters on Data Handling, some valuable ideas and data sets were gleaned from the following sources:
http://www.statssa.gov.za/censusatschool/docs/Study_guide.pdf
http://www.statssa.gov.za/censusatschool/docs/Census_At_School_2009_Report.pdf

Illustrations and computer graphics:
Leonora van Staden, Lisa Steyn Illustration, Ian Greenop
Zhandré Stark, Lebone Publishing Services
Computer graphics for chapter frontispieces: Piet Human

Cover illustration: Leonora van Staden

Text design: Mike Schramm

Layout and typesetting: Lebone Publishing Services

Printed by: [printer name and address]

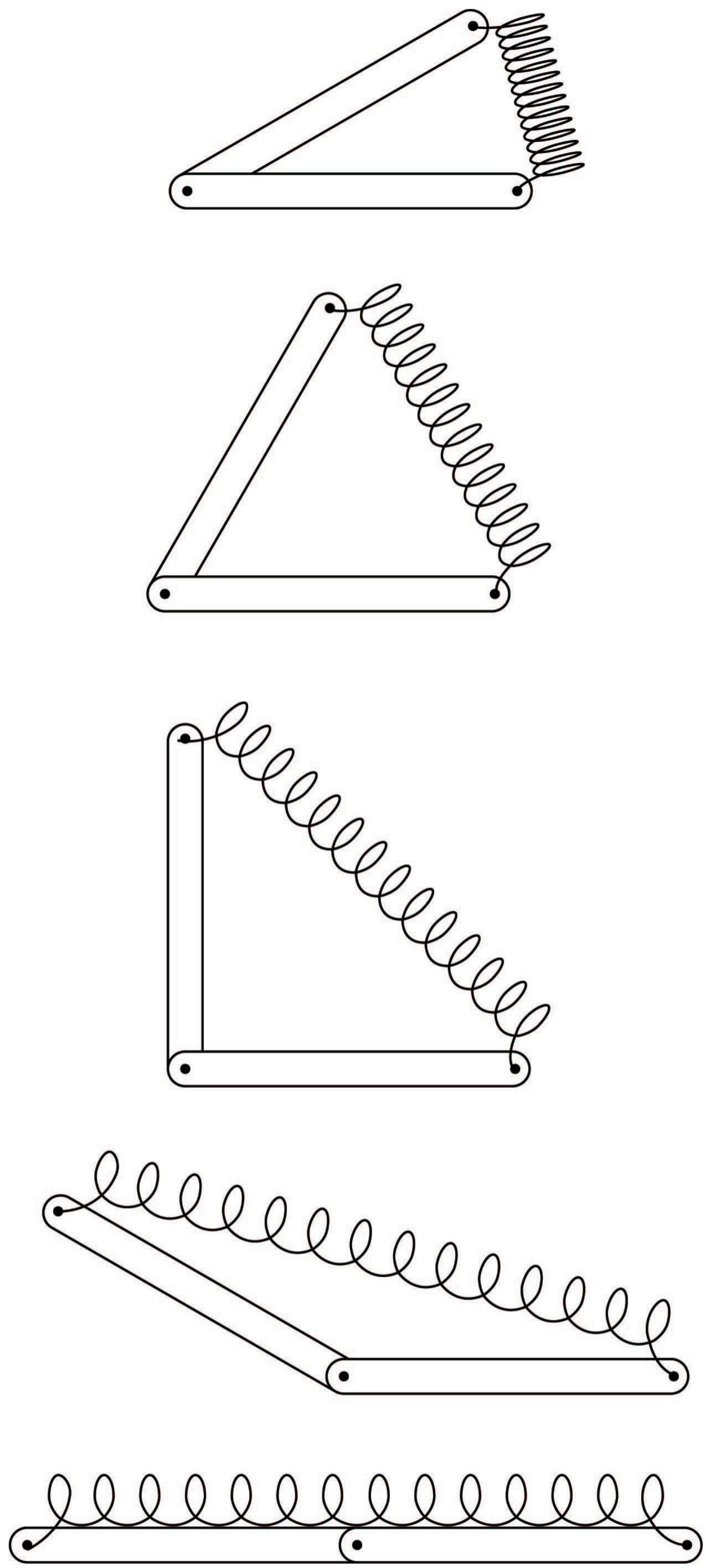
Table of contents

Term 3

Term 4

Chapter 1
Functions

In this chapter you will work with relationships between variables. You will find values of the output variable by doing calculations described in algebraic language. You will represent relations between variables in different ways, by describing them in words, in flow diagrams, in algebraic language and by means of tables and graphs. You will learn to recognise how the same relationship can be represented in different ways.



1 Functions

1.1 From formulas to words, tables and graphs

THE SAME INSTRUCTIONS IN WORDS AND IN SYMBOLS

1. Each of the formulas below indicates a relationship between two sets of numbers that may be called the *input numbers* and the *output numbers*. For each formula, calculate the output numbers that correspond to the input numbers 0; 1; 2 and 10.

   (a) $y = 3x + 5$ ....

   (b) $y = 3(x + 5)$ ....

   (c) $y = 3x + 5x$ ....

   (d) $y = 3x^2 + 5$ ....

   (e) $y = 3x^2 + 5x$ ....

   (f) $y = 3x(x + 5)$ ....

2. The information provided in the formula $y = 5x^2 - 3x$ can also be represented in words, for example: *To get the output number, you have to subtract 3 times the input number from 5 times the square of the input number.*

   Represent each of the formulas in question 1 in words:

   (a) $y = 3x + 5$

   ....

   (b) $y = 3(x + 5)$

   ....

   (c) $y = 3x + 5x$

   ....

   ....

   (d) $y = 3x^2 + 5$

   ....

   ....

   (e) $y = 3x^2 + 5x$

   ....

   ....

(f) $y = 3x(x + 5)$

....

....

3. For each set of instructions write a formula that provides the same information:
   (a) *multiply the input number by 10, then subtract 3 to get the output number* ....
   (b) *subtract 3 from the square of the input number, then multiply by 10 to get the output number* ....
   (c) *multiply the square of the input number by 10, then add 5 times the input number to get the output number* ....
   (d) *subtract 7 times the square of the input number from 100, then multiply by 3 to get the output number* ....
   (e) *add 4 to the input number, then subtract the answer from 50 to get the output number* ....
   (f) *multiply the input number by 3, then subtract the answer from 15 to get the output number* ....

4. To check your answers for question 3, use the table below. First apply the verbal instructions for the input numbers 1, 5 and 10 in each case. Then choose another input number and do the same thing. Next use the formula you have written to calculate the output numbers. Do corrections where there are differences.

		1	5	10	
(a)	verbal description				
	formula				
(b)	verbal description				
	formula				
(c)	verbal description				
	formula				
(d)	verbal description				
	formula				
(e)	verbal description				
	formula				
(f)	verbal description				
	formula				

5. In certain cases, flow diagrams can be used to provide instructions on how output numbers can be calculated. For each flow diagram below, represent the information in a formula and also in words.

(a) → [× 3] → [+ 17] → ....

....

(b) → [+ 5] → [× 3] → [+ 2] → ....

....

(c) → [– 2] → [× 3] → [+ 23] → ....

....

(d) → [× 2] → [+ 3] → [× 5] → [+ 4] → ....

....

....

(e) → [+ 3] → [× 2] → [+ 4] → [× 5] → ....

....

....

(f) → [× 10] → [+ 19] → ....

....

(g) → [+ 5] → [× 10] → ....

....

6. (a) Complete the following table.

x	0	1	2	3
y according to your formula for 5(a)				
y according to your formula for 5(b)				
y according to your formula for 5(c)				

(b) If your output numbers for 5(a), 5(b) and 5(c) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

7. (a) Complete the following table.

x	–3	–2	–1	0
y according to your formula for 5(d)				
y according to your formula for 5(e)				
y according to your formula for 5(f)				
y according to your formula for 5(g)				

(b) If your output numbers for 5(d) and 5(f) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

(c) If your output numbers for 5(e) and 5(g) are not the same, you have made a mistake somewhere. If this is the case, find your mistake and correct it.

8. Explain why the output numbers in 5(a), 5(b) and 5(c) are the same.

....

....

1.2 Tables and graphs

1. Complete the table to show some of the input and output numbers of the relationship described by the formula $y = 2x - 3$.

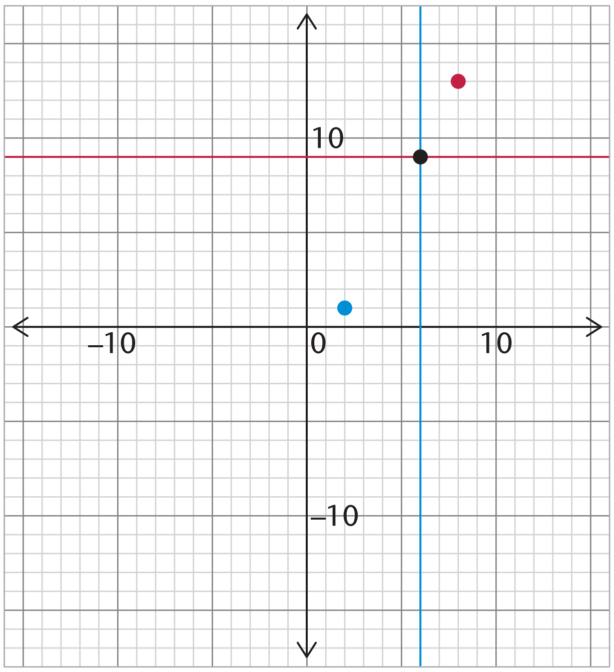
input numbers	–5	0	2	4	6	8
output numbers						

The vertical blue line on this graph represents the input number 6.

The heavy horizontal red line represents the output number 9.

The black point where the blue and red lines intersect indicates that the input number 6 is associated with the output number 9.

We also say the black point represents the **ordered number pair** (6; 9).

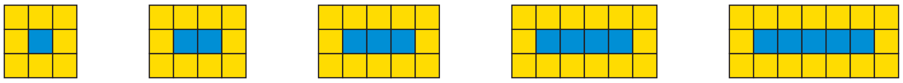


2. (a) Which ordered number pair does the red point on the graph represent?

....

(b) Which ordered number pair does the blue point on the graph represent?

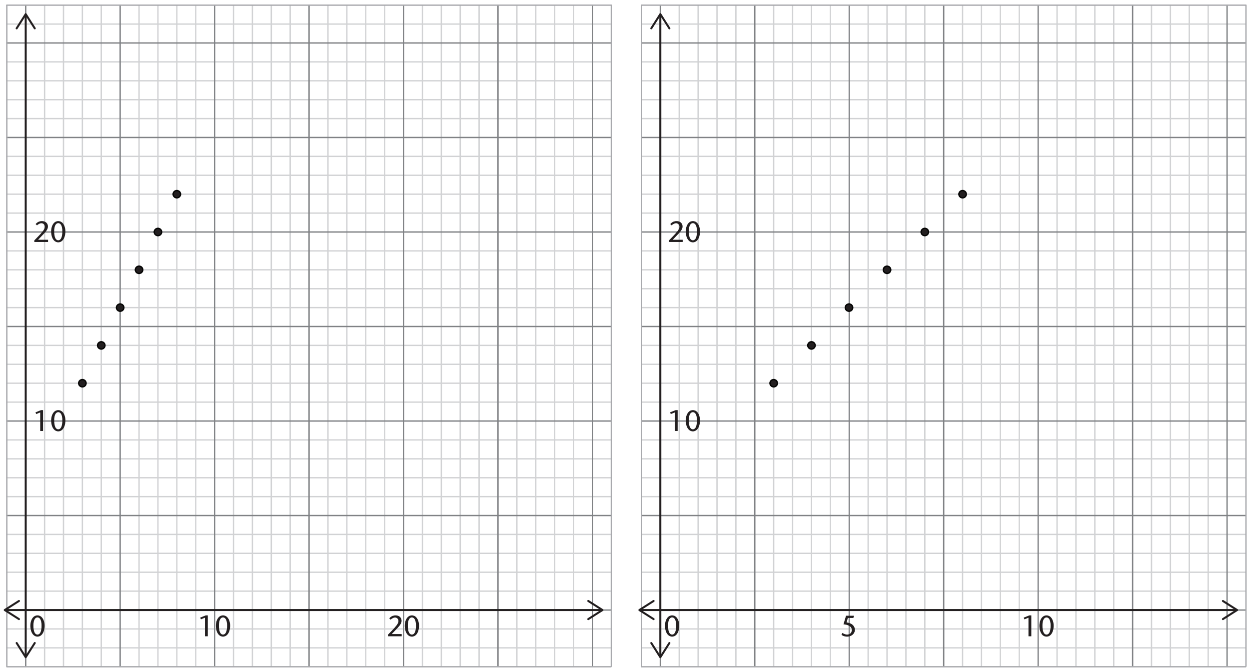
....



A relationship between two variables can be represented by a table of some values of the independent and dependent variables (input and output numbers):

values of the independent variable	3	4	5	6	7	8
values of the dependent variable	12	14	16	18	20	22

The same information can also be shown on a graph:



3. Do the two graphs show the same relationship, or different relationships between two variables?

. . . .

. . . .

. . . .

4. How do the two graphs differ?

. . . .

5. Use one of the graphs to find out how many yellow squares there will be, in an arrangement like those at the top, with 12 blue squares. . . . .

6. Does the table below represent the same relationship as the table at the top of the page? Explain your answer.

values of the independent variable	0	5	10	15	20	25
values of the dependent variable	8	18	28	38	48	58

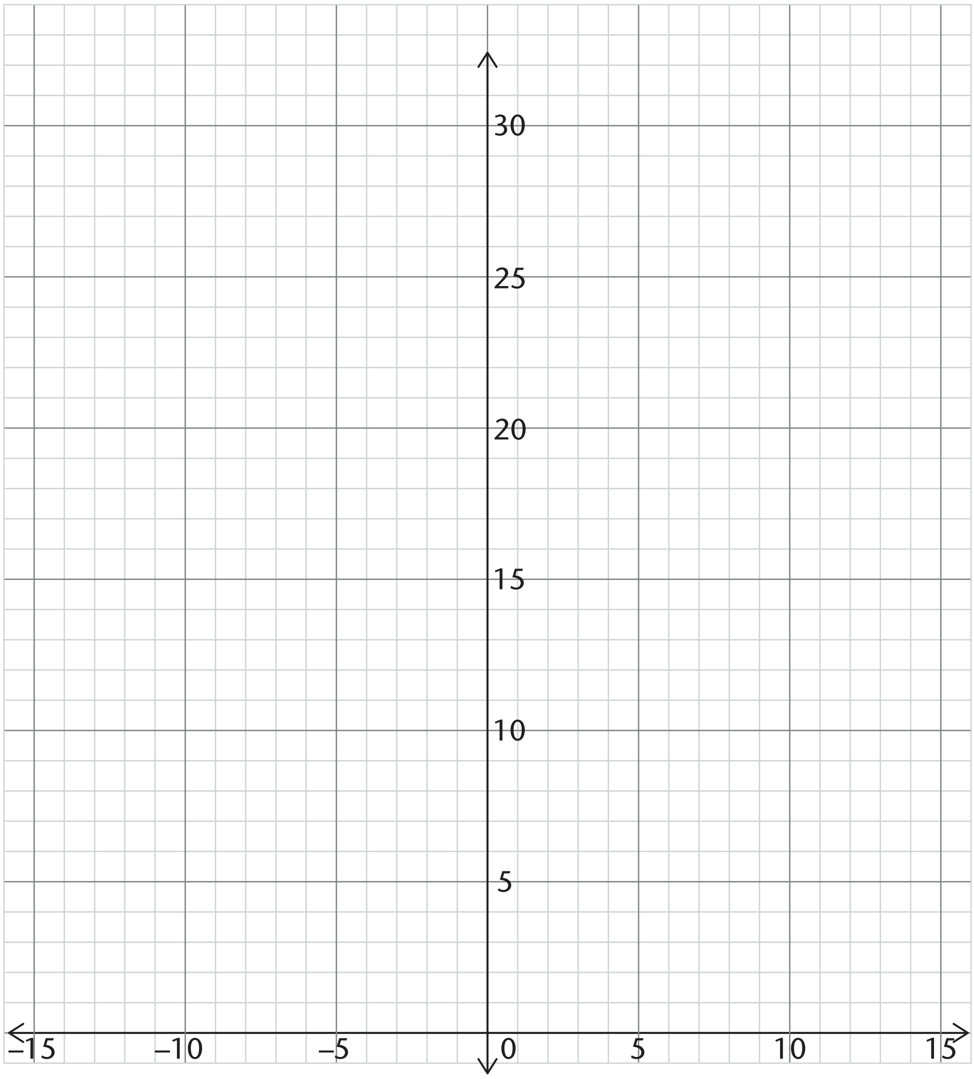
. . . .

. . . .

7\. (a) Complete the following table for the relationship described by $y = x^2$.

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y											

(b) Represent the ordered number pairs in the table on the graph sheet below.



8\. Complete the table for the relationship $y = 15 + x$. Represent the ordered number pairs on the graph sheet above.

x	–15	–10	–5	0	5	10	15
$15 + x$							

9\. Complete the table for the relationship $y = 15 – x$. Represent the ordered number pairs on the graph sheet above.

x	–15	–10	–5	0	5	10	15
$15 – x$							

10. (a) The output values for $y = x^2$ and $y = 15 + x$ show patterns. Describe in words how the patterns differ. Use the words *increase* and *decrease* in your description.

(b) Describe in words how the graphs of $y = x^2$ and $y = 15 + x$ differ.

11. (a) Describe in words how the patterns in the output values for $y = 15 + x$ and $y = 15 - x$ differ. Use the words *increase* and *decrease* in your description.

(b) Describe in words how the graphs of $y = 15 + x$ and $y = 15 - x$ differ.

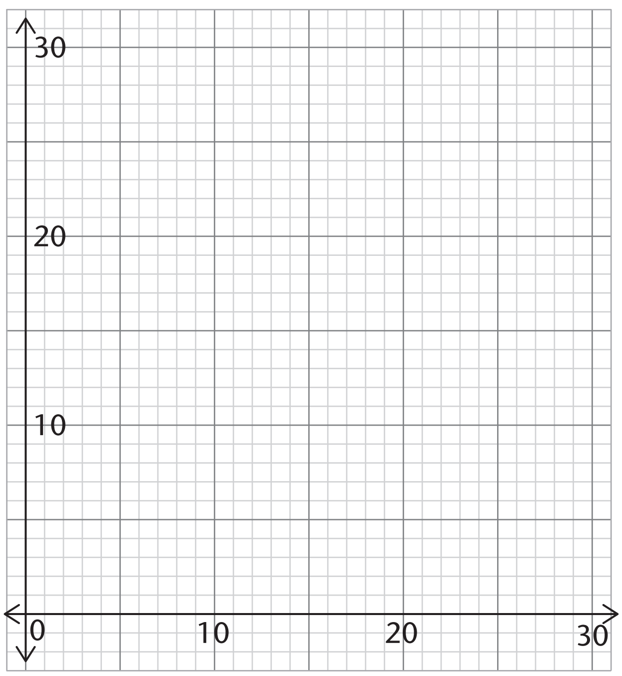
12. Complete each of the following tables by extending the pattern in the output numbers. Also represent the relationship on the graph sheets below.

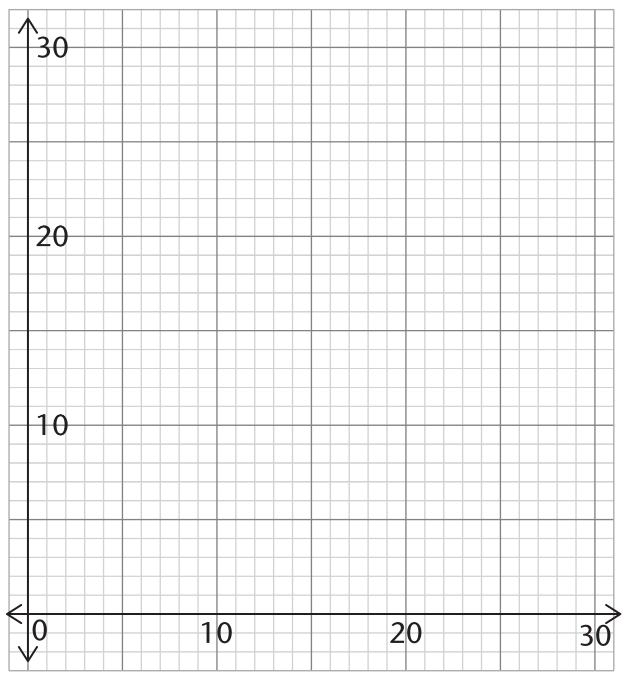
(a)

input numbers	0	5	10	15	20	25	30
output numbers	0	4	8	12			

(b)

input numbers	0	5	10	15	20	25	30
output numbers	0	2	4	6			





13. How do the patterns in 12(a) and (b) differ, and how do the graphs differ?

....

....

14. Each table below shows some values for a relationship represented by one of these rules:

$y = -2x + 3$	$y = 2x - 5$	$y = -3x + 5$	$y = -3(x + 2)$
$y = 3x + 2$	$y = 5(x - 2)$	$y = 2x + 3$	$y = 2x + 5$
$y = -3x + 6$	$y = 5x + 10$	$y = 5x - 10$	$y = -x + 3$

(a) Complete the tables below by extending the patterns in the output values.

(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

A. ....

B. ....

C. ....

D. ....

E. ....

F. ....

G. ....

A.

x	0	1	2	3	4	5	6	7
y	2	5	8					

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3				

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5				

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1					

E.

x	0	1	2	3	4	5	6	7
y	6	3	0					

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

G.

x	0	1	2	3	4	5	6	7
y	3	5	7					

AN INVESTIGATION: PATTERNS IN DIFFERENCES

1. Complete the tables for $y = x^2$, $z = x^2 + 1^2$, $w = x^2 + 2^2$ and $s = x^2 + 3^2$.

x	1	2	3	4	5	6	7	8	9	10
y										
z										
w										
s										

2. Complete the tables for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$.

(a)

x	1	2	3	4	5	6	7	8	9	10
p										
y										
$p - y$										

(b)

x	1	2	3	4	5	6	7	8	9	10
q										
y										
$q - y$										

(c)

x	1	2	3	4	5	6	7	8	9	10
r										
y										
$r - y$										

3. In each of the following cases, you should have different output values for the two relationships. If your output values are the same, find your mistakes and correct your work.
   (a) $z = x^2 + 1^2$ and $p = (x + 1)^2$
   (b) $w = x^2 + 2^2$ and $q = (x + 2)^2$
   (c) $s = x^2 + 3^2$ and $r = (x + 3)^2$

4. Complete the tables, for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$.

(a)

x	1	2	3	4	5	6	7	8	9	10
$p - y$										
$q - y$										
$r - y$										

(b)

x	10	11	12	13	14	15	16	17
$p - y$								
$q - y$								
$r - y$								

5. (a) Complete the table.

x	1	2	3	4	5	6	7	8	9	10
$2x + 1$										
$4x + 4$										
$6x + 9$										

(b) What are the constant differences in the sequences of values of $2x + 1$, $4x + 4$ and $6x + 9$, for $x = 1;\ 2;\ 3;\ 4 \ldots\ldots$? ....

(c) Do you have an idea whether the corresponding sequence for $12x + 36$ will also have a constant difference and what the constant difference may be?

....

(d) There are certain patterns in the coefficients and constant terms in the expressions in the above table. Continue the patterns to make some more similar expressions and complete the table below for your expressions.

x	1	2	3	4	5	6	7	8	9	10

6. (a) If your answers for the tables in 4(a) and 5(a) are correct, they will be the same. Try to explain why they are the same.

....

....

....

(b) What expressions, similar to those in question 5(a), may have the same values as $(x + 4)^2 - x^2$ and $(x + 5)^2 - x^2$ respectively?

....

Maths2_Gr9_LB_Book.indb 12 2014/09/08 09:06:07 AM

Chapter 2
Algebraic expressions

$(8 \times 23 - 5) \times 3 \times 23 - (4 \times 23 + 3) \times 6 \times 23$

$= \mathbf{51}$

$3 \times 23 = \mathbf{51}$

$(8 \times 25 - 5) \times 3 \times 25 - (4 \times 25 + 3) \times 6 \times 25$

$= \mathbf{75}$

$3 \times 25 = \mathbf{75}$

$(8 \times 17 - 5) \times 3 \times 17 - (4 \times 17 + 3) \times 6 \times 17$

$= \mathbf{69}$

$3 \times 17 = \mathbf{69}$

$(8 \times 27 - 5) \times 3 \times 27 - (4 \times 27 + 3) \times 6 \times 27$

$= \mathbf{81}$

$3 \times 27 = \mathbf{81}$

2 Algebraic expressions

2.1 Introduction

MANIPULATING EXPRESSIONS

The process of writing a polynomial as a product is called factorisation. This is the inverse of expansion.

$$x^2 + 5x + 6 \underset{\text{expansion}}{\overset{\text{factorisation}}{=}} (x + 2)(x + 3).$$

A numerical or algebraic expression that requires multiplication as a last step is called a **product**. For example, $12(37 + 63)$, $2x(x - 5)$ and xyz are called products. A product is a monomial.

Each part of a product is called a **factor** of the expression. If $c = ab$, then a and b are factors of c. $x + 2$ and $x + 3$ are the factors of $(x + 2)(x + 3)$. Since $x^2 + 5x + 6 = (x + 2)(x + 3)$, $x + 2$ and $x + 3$ are the factors of $x^2 + 5x + 6$.

1. Calculate the value of each of the following expressions for $x = 12$.

   (a) $\dfrac{(x+2)(x+5)}{x+2}$ (b) $\dfrac{(x-3)(x-4)}{x-4}$

   ....

   ....

   ....

   ....

   ....

   (c) $\dfrac{x(2x+1)}{2x+1}$ (d) $\dfrac{(x+5)(x-5)}{x-5}$

   ....

   ....

2. Check whether the following statements are identities by expanding the expressions on the right.

   A statement like $2(x + 3) = 2x + 6$, which is true for all values of x you can think of, is called an **identity**.

   (a) $x^2 - 9 = (x + 3)(x - 3)$ (b) $x^2 + x - 6 = (x + 3)(x - 2)$

   ....

   (c) $x^2 + 4x + 3 = (x + 3)(x + 1)$ (d) $x^2 + 3x = x(x + 3)$

   ....

Maths2_Gr9_LB_Book.indb 15 2014/09/08 09:06:10 AM

3. Write down the factors of each of the following expressions.

(a) $x^2 + x - 6$ .... (b) $x^2 + 3x$ ....

(c) $x^2 + 4x + 3$ .... (d) $x^2 - 9$ ....

4. Simplify the following quotients (algebraic fractions).

(a) $\frac{x^2 - 9}{x + 3}$ (b) $\frac{x^2 + x - 6}{x + 3}$

....

(c) $\frac{x^2 + x - 6}{x - 2}$ (d) $\frac{x^2 + 4x + 3}{(x + 3)(x + 1)}$

....

5. (a) Suppose you have to find the value of the expression for $x = 15$. Which expression will be the least amount of work? $\frac{x^2 - 9}{x + 3}$ or $x - 3$? ....

(b) Are you sure that you will get the same answers for the two expressions?

In the following sections you will learn how to factorise certain types of expressions. The following identities are useful for the purposes of factorisation:

$a(b + c) = ab + ac$ $\quad (x + a)(x + b) = x^2 + (a + b)x + ab$ $\quad (a + b)(a - b) = a^2 - b^2$

2.2 Factors of expressions of the form *ab* + *ac*

THE GREATEST COMMON FACTOR

1. (a) Is 5 a factor of 20? (b) Is 5 a factor of 30?

....

(c) Is 5 a factor of 30 + 20? (d) Is 5 a factor of 30 – 20?

....

2. (a) Is a a factor of ab? (b) Is a a factor of ac?

....

(c) Is a a factor of $ab + ac$? (d) Find another factor of $ab + ac$.

....

(e) Now try and simplify: $\frac{ab + ac}{a}$ ....

Suppose you have to factorise $4x^3 + 2x^2 - 6x$: It is clear that $2x$ is a factor of every term, hence it is a factor of $4x^3 + 2x^2 - 6x$.

By division we get $\frac{4x^3 + 2x^2 - 6x}{2x} = 2x^2 + x - 3$. Hence $4x^3 + 2x^2 - 6x = 2x(2x^2 + x - 3)$.

It is always a good idea to check factorisation by expanding the answer and making sure that the result is equal to the original expression.

3. Complete the table.

For each expression, find:	$3x + 6y$	$4a^3 + 2a$	$5x - 2x^2$	$ax^2 - bx^3$	$12a^2b + 18ab^2$
the factors of the first term	3; x				
the factors of the second term	2; 3; y				
the greatest common factor of the two terms	3				
Write the expression in factor form	$3(x + 2y)$				

4. Study the example and then factorise the expressions that follow.

$$(a - b)x + (b - a)y = (a - b)x - (a - b)y$$
$$= (a - b)(x - y)$$

Note that:
$b - a = -a + b = -(a - b)$

(a) $(a - b)x + a - b$

....

....

(b) $(a - b)x - a + b$

....

....

(c) $(a + b)^2 - (a + b)$

....

....

(d) $(a + b)x - a - b$

....

....

(e) $3x(2x - 3) - (3 - 2x)$

....

....

(f) $(y^2 - 4y) + (3y - 12)$

....

....

SOMETHING IN BETWEEN

1. By completing the tables below you will learn something that will help you to find the factors of expressions of the form $x^2 + (b + c)x + bc$, for example $x^2 + 17x + 30$.

b	1	2	3	5	–1	–2	–3	–5
c	30	15	10	6	–30	–15	–10	–6
$b + c$								
bc								

b	–1	–2	–3	–5	1	2	3	5
c	30	15	10	6	–30	–15	–10	–6
$b + c$								
bc								

2. For each case below find two numbers x and y so that their product xy is 30 and their sum $x + y$ is the given number.

(a) $xy = 30$ and $x + y = 13$ ....

(b) $xy = 30$ and $x + y = -17$ ....

(c) $xy = 30$ and $x + y = -11$ ....

(d) $xy = 30$ and $x + y = 11$ ....

3. Find x and y in each case.

You may use the tables you completed in question 1 to find the answers to some of these questions.

(a) $xy = -30$ and $x + y = -13$ ....

(b) $xy = 30$ and $x + y = -13$ ....

(c) $xy = -30$ and $x + y = 13$ ....

(d) $xy = -30$ and $x + y = -1$ ....

(e) $xy = -30$ and $x + y = 1$ ....

4. Find x and y in each case.

(a) $xy = 36$ and $x + y = 15$ ....

(b) $xy = 40$ and $x + y = 22$ ....

(c) $xy = 36$ and $x + y = 20$ ....

(d) $xy = -40$ and $x + y = 18$ ....

(e) $xy = 36$ and $x + y = -20$ ....

(f) $xy = -40$ and $x + y = -18$ ....

5. Evaluate each expression for $x = 2$. Also expand each expression.

(a) $(x + 5)(x - 2)$ ....
....

(b) $(x + 5)(x + 2)$ ....
....

(c) $(x - 5)(x - 2)$ ....
....

(d) $(x - 5)(x + 2)$ ....
....

6. Evaluate each polynomial you formed in question 5 for $x = 2$. Compare the answers with the values of the corresponding product expressions in question 1. In cases where the values differ, you have made a mistake somewhere. Sort out any mistakes completely before you continue with question 7.

....

Maths2_Gr9_LB_Book.indb 18 2014/09/08 09:06:12 AM

7. Expand each product.

(a) $(x + 3)(x + 8)$ ....

(b) $(x + 2)(x + 12)$ ....

(c) $(x + 4)(x + 6)$ ....

(d) $(x + 1)(x + 24)$ ....

(e) $(x + 3)(x - 8)$ ....

(f) $(x + 2)(x - 12)$ ....

(g) $(x + 4)(x - 6)$ ....

(h) $(x + 1)(x - 24)$ ....

2.3 Factors of expressions of the form $x^2 + (b + c)x + bc$

The expanded form of a **product of two linear binomials** like $(x + 3)(x + 8)$ or $(x + 3)(x - 8)$ is a **quadratic trinomial** like $x^2 + 11x + 24$ or $x^2 - 5x - 24$ with

- a term in x^2,
- a term in x that is called the **middle term**, which is $+11x$ in $x^2 + 11x + 24$ and $-5x$ in $x^2 - 5x - 24$, and
- a constant term also called the **last term**, which is $+24$ in $x^2 + 11x + 24$, and -24 in $x^2 - 5x - 24$.

To factorise an expression like $x^2 + 5x + 6$ means to reverse the process of expansion. This means that we have to find out which binomials will produce the trinomial when the product of the binomials is expanded, for example:

$$x^2 + 5x + 6 = (\textbf{?} + \textbf{?})(\textbf{?} + \textbf{?})$$

expansion $(x + 2)(x + 3)$ **factorisation**

$= x^2 + 5x + 6$

TRY TO FIND THE FACTORS

1. Fill in the missing parts of the factors in each of the following cases.

(a) $(x + 3)(x \ldots\ldots) = x^2 + 9x + 18$

(b) $(x + 2)(x \ldots\ldots) = x^2 + 11x + 18$

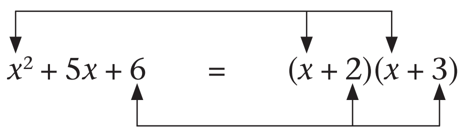
(c) $(x + 3)(x - \ldots\ldots) = x^2 + 9x - 18$

(d) $(\dots + \dots)(x + 2) = x^2 + 5x + 6$ (e) $x^2 - x - 6$

....

2. Expand each product:
   (a) $(x + p)(x + q)$ ....
   (b) $(x + p)(x - q)$ ....
   (c) $(x - p)(x + q)$ ....
   (d) $(x - p)(x - q)$ ....

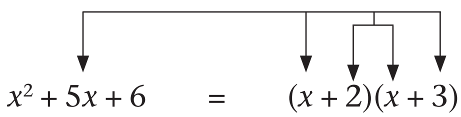
The product of the first terms of the factors must be equal to the x^2 term of the trinomial.



Meaning: $x \times x = x^2$

Meaning: $2 \times 3 = 6$

The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial. The sum of the inner and outer products must be equal to the term in x (the middle term) of the trinomial.



Meaning: $2x + 3x = (2 + 3)x = 5x$

$(x + a)(x + b) = x \times x + ax + bx + a \times b = x^2 + (a + b)x + ab$

3. Try to factorise the following trinomials.
   (a) $x^2 + 8x + 12$ (b) $x^2 - 8x + 12$

....

PRACTICE MAKES PERFECT

1. Factorise the following trinomials: (Remember to check your answer by expanding the factors to test if you do get the original expression.)

(a) $a^2 + 9a + 14$

(b) $x^2 + 3x - 18$

(c) $x^2 - 18x + 17$

(d) $y^2 + 17y + 30$

(e) $y^2 - 13y - 30$

(f) $y^2 + 7y - 30$

(g) $x^2 + 2x - 15$

(h) $m^2 + 4m - 21$

(i) $x^2 - 6x + 9$

(j) $b^2 + 15b + 56$

(k) $a^2 - 2a - 63$

(l) $a^2 - ab - 30b^2$

(m) $x^2 - 5xy - 24y^2$

(n) $x^2 - 13x + 40$

An alternative method

2. Study the example and then factorise the expressions that follow.

Example: Factorise $ac + bc + bd + ad$

$ac + bc + bd + ad$	$= (ac + bc) + (bd + ad)$	*Order and group terms with common factors*
	$= c(a + b) + d(b + a)$	*Take out the common factor*
	$= (a + b)(c + d)$	*Write expression as a product*

(a) $px + py + qx + qy$

(b) $9x^3 - 27x^2 + x - 3$

(c) $4a + 4b + 3ap + 3bp$

(d) $a^4 + a^3 + 3a + 3$

....

....

(e) $xy + x + y + 1$

(f) $ac - ad - bc + bd$

....

....

Yet another method

Example 1:	**Example 2:**	**Action:**
$x^2 + 4x + 3$	$x^2 + 3x - 4$	
$= x^2 + x + 3x + 3$	$= x^2 - x + 4x - 4$	*Re-writing middle term as sum of two terms.*
$= (x^2 + x) + (3x + 3)$	$= (x^2 - x) + (4x - 4)$	*Grouping.*
$= x(\mathbf{x + 1}) + 3(\mathbf{x + 1})$	$= x(\mathbf{x - 1}) + 4(\mathbf{x - 1})$	*Taking out the GCF of each group.*
$= (\mathbf{x + 1})(x + 3)$	$= (\mathbf{x - 1})(x + 4)$	*Write it as a product.*

3. Factorise:

(a) $x^2 + 7x + 12$

(b) $x^2 - 7x + 12$

....

The challenge is to re-write the middle term as the sum of two terms in a way that you are able to take out the common factor.

2.4 Factors of expressions of the form $a^2 - b^2$

PRELIMINARY WORK

1. Complete the following table and see if you can notice a pattern (rule) whereby you can predict the answers to the first column's calculations without squaring it:

(a)	$3^2 - 2^2$	$3 + 2$	$3 - 2$	$(3 + 2)(3 - 2)$
(b)	$4^2 - 3^2$	$4 + 3$	$4 - 3$	$(4 + 3)(4 - 3)$
(c)	$6^2 - 4^2$	$6 + 4$	$6 - 4$	$(6 + 4)(6 - 4)$
(d)	$9^2 - 3^2$	$9 + 3$	$9 - 3$	$(9 + 3)(9 - 3)$

2. Do you notice a pattern (rule) whereby you can predict the answers to such calculations?

....

....

3. Now predict the answers to each of the following without squaring. Check your answers where necessary. Does the rule that you discovered in question 2 also hold for the following cases?

(a) $17^2 - 13^2$ (b) $54^2 - 46^2$ (c) $28^2 - 22^2$

....

....

4. Formulate your rule in symbols:

$a^2 - b^2 =$ ....

5. Can you explain why factors of $a^2 - b^2$ have this form?

....

Stated differently: If p and q are perfect squares, also "algebraic squares", then:

$$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$$

$$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$$

$$= (3x^2 + 2y)(3x^2 - 2y)$$

(Note the operations within the brackets differ.)

An expression of the form $a^2 - b^2$ is called the **difference between two squares**.

To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

FACTORISING DIFFERENCE BETWEEN TWO SQUARES EXPRESSIONS

1. Use the skills you learnt in the previous exercises to factorise the following:

(a) $4a^2 - b^2$ (b) $m^2 - 9n^2$

....

(c) $25x^2 - 36y^2$ (d) $121x^2 - 144y^2$

....

(e) $16p^2 - 49q^2$ (f) $64a^2 - 25b^2c^2$

....

(g) $x^2 - 4$ (h) $16x^2 - 36y^2$

....

Always factorise completely.
Always take out the greatest common factor if there is one.
One is a perfect square: $1 = 1^2$ and $1^m = 1$.
The exponential law: $a^m.a^n = a^{m+n}$.

2. Factorise.

(a) $x^4 - 1$

....

(b) $16a^4 - 81$

....

(c) $1 - a^2b^2c^2$

....

(d) $25x^{10} - 49y^8$

....

(e) $2x^2 - 18$

....

....

(f) $200 - 2b^2$

....

....

(g) $3xy^2 - 48xa^2$

....

....

(h) $5a^4 - 20b^2$

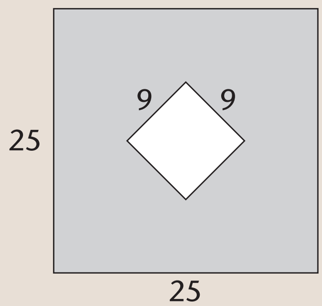
....

....

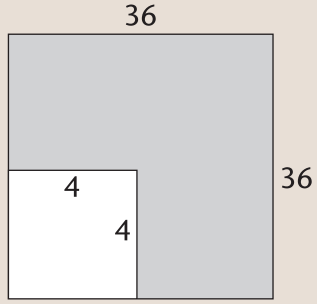
IN EACH CASE CALCULATE THE AREA OF THE SHADED PART.

Use the shortest possible method.

(a)



(b)



THIS IS HOW FACTORISATION CAN MAKE CALCULATION EASY!

2.5 Simplification of algebraic fractions

WORKING WITH ALGEBRAIC FRACTIONS

Liza and Madodo have to determine the value of $\frac{x^2 - 2x - 3}{x - 3}$ for $x = 4,6$.

Liza's solution:		Madoda's solution:	
$\frac{x^2 - 2x - 3}{x - 3}$		$\frac{x^2 - 2x - 3}{x - 3}$	
$= \frac{4,6^2 - 2(4,6) - 3}{4,6 - 3}$	Substitute $x = 4,6$	$= \frac{(x - 3)(x + 1)}{x - 3}$	Factorise the numerator
$= \frac{21,16 - 9,2 - 3}{4,6 - 3}$		$= x + 1$	Simplify the expression
$= \frac{8,96}{1,6}$		$= 4,6 + 1$	Substitute $x = 4,6$
$= 5,6$		$= 5,6$	

1. Which solution do you prefer? Why?

....

It is useful to manipulate quotient expressions like $\frac{x^2 + 5x + 6}{x + 2}$ into simpler but equivalent sum expressions, like $x + 3$ in this case. It makes substitution and the solving of equations easier.

2. Solve the following problems.

   (a) Evaluate $\frac{x^2 + 5x + 6}{x + 2}$ if $x = 23$.

   (b) Solve $\frac{x^2 + 5x + 6}{x + 2} = 19$.

....

....

....

....

3. Determine the value of each of the following expressions if $x = 36$. See if you can use the shortest possible method.

   (a) $\frac{x^2 - 9}{x + 3}$

   (b) $\frac{x^2 + x - 6}{x + 3}$

....

....

....

....

HOW IS IT POSSIBLE THAT 2 = 1?

What went wrong in the following argument?

Let:		$a = b$	(If: $b \neq 0$)
$\times\ a$:	$\Leftrightarrow$	$a^2 = ab$	
$-\ b^2$:	$\Leftrightarrow$	$a^2 - b^2 = ab - b^2$	
Factorise:	$\Leftrightarrow$	$(a + b)(a - b) = b(a - b)$	
$\div\ (a - b)$:	$\Leftrightarrow$	$a + b = b$	
But $a = b$:	$\Leftrightarrow$	$b + b = b$	
Add terms:	$\Leftrightarrow$	$2b = b$	
$\div\ b$:	$\Leftrightarrow$	$2 = 1$	

Explain what went wrong and why it is wrong?

DIVIDING BY ZERO CANNOT BE DONE

1. Complete the following table by evaluating the value of the expression $\frac{x+2}{x-2}$ for the x-values given in the top row:

x	-2	0	2	4
$\frac{x+2}{x-2}$				

2. If $x = 2$ then $\frac{x+2}{x-2}$ will have the value $\frac{4}{0}$. What is the value of $\frac{4}{0}$?

   ....

3. One way to determine the value of $\frac{4}{0}$, you can set it as $\frac{4}{0} = a$. Then $4 = 0 \times a$. Which values of a will make this statement true?

   ....

4. What is the result of the calculation of $4 \div 0$ on your calculator? Can you explain the message on your calculator?

   ....

Division by 0 is not possible. The algebraic fraction $\frac{x+2}{x-2}$ cannot have a value a value if the denominator $(x - 2)$ is equal to 0. We may say the expression $\frac{x+2}{x-2}$ is **undefined** for $x - 2 = 0$ i.e. for $x = 2$. We also say $x = 2$ is an **excluded value** of x for $\frac{x+2}{x-2}$.

DEFINING THE UNDEFINED

1. Is the following statements true? If not, correct the statement.
   (a) $\frac{x}{x} = 1$ for all values of x.

   ....

   ....

   (b) $\frac{x^3}{x^2} = x$ for all values of x.

   ....

   ....

   (c) $\frac{x-3}{x-3} = 1$ for all values of x.

   ....

   ....

   (d) $\frac{x^2+x}{x(x+1)} = 1$ for all values of x.

   ....

   ....

   ....

2. For which values of the variables will each expression be undefined?
   (a) $\frac{7(y+5)}{y+2}$ ....
   (b) $\frac{3x+2}{x+4}$ ....
   (c) $\frac{2x+1}{x^2-1}$ ....
   (d) $\frac{2x^2-1}{(x-2)(x+3)}$ ....

SIMPLIFYING ALGEBRAIC FRACTIONS

To simplify an algebraic fraction that contains a polynomial as numerator or denominator, the polynomial should be factorised first.

To prevent division by zero, the excluded values must be stated.

1. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. Give excluded values.

(a) $\frac{3xy + y^2}{3x + y}$

(b) $\frac{a^2b + ab^2}{a + b}$

....

....

(c) $\frac{3x^2y - 6x^2y^2}{3xy}$

(d) $\frac{10x^4 + 15x^3}{5x^2}$

....

....

2. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. (See if you can factorise the trinomials.)

(a) $\frac{x^2 + 5x + 6}{x + 2}$

(b) $\frac{x^2 + 2x - 8}{x - 2}$

....

....

(c) $\frac{x^2 - 5x - 50}{x + 5}$

(d) $\frac{x^2 - 16x + 15}{x - 15}$

....

....

3. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$.

(a) $\frac{x^2 - 4}{x - 2}$

(b) $\frac{4x^2 - 1}{2x + 1}$

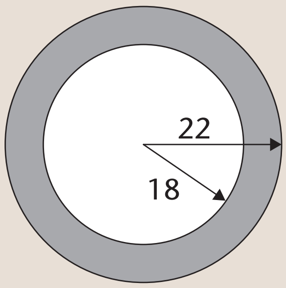
....

....

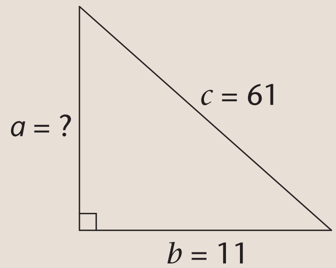
FACTORISATION CAN REDUCE CALCULATIONS

In each case, use the shortest possible method to get to your answer.

(a) Calculate the shaded area.
(Area = πr^2 and use $\pi = 3.142$)



(b) Calculate the length of side a
(Pythagoras: $c^2 = a^2 + b^2$)



THIS IS HOW FACTORISATION CAN SAVE YOU TIME!

MORE PRACTICE

1. Factorise the following expressions completely.

(a) $4a + 6b$

(b) $x^2 + 8x + 7$

(c) $c^2 - 9$

(d) $y^2 - 8y + 15$

(e) $-3ab + b$

(f) $-3a(b - 1) + (b - 1)$

(g) $dfg^2 + d^2g - df^2g$

(h) $x^2 + 6x + 8$

(i) $a^2 + 5a + 6$

(j) $x^2 - 8x - 20$

(k) $x^5y^3 - x^3y^5$

(l) $x^3y - xy^3$

(m) $4 - 4y + y^2$

(n) $3a^2 + 18a - 21$

(o) $6a^2 - 54$

(p) $-a^2 - 11a - 30$

(q) $2a^2 + 10a - 72$

(r) $5x^3 - 15x^2 - 200x$

(s) $(x + 2)^2 - y^2$

(t) $(x + y)^2 - a^2$

(u) $(a^2 - 2a + 1) - b^2$

(v) $1 - (a^2 - 2ab + b^2)$

(w) $(a - b)x + (b - a)y$

(x) $a(2x - y) + (y - 2x)$

(y) $2x^2y^{10} - 8x^{10}y^2$

(z) $(a + b)^3 - 4(a + b)$

(aa) $(a + b)^2 - a - b$

(ab) $(x + y)(a - b) + (-x - y)(b - a)$

2. Simplify each of the following algebraic fractions as far as possible.

(a) $\frac{16 - 9x^2}{4 + 3x}$

(b) $\frac{25x^2 - 36}{5x^2 + 6x}$

(c) $\frac{x^3 + x^2 - 30x}{x + 6}$

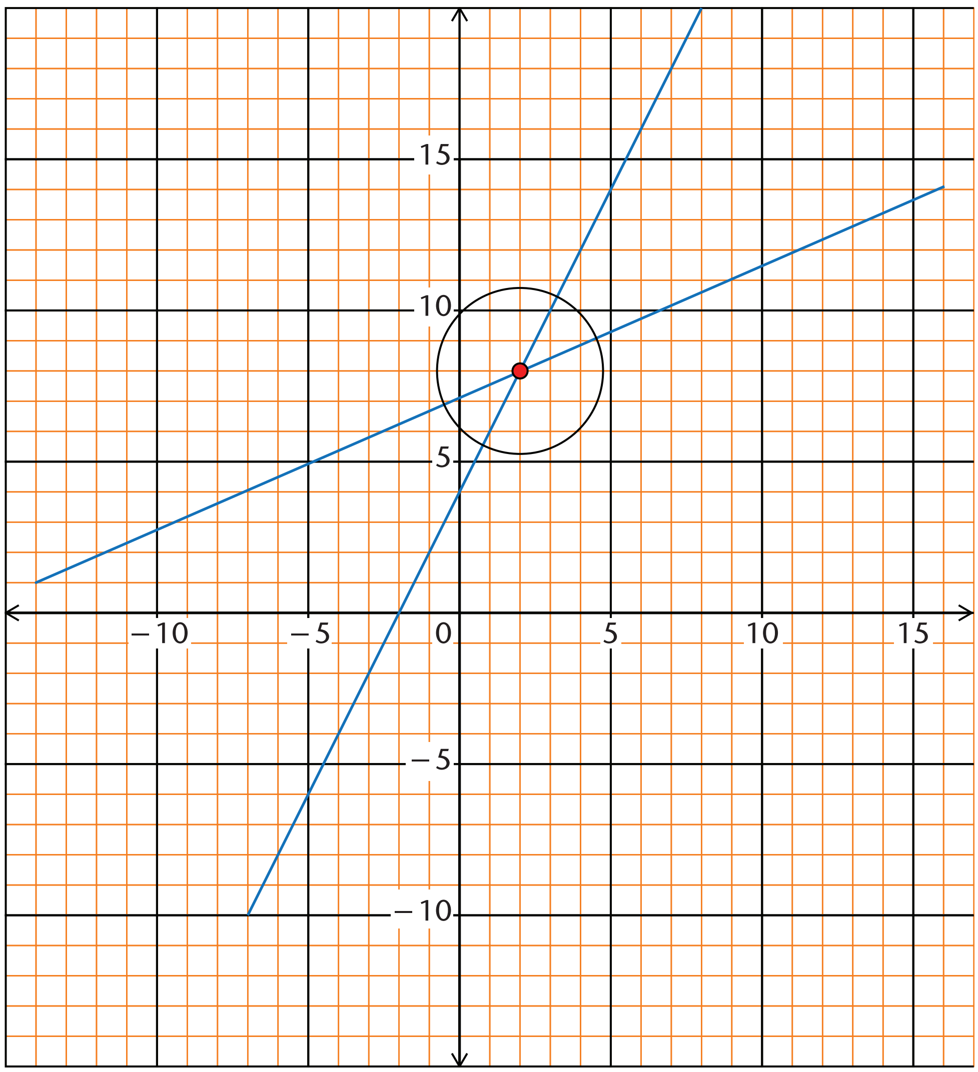
(d) $\frac{2x^2 + 5x + 3}{2x + 3}$

(e) $\frac{ab + bc}{abc}$

(f) $\frac{pa + pb}{a + b}$

Chapter 3
Equations

You have already solved equations by inspection and inverse operations in the first term. In this chapter you will first revise this work. Then you will work with equations which contain product expressions, like $2x(x - 2)$ and $(x - 5)(x + 3)$. You will learn new methods to solve these equations, based on the fact that if the product of two expressions (or numbers) equals zero, one or both of the expressions or numbers must be zero. You will use factorisation to write equations in the form $pq = 0$ so that you can solve them.


15
10
5
−10
−5
0
5
10
15
−5
−10

3 Equations

3.1 Introduction

SOLUTION BY INSPECTION

1. Complete the following table. Substitute the given x-values into the equation until you find the value that makes the equation true.

You can read the solutions of an equation from a table.

	Equation	LHS if $x = 4$	Is LHS = RHS ?	LHS if $x = 5$	Is LHS = RHS ?	LHS if $x = 6$	Is LHS = RHS ?	Correct solution
(a)	$3x - 4 = 11$							$x =$
(b)	$2x + 7 = 19$							$x =$
(c)	$13 - 5x = -7$							$x =$

(LHS = Left-hand side and RHS = Right-hand side)

2. In the following table, you are given equations with their solutions. Insert + or – or = signs between each term to make the equations true for the solution given.

The "searching" for the solution of an equation is referred to as solving the equation by **inspection**.

	Equation	Solution
(a)	$2x \quad 7 = 15$	$x = 4$
(b)	$3 \quad 2x = 11$	$x = -4$
(c)	$-x \quad 7 = 3$	$x = 4$
(d)	$28 \quad 5x = 3$	$x = 5$

Statements like $21 - x = 2x + 3$ and $(x - 3)(x - 5) = 0$, which are true for only some values of x are called **equations**.

A statement like $2(x + 3) = 2x + 6$, which is true for all values of x you can think of, is called an **identity**.

A statement like $2(x + 3) = 2x + 3$, where there are no values of x for which it is true, is called an **impossibility**.

SOLVING EQUATIONS THROUGH INVERSE OPERATIONS

In this section you are going to explore a different way of solving equations.

1. Complete the following calculations.
   (a) $3 - 3$ (b) $-9\ 765 + 9\ 765$
   ....
   (c) $-a + a$ (d) $13a - 13a$
   ....

2. What do you notice?
   ....

3. Complete the following calculations.
   (a) $3 \div 3$ (b) $3 \times \frac{1}{3}$
   ....
   (c) $\frac{1}{x} \times x$ (d) $\frac{x}{3} \times \frac{3}{x}$
   ....

4. What do you notice?
   ....

We can start with a solution as an equation and then apply some operations to it to turn it into an equivalent but more complicated equation.

Two equations are **equivalent** if they have the same solution.

Building an equation

Action on both sides	Equivalent equations
Solution (1)	$x = 3$
$\times 3$	$3x = 9$
$+ 2$	$3x + 2 = 11$
Solution (2)	$x = -9$
$\times 2$	$2x = -18$
$+ 6$	$2x + 6 = -12$
$+ x$	$3x + 6 = x - 12$
factorise	$3(x + 2) = x - 12$

Solving an equation

Action on both sides	Equivalent equations
Equation (1)	$3x + 2 = 11$
$- 2$	$3x = 9$
$\div 3$	$x = 3$
Equation (2)	$3(x + 2) = x - 12$
remove brackets	$3x + 6 = x - 12$
$- x$	$2x + 6 = -12$
$- 6$	$2x = -18$
$\div 2$	$x = -9$

Building an equation		Solving an equation	
Action on both sides	Equivalent equations	Action on both sides	Equivalent equations
Solution (3)	$x = 1$	Equation (3)	$\frac{(x+3)}{2} = 1 + x$
$\times -1$	$-x = -1$	$\times 2$	$x + 3 = 2 + 2x$
$+ 3$	$-x + 3 = 2$	$- 2x$	$-x + 3 = 2$
$+ 2x$	$+x + 3 = 2 + 2x$	$- 3$	$-x = -1$
$\div 2$	$\frac{(x+3)}{2} = 1 + x$	$\div -1$	$x = 1$

Try making up your own equations and then solving them. Did you get the "solution" that you started with?

When you solve an equation, you actually reverse the making of the equation.

5. Solve for x:

(a) $2(x + 4) + 9 = 15$

....

....

....

....

(b) $5(x - 2) = 7(2 - x)$

....

....

....

....

(c) $\frac{2x}{3} - 2 = 12$

....

....

....

....

(d) $\frac{3y-3}{2} + \frac{5}{2} = \frac{5y}{3}$

....

....

....

....

Up to now you have only dealt with equations of the **first degree**. That means they contained only *first powers* of the unknown (x), for example $3x - 2 = 5x + 7$. In the following sections you will solve equations of the **second degree**, where the expressions contain second powers. This is an equation of the second degree:

$x^2 + 1 = x + 13$.

When the expression part of the equation is written as the product of a monomial and a binomial, e.g. $x(x - 2) = 0$; or the product of two binomials, e.g. $(x - 2)(x + 3) = 0$ the result is also an equation of the second degree.

3.2 Solving by factorisation (Part 1)

DEVELOPING A STRATEGY: MULTIPLYING BY ZERO

1. Can you find two numbers x and y so that if you multiply them the answer is 0, i.e. $xy = 0$?

....

....

....

Each part of a product is called a **factor** of the expression.
If $c = ab$, then a and b are factors of c.
If $x^2 + 5x + 6 = (x + 2)(x + 3)$, then $x + 2$ and $x + 3$ are factors of $x^2 + 5x + 6$.

2. Complete the following table:

	Equation	Factors	Product	First possible solution	Second possible solution
Example	$x(x - 2) = 0$	x and $(x - 2)$	0	$x = 0$	$x - 2 = 0$ $x = 2$
(a)	$x(x + 5) = 0$	....	....	....	....
(b)	$2x(3x - 12) = 0$	....	....	....	....
(c)	$0 = (x + 2)(x - 2)$	....	....	....	....

You can rewrite an equation so that it is in the form *expression* = 0; for example you can write $x^2 - 2x = 3x + 6$ as $x^2 - 5x - 6 = 0$.

You can factorise $x^2 - 5x - 6$ and then use the zero-product property to solve the equation.

$x^2 - 5x - 6 = 0$

$(x - 6)(x + 1) = 0$

$x = 6$ or $x = -1$

Zero-Product Property

If: $a \times b = 0$

Then: $a = 0$ or

$b = 0$ or

$a = 0$ and $b = 0$

In a later section you will solve equations like the above example. You have to write the equation in the form, *expression* = 0, then factorise the left-hand side and then use the zero-product property.

TAKING OUT THE HIGHEST COMMON FACTOR

The process of writing a sum expression (polynomial) as a product (monomial) is called **factorisation**. This is the inverse of **expansion**.

Look at the expression $2x^2 - 6x$.

$2x$ is a factor of both terms, therefore it is a factor of $2x^2 - 6x$.

By division we get $\frac{2x^2 - 6x}{2x} = x - 3$.

Hence $2x^2 - 6x = 2x(x - 3)$.

It is unnecessary to write out the division step of this method. After finding the common factor, we write down the product form directly.
$2x^2 - 6x = 2x(\quad\quad)$

Determine the values of x which will make the following statements true:

1. $x^2 = -3x$

....
....
....
....
....
....
....

2. $x^2 + 2x^2 = 6x$

....
....
....
....
....
....
....

3. $\frac{6x}{3} + x = -4x^2$

....
....
....
....
....
....
....
....

4. $x = x(2 - x)$

....
....
....
....
....
....
....
....

3.3 Solving by factorisation (Part 2)

SOLVING BY FACTORISING TRINOMIALS

The product of the first terms of the factors must be equal to the x^2 term of the trinomial. The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial.

$x^2 + 5x + 6 \quad = \quad (x + 2)(x + 3)$ Meaning: $x.x = x^2$

Meaning: $2.3 = 6$

The sum of the inner and outer products must be equal to the x term of the trinomial.

$x^2 + 5x + 6 \quad = \quad (x + 2)(x + 3)$ Meaning: $(2 + 3)x = 5x$

The factors are of the form: $(x.x) + (a + b)x + (a.b) = (x + a)(x + b)$.

Determine the values of x which will make the following statements true.
Remember to write the equation in the form *expression* = 0 so that you can use the zero-product property.

1. $x^2 + 9x = -14$

....

....

....

....

2. $x^2 + 3x = 18$

....

....

....

....

3. $x^2 - 18x = -17$

....

....

....

....

4. $x^2 + 30 = 11x$

....

....

....

....

5. $x^2 = 13x + 30$

....

....

....

....

6. $x^2 + 7x = 30$

....

....

....

....

Maths2_Gr9_LB_Book.indb 38 2014/09/08 09:06:27 AM

SOLVING BY FACTORISING THE DIFFERENCE BETWEEN TWO SQUARES

Remember from the previous chapter:

If p and q are perfect squares, also "algebraic squares", then:

$$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$$

$$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$$
$$= (3x^2 + 2y)(3x^2 - 2y)$$

An expression of the form $a^2 - b^2$ is called the **difference between two squares**.

To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

Determine the values of the unknown (x or a or n, etc.) which will make the following statements true.

Remember to write the equation in the form *expression* = 0 so that you can use the zero-product property.

1. $x^2 = 4$
2. $x^2 = 16$
3. $4a^2 = 9$
4. $81 = 9n^2$
5. $25x^2 = 36$
6. $121x^2 = 144$
7. $16p^2 = 49$
8. $64a^2 = 25$

Maths2_Gr9_LB_Book.indb 39 2014/09/08 09:06:28 AM

3.4 Solving by factorisation (Part 3)

SOLVING BY USING PROPERTIES OF EXPONENTS

1. Write the following numbers as the product of their prime factors.

(a) 128 (b) 243

…………………… ……………………

(c) 125 (d) 2 401

…………………… ……………………

All numbers can be written as the product of their prime factors:
16 $= 4 \times 4 = 2 \times 2 \times 2 \times 2 = \mathbf{2^4}$
Factorise the number until all the factors are prime numbers.

2. Determine the values of x which will make the following statements true.

(a) $2^x = 2^7$ (b) $3^x = 3^5$

…………………… ……………………

(c) $5^x = 5^3$ (d) $7^x = 7^4$

…………………… ……………………

If the base of the LHS is the same as the base of the RHS, then the exponent on the LHS must be equal to the exponent on the RHS.
If $a^x = a^y$, then $x = y$.

3. Determine the values of x which will make the following statements true.

(a) $2^x = 128$ (b) $3^x = 243$

……………………………… ………………………………

……………………………… ………………………………

(c) $5^x = 125$ (d) $7^x = 2\ 401$

……………………………… ………………………………

……………………………… ………………………………

(e) $2^x + 9 = 25$ (f) $27(3^x) = 3$

……………………………… ………………………………

……………………………… ………………………………

……………………………… ………………………………

In the equation $2^x = 16$, the letter symbol (x) is the exponent. Equations with the letter symbol as an exponent are referred to as **exponential equations**.

MIXED EXERCISES FOR MORE PRACTICE

Determine the values of the unknown (*x* or *m* or *b*, etc.) which will make the following statements true.

1. $\frac{6x}{3} + x = -4x^2$

....
....
....
....
....

2. $x = x(2 - x)$

....
....
....
....
....

3. $x^2 + 2x = 15$

....
....
....
....

4. $m^2 + 4m = 21$

....
....
....
....

5. $x^2 + 3 = 4x$

....
....
....
....

6. $b^2 - 16b = -15$

....
....
....
....

7. $1 = a^2$

....
....
....
....

8. $25x^2 = 49$

....
....
....
....

9. $2^x - 25 = -9$

....
....
....

10. $81(3^x) = 3$

....
....
....

3.5 Set up equations to solve problems

THE MATHEMATICAL MODELLING PROCESS

Consider this problem involving a practical situation.

Printing shop A charges 45c per page and R12 for binding a book.

Printing shop B charges 35c per page and R15 for binding a book.

For a book with how many pages will the two shops charge the same?

You can write an equation to describe the problem.

Let the number of pages for which the work costs the same be x. Then

$45x + 1\,200 = 35x + 1\,500$.

The equation represents a mathematical problem that can be solved without necessarily keeping the practical situation in mind. It is called a **mathematical model** of the practical situation.

Now solve the equation.

$$\begin{aligned} 45x + 1\,200 &= 35x + 1\,500 \\ 45x - 35x &= 1\,500 - 1\,200 \\ 10x &= 300 \\ x &= 30 \end{aligned}$$

We describe this as **analysing** the mathematical model, to produce a **mathematical solution**.

We may now ask what the solution to the mathematical problem ("$x = 30$") means in terms of the practical situation. When the equation was set up above, the symbol x was used as a placeholder for the number of pages in a book for which the two shops would charge the same. So – what does the solution tell you?

The mathematical solution may be **interpreted** to establish what it means in the practical situation.

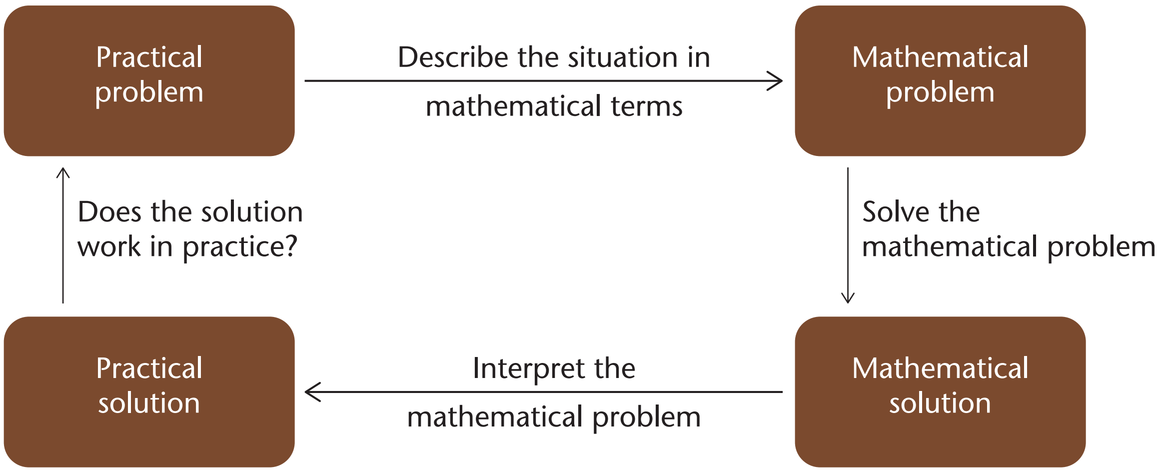
Now check whether the two shops will charge the same for a book with 30 pages.

At shop A 30 pages will cost 30×45c = 1350c = R13,50. Binding is R12, total cost is R25,50.

At shop B 30 pages will cost 30×35c = 1050c = R10,50. Binding is R15, total cost is R25,50.

The solution to the mathematical problem is also a solution to the practical problem.

The mathematical solution should be **tested in the practical situation**, because mistakes may have been made.



When people work like this, we say they do **mathematical modelling**.

PRACTICE YOUR MODELLING SKILLS

For each situation in questions 1 to 3, the mathematical model is outlined and some clues are provided. Fill in the missing information.

1. Louis is 6 years older than Karin and Karin is 4 years older than Heidi. The sum of their ages is 53 years. How old is Heidi?

Model: Let x be: Heidi's age
Then: Karin's age will be ....
And: ....
Hence: .... = 53

Analysis: $x + (x + 4) + (x + 10) = 53$

....

Interpretation: So Heidi is: ....

2. The sum of two numbers is 15. Three times the smaller number is 5 more than the larger number. Calculate the two numbers. (**Hint:** Let the smaller number be x.)

Model: Let x be: ....
Then: .... is the larger number
Hence: ....

Analysis: ....

Interpretation: So the smaller number is: ....
And the larger number is: ....

3. The sum of three consecutive even numbers is 108. What are the numbers?
Hint: Consecutive numbers are numbers that follow on each other.
We define an even number as a number of the form $2n$ where n is a counting number.

Model: Let the first number be: ....
Then: ....
....
Hence: ....

Analysis: ....
....

Interpretation: So the first number is: ....
And the second number is: ....
And the third number is: ....

4. Firm A calculates the cost of a job using the formula Cost = $500 + 30t$, where t is the number of days it takes to complete the job.
Firm B calculates the cost of the same job using the formula Cost = $260 + 48t$, where t is the number of days needed to complete the job.
(a) What would Firm A charge for a job that takes 10 days?
....
....
....
....
(b) How long would Firm B take to complete a job for which their charge is R596?
....
....
....
....
....
(c) Here is a specific job for which firms charge the same and take the same time to complete. How long does this job take?
....
....
....
....
....

3.6 Equations and ordered pairs

WHEN UNKNOWNS BECOME VARIABLES

In the previous sections we dealt with equations which had fixed or limited solutions. They only had one letter symbol, which in this case acted as a placeholder for the value/s which will make the statement true.

Study the equation: $y = 5x + 2$

1. How many letter symbols does the equation have? (List them.)

   ....

2. Is it possible to solve this "equation"?

   ....

3. Complete the table.

x	12	10	20	5	6	–5	–10
$5x + 2$							

FUNCTIONS AS SETS OF ORDERED PAIRS

A specific input number, for example 10, and the output number associated with it (52 in the case of the function described by $y = 5x + 2$) is called an *ordered pair*. Ordered pairs can be represented in a table like the one you completed in question 3 above.

Ordered pairs can also be written in brackets: (input number; output number).

For example the ordered pairs you entered into the table in 3 can be written as (12; 62), (10; 52), (20; 102), (5; 27), (6; 32), (–5; –23), (–10; –48)

In the function indicated by $y = 5x + 2$ the letter symbol in the formula (x) represents the **input** or **independent** variable while the other letter symbol (y) represents the **output** or **dependent** variable.

If there is precisely one value of y for each value of x, we say that **y is a function of x.**

1. Complete each table by writing the ordered pairs in brackets below the table, in the table as shown in the example. Then choose two more input numbers and write down two additional ordered pairs that belong to each given function.

   For the function with the rule $y = 4x + 5$

x	–2	0	1	2	5
y	–3	5	9	13	25

(–2; –3), (0; 5), (1; 9), (2; 13), (5; 25), and (10; 45) and (20; 85)

(a) For the function with the rule $y = x^2 + 9$

x	5		0	–3	
y		18			34

(5;34), (3; 18), (0; 9), (–3; 18), (–5; 34), and (. . ; . . .), and (. . ; . . .)

(b) For the function with the rule $y = 3x – 2$

x	5	1	0	–3	
y					–17

(5; 13), (1; 1), (0; –2), (–3; –11), (–5; –17), and (. . . ; . . .) and (. . ; . . .)

(c) For the function with the rule $y = 5x – 4$

x	–5	–3	1	2	
y					21

(–5; –29), (–3; –19), (1; 1), (2; 6), (5; 21), and (. . . ; . . .) and (. . ; . . .)

(d) For the function with the rule $y = 12 – 3x$

x	1	2	3	4	
y					–3

(1; 9), (2; 6), (3; 3), (4; 0), (5; –3), and (. . . ; . .) and (. . ; . . .)

(e) For the function with the rule $y = x^2 + 2$

x	–12	–7	–2	3	
y					102

(–12; 146), (–7; 51), (–2; 6), (3; 11), (10; 102), and (. . . ; . . .) and (. . ; . . .)

(f) For the function with the rule $y = 2^x + 2$

x	0	1	2	3	
y					18

(0; 3), (1; 4), (2; 6), (3; 10), (4; 18) and (. . . ; . . .) and (. . ; . . .)

3. (a) Which ordered pair belongs to both $y = 3x – 2$ and $y = 5x – 4$? . . . .

(b) Which ordered pair belongs to both $y = 12 – 3x$ and $y = 5x – 4$? . . . .

4. Which ordered pair belongs to both $y = 5x + 7$ and $y = 3x + 25$?

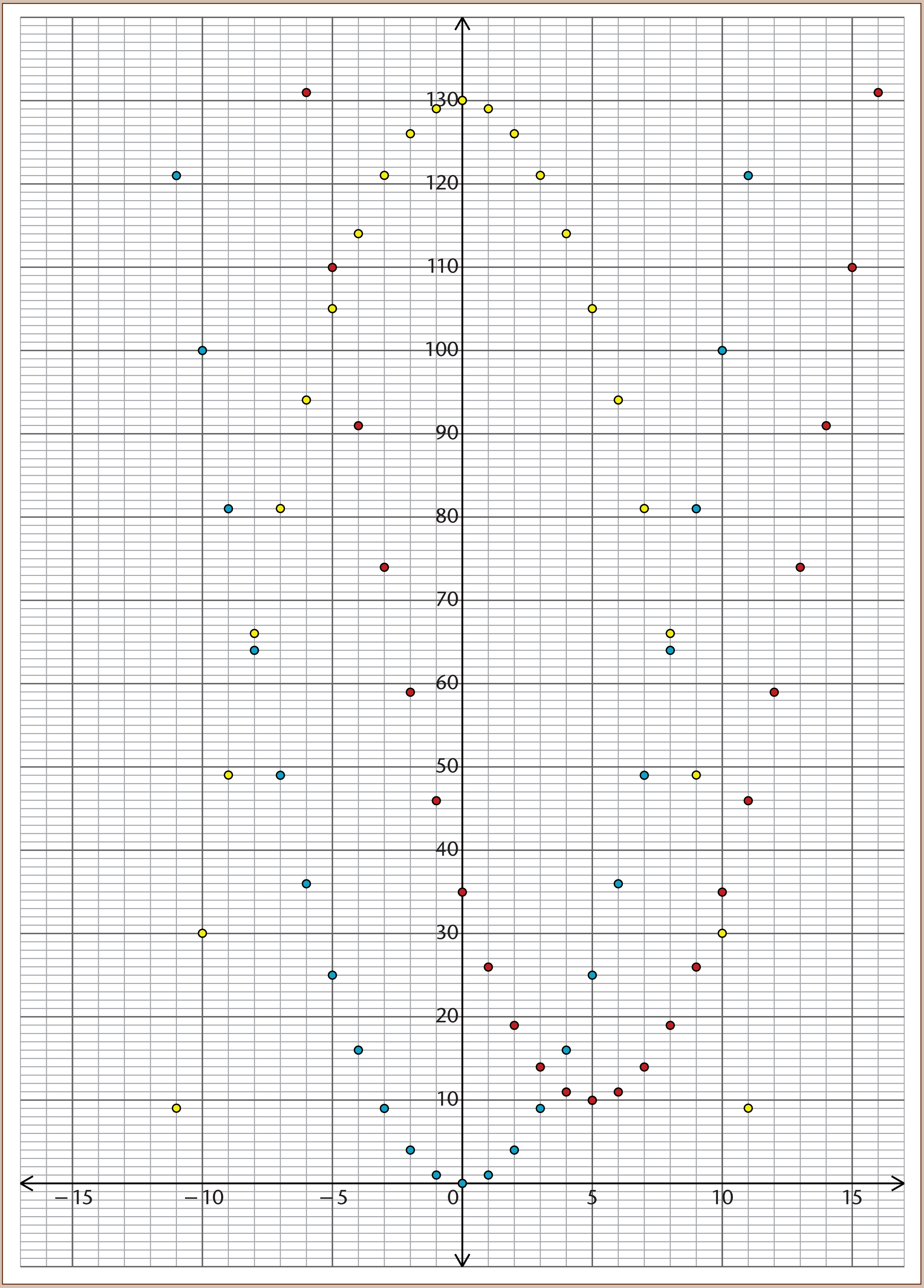
. . . .

. . . .

Maths2_Gr9_LB_Book.indb 46 2014/09/08 09:06:29 AM

Chapter 4
Graphs

In this chapter you will learn more about making graphs to show how quantities change, and about interpreting graphs. Graphs can show how quantities increase and decrease, how rapidly they increase and decrease, and where they have maximum and minimum values. You will pay special attention to graphs of quantities which change at a constant rates. These graphs are straight lines.


130
120
110
100
90
80
70
60
50
40
30
20
10
−15
−10
−5
0
5
10
15

4 Graphs

4.1 Global graphs

DISCRETE AND CONTINUOUS VARIABLES

Sibongile collects honey on his farm and puts it in large jars to sell. His business is doing so well that he can no longer do all the work himself. He needs to get some help. Sibongile knows that one person can normally fill 2 jars in 3 days. He sets up this table to help him determine how many full-time workers he should employ to fill different numbers of jars in a five-day week.

Jars per week	$3\frac{1}{3}$	$6\frac{2}{3}$	10	$13\frac{1}{3}$	$16\frac{2}{3}$	20	$23\frac{1}{3}$
Workers	1	2	3	4	5	6	7

1. (a) If Sibongile needs to produce 40 jars a week, how many workers does he need?

   ....

   (b) How many jars can 9 workers fill in a week? ....

   (c) How many workers does Sibongile need to produce 15 jars per week?

   ....

   (d) What are the two variables in the above situation?

   ....

In a situation like the above, one can have any number of jars, as well as fractions of a jar. One can have a whole number of jars (for example 4 jars) or a fractional quantity of jars (for example $6\frac{2}{3}$ or 4,45 jars). The other variable in the above situation, the number of full-time employees, is different. Only whole numbers of people are possible.

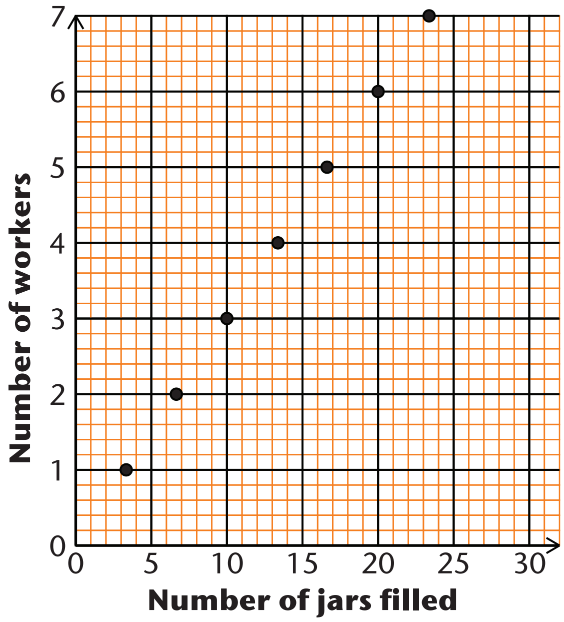
Quantities like the quantity of jars of honey, which can include any fraction, are sometimes called "continuous quantities" or "continuous variables". Quantities that can be counted, like a number of people or a number of motor cars or rivers or towns, are sometimes called "discrete quantities" or "discrete variables".

When a graph of a discrete variable is drawn, it does not normally make sense to join the dots with a line, but for some purposes it may be useful.

2. Can you use the second graph on the next page to find out how many workers are needed to fill 30 jars in a week, and how many to fill 40 jars? Check your answers by doing calculations.

   ....

Here is a graph of the information in Sibongile's table.



Here is another graph of the same information.

3. In what way are these two graphs different?

....

....

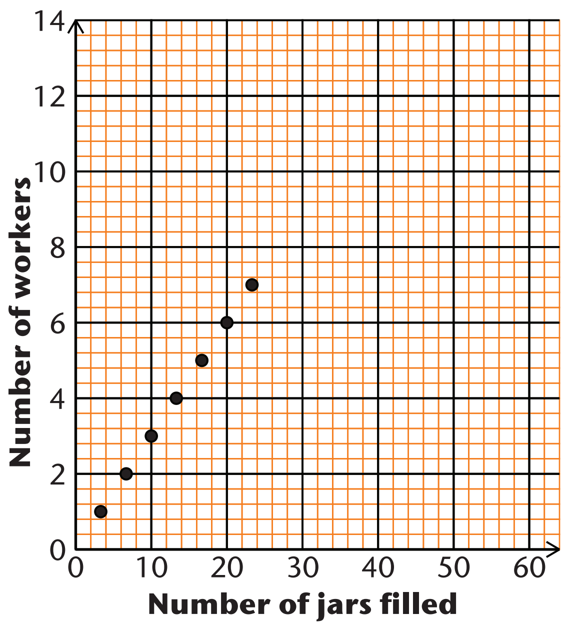
....

....

....

....

....



4. In each case say whether the variables are "discrete" or "continuous".
   (a) You order pizzas for a class party and you need 1 pizza for every 3 learners.

   ....

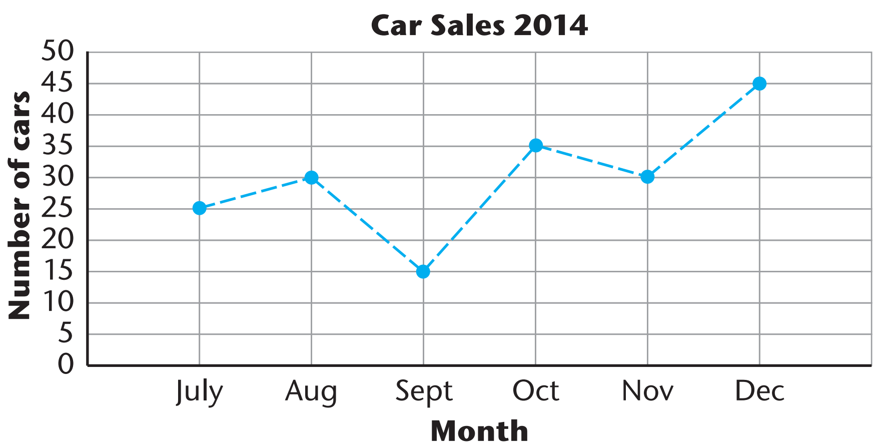
   (b) Your height measured at different stages as you grew up.

   ....

   (c) The speed the car is travelling as you drive to town.

   ....

5. The line graph shows the number of cars that a company sold between July and December of 2014.



   (a) Is the data shown in the graph discrete or continuous? Explain your answer

   ....

   (b) How many cars were sold in August? ....

   (c) During which months were the maximum and minimum number of cars sold?

   ....

   (d) How many more cars were sold in November than in July? ....

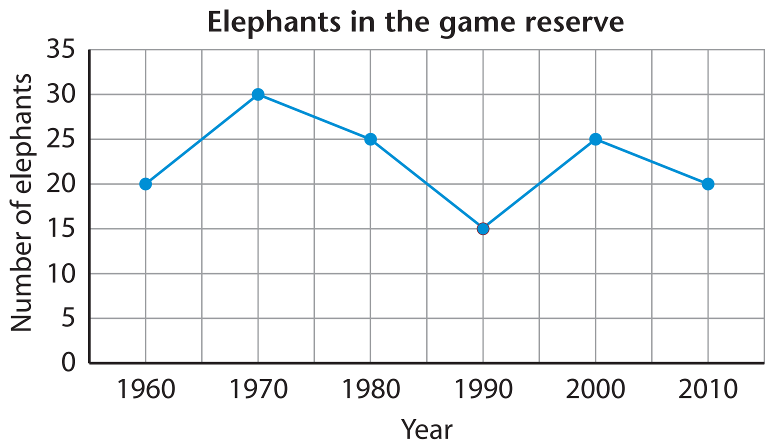
   (e) During which months did the car sales decrease?

   ....

   (f) Would you say that the car sales generally improved over the 6 months? Explain your answer.

   ....

6. The graph below shows the population of elephants at a game reserve in South Africa between 1960 and 2010. Study the graph and answer the questions that follow.



(a) Did the elephant population increase or decrease between 1970 and 1990?

....

(b) Between which years did the elephant population increase?

....

(c) In which year were there the most elephants on the game farm?

....

(d) Is the data in this graph discrete or continuous?

....

(e) How many elephants do you think there were on the game reserve in 1995?

....

(f) The following data shows the number of elephants at a different game reserve. Plot this information on the grid above.

Year	1960	1970	1980	1990	2000	2010
Elephants	30	25	20	15	20	35

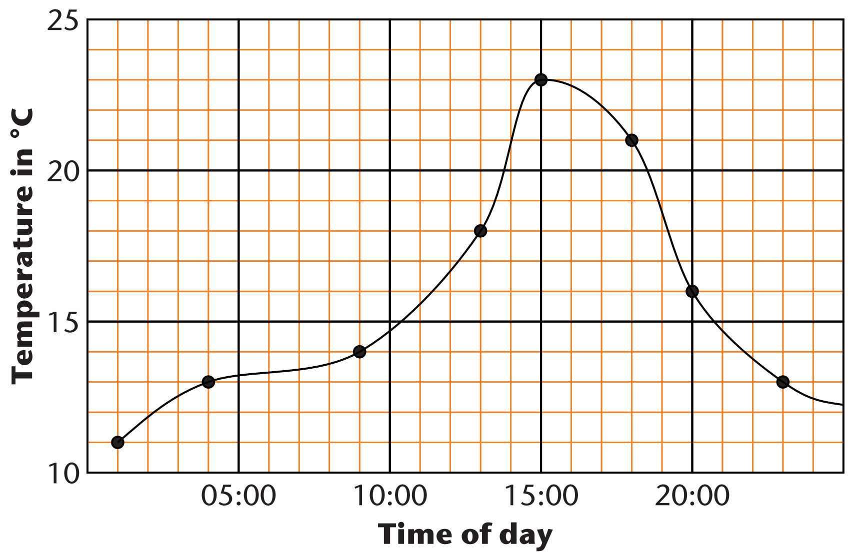
(g) Would you say that the second game reserve had more elephants than the first game reserve between 1960 and 2010? Explain your answer.

....

....

SHOWING INCREASE AND DECREASE ON GRAPHS

The graph below shows the temperature over a 24-hour period in a town in the Free State. The graph was drawn by connecting the points that show actual temperature readings.



1. (a) Do you think the above temperatures were recorded on a summer day or a winter day?

....

....

(b) At what time of the day was the highest temperature recorded, and what was this temperature?

....

(c) During what part of the day did the temperature rise, and during what part did the temperature drop?

....

....

(d) During what part of the period when the temperature was rising did it rise most rapidly?

....

(e) During what part of the day did the temperature drop most rapidly?

....

....

....

2. Here are descriptions of the temperature changes on five different days.

Day A: It is already warm early in the morning. The temperature does not change much during the day but late in the afternoon a breeze causes the temperature to drop quite sharply.

Day B: It is very cold early in the morning but it gets quite hot soon after the sun gets up. By midday a cold wind comes up and the temperature drops till late in the afternoon. The wind then stops and it gets warmer again into the evening.

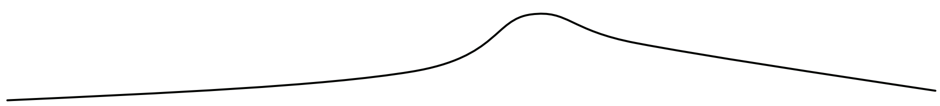
Day C: It is warm in the early morning and the temperature remains about the same till midday. then the temperature drops slowly during the afternoon.

Day D: It is cold in the early morning and it remains cold for the whole day, except for a short time after lunch when the sun comes out for a while.

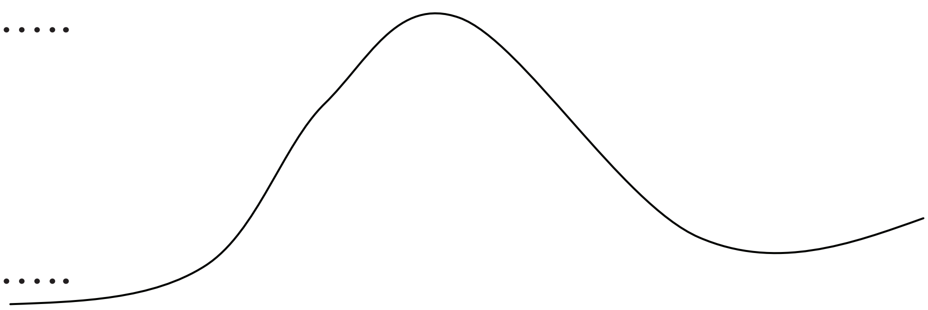
Day E: It is warm early in the morning, but the temperature drops sharply soon after sunrise and remains low until mid-afternoon, when it slowly warms up a little.

The shapes of some temperature graphs for 24-hour periods, starting early in the morning, are given below. Below each graph, write which of the above days is possibly represented by the graph.

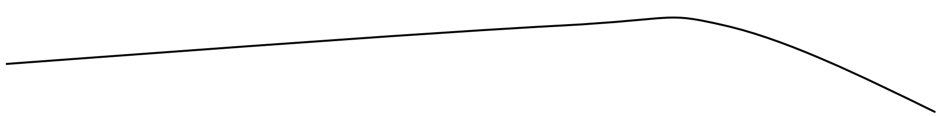
....



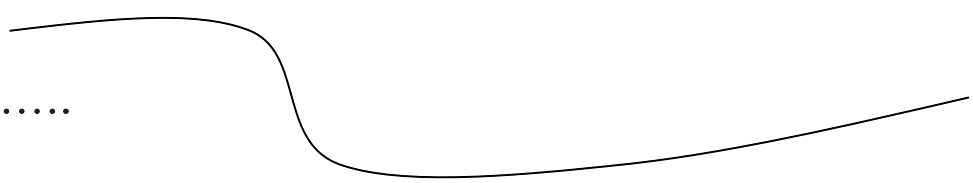
....



....

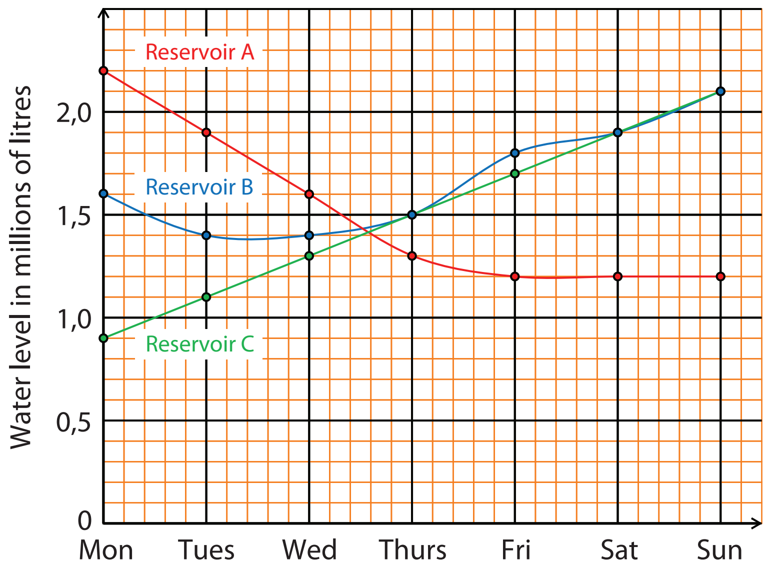


....



....

Water is supplied to a township from three reservoirs. The amount of water in each reservoir is measured each day at 08:00 am. The water level in reservoir A is represented in red on the graph below, and the water levels in reservoirs B and C are represented in blue and green respectively.



The daily water levels in the three reservoirs, in millions of litres, are also given in the table below.

	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
Reservoir A	2,2	1,9	1,6	1,3	1,2	1,2	1,2
Reservoir B	1,6	1,4	1,4	1,5	1,8	1,9	2,1
Reservoir C	0,9	1,1	1,3	1,5	1,7	1,9	2,1

3. You may use the graph or the table, or both, to find the answers to the questions below.
   (a) On which days does the water level in reservoir B increase from one day to the next?

   ....

   (b) On which of these days does the water level in reservoir B increase most, and by how much does it increase from that day to the next?

   ....

   (c) By how much does the water level in reservoir B change each day?

   ....

   (d) By how much does the water level in reservoir C change each day?

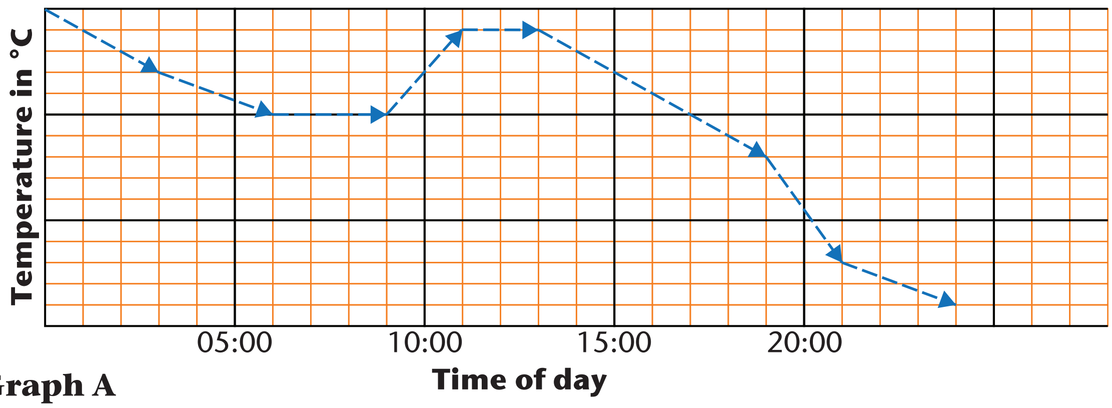
   ....

   (e) Describe the water level situation from Friday to Sunday, in reservoir A.

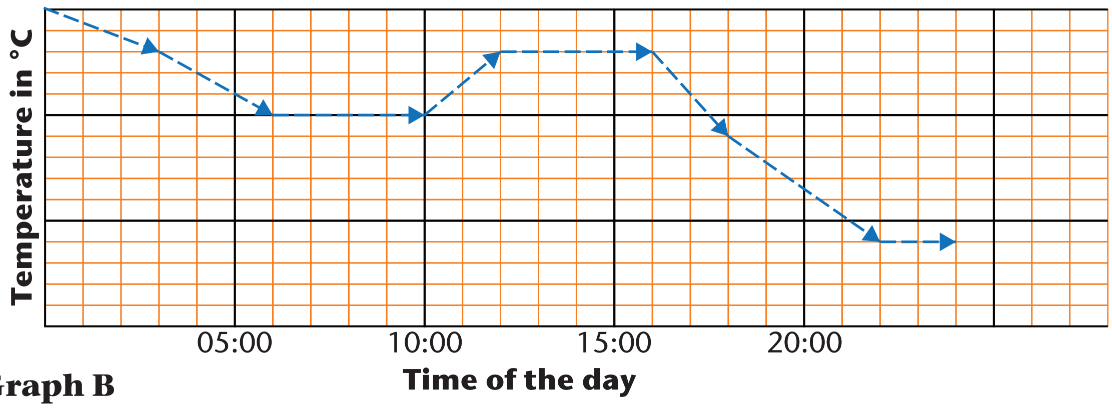
   ....

4. During a certain day, these changes occur in the temperature at a certain place.
Between 00:00 and 03:00, the temperature drops by 2°C.
Between 03:00 and 06:00, the temperature drops by 3°C.
Between 06:00 and 10:00, the temperature remains constant.
Between 10:00 and 12:00, the temperature rises by 3°C.
Between 12:00 and 16:00, the temperature remains constant.
Between 16:00 and 18:00, the temperature drops by 4°C.
Between 18:00 and 22:00, the temperature drops by 5°C.
Between 22:00 and 24:00, the temperature remains constant.

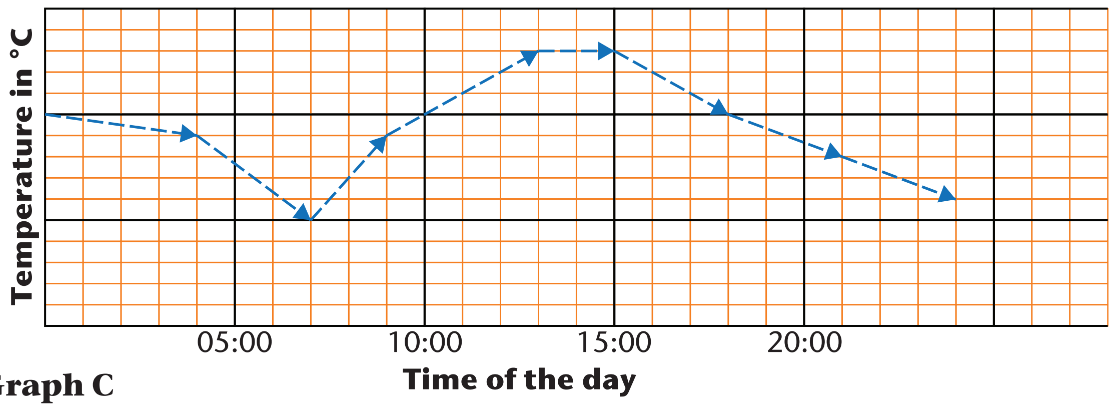
Which of the graphs below show the above temperature changes? ....



Graph A



Graph B



Graph C

5. Write a verbal description, like in question 4, of the temperature changes shown in graph A in question 4.

Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....

6. Write a verbal description, like in question 4, of the temperature changes shown in graph C in question 4.

Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....
Between .... and ...., the temperature ....

7. Look at graph A in question 4.
   (a) By how much does the temperature drop from 13:00 to 19:00? ....
   (b) By how much does the temperature drop from 19:00 to 21:00? ....
   (c) When does the temperature drop most rapidly, from 13:00 to 19:00 or from 19:00 to 21:00? Explain your answer.

....

....

....

8. Look at graph C in question 4.
   (a) By how much does the temperature increase from 07:00 to 09:00? ....
   (b) By how much does the temperature increase from 09:00 to 13:00? ....
   (c) When does the temperature increase more rapidly, from 07:00 to 09:00 or from 09:00 to 13:00? Explain your answer.

....

....

....

9. Look at graph B in question 4.
   (a) By how much does the temperature drop from 16:00 to 18:00? ....
   (b) By how much does the temperature drop from 18:00 to 22:00? ....
   (c) When does the temperature drop more rapidly, from 16:00 to 18:00 or from 18:00 to 22:00? Explain your answer.

....

....

4.2 Change at different rates

1 kilolitre = 1 000 litre

The water levels in kilolitres (kl) in different water storage tanks over a period of 30 hours are represented on the graphs below and on the next page.

1. (a) In which tanks does the water level rise during the 30-hour period? ....
   (b) In which tanks does the water level drop during the 30-hour period? ....
2. How much water is there at the start of the 30-hour period, in each of the tanks?

....

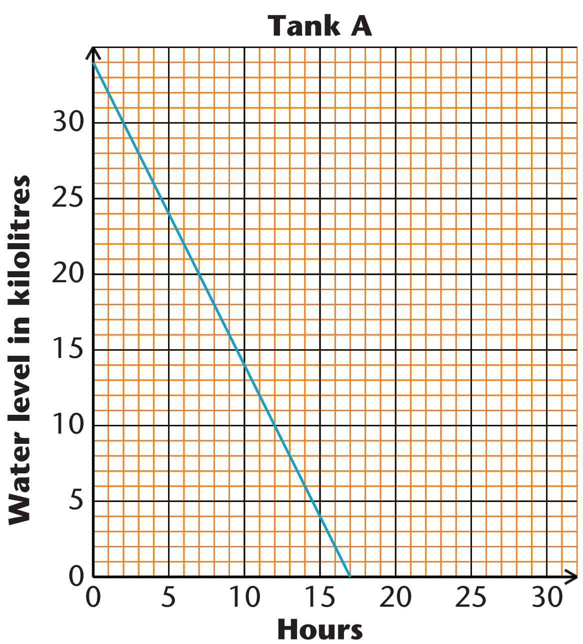
....

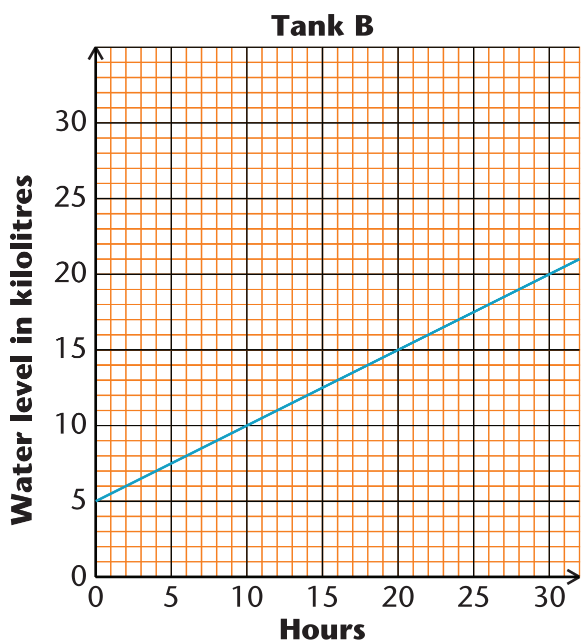
3. (a) Which tank is losing water most rapidly? Explain your answer.

....

   (b) Which tank is gaining water most slowly? Explain your answer.

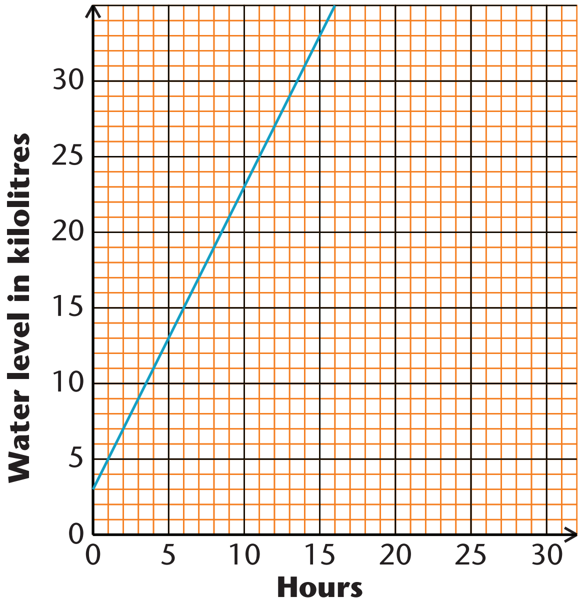
....



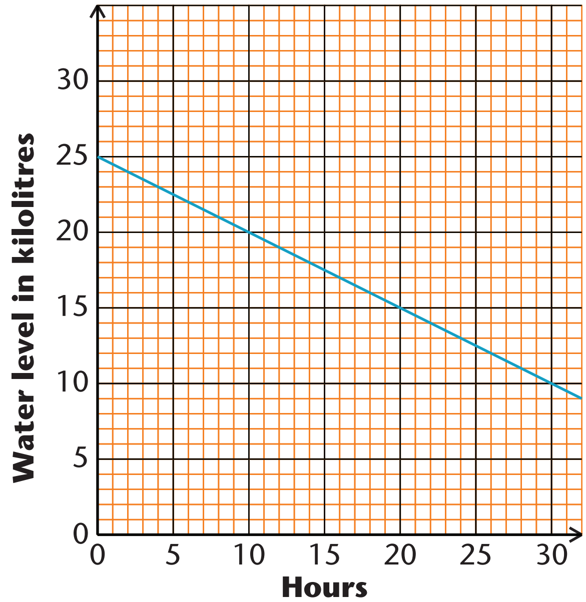


Maths2_Gr9_LB_Book.indb 58 2014/09/08 09:06:33 AM

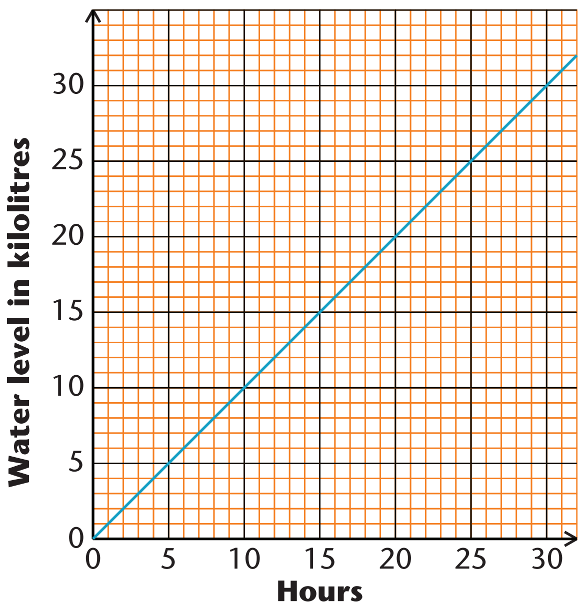
Tank C



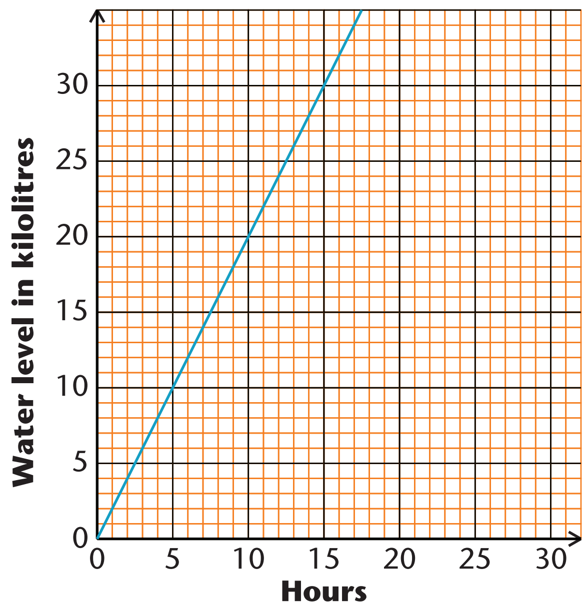
Tank D



Tank E



Tank F



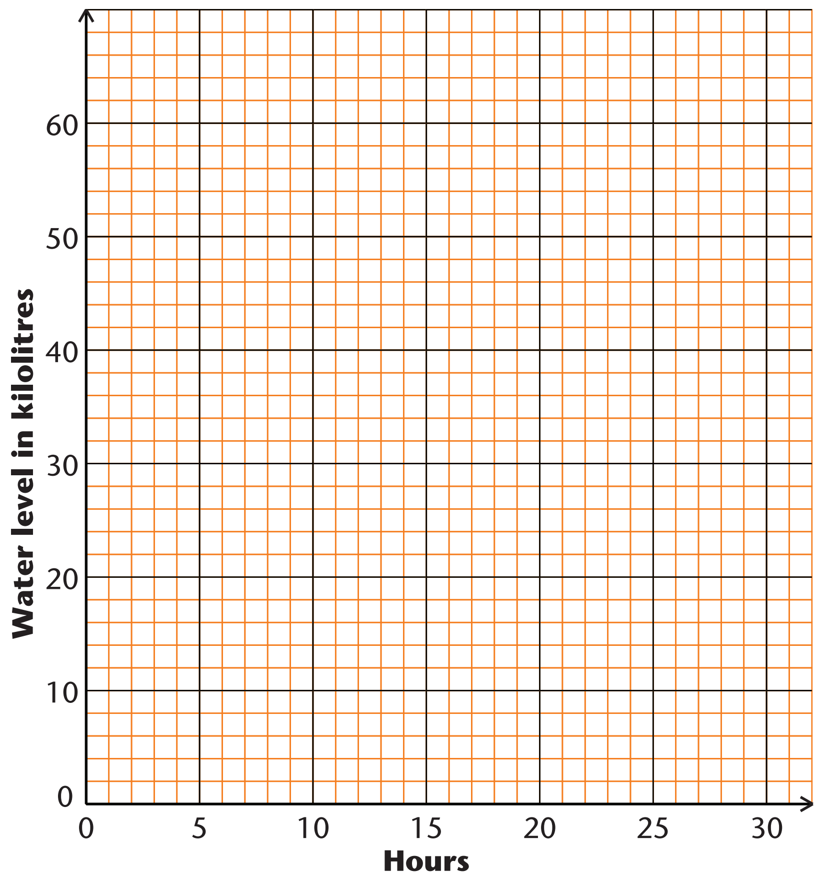
3. Complete the table. Use negative numbers for decreases.

	Change over each hour	Change over any period of 5 hours
Tank A		
Tank B		
Tank C		
Tank D		
Tank E		
Tank F		

If a constant stream of water is pumped into a tank so that the water level is increased by 3 kilolitre in each hour, we say:

Water is pumped into the tank at a **constant rate** of **3 kilolitres per hour**.

4. (a) Tank G contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into it at a constant rate of 3 kilolitres per hour. Draw a dotted line graph to show the water level in Tank G on the graph sheet below.
   (b) Tank H also contains 12 kilolitres at the beginning of a 30-hour period. Water is then pumped into at a constant rate of 1,5 kilolitres per hour. Draw a solid line graph to show the water level in Tank H on the graph sheet below.



5. Complete the table for tanks G and H over the 30-hour period.

Hours	0	5	10	15	20	25	30
Kilolitres in tank G	12						
Kilolitres in tank H	12						

4.3 Draw graphs from tables of ordered pairs

A "coordinate" graph shows the relationship between two variables, the dependent and independent variable in a function. The value of the dependent variable depends on the value given to the independent variable, hence its name. Sometimes there is no pattern to the relationship between the two variables and sometimes there is. In Grade 9 we will focus on graphs where there is a pattern to the relationship. Specifically, we will focus on graphs of linear functions. The graph of a linear function is a straight line.

GRAPHS OF FUNCTIONS WITH CONSTANT DIFFERENCES

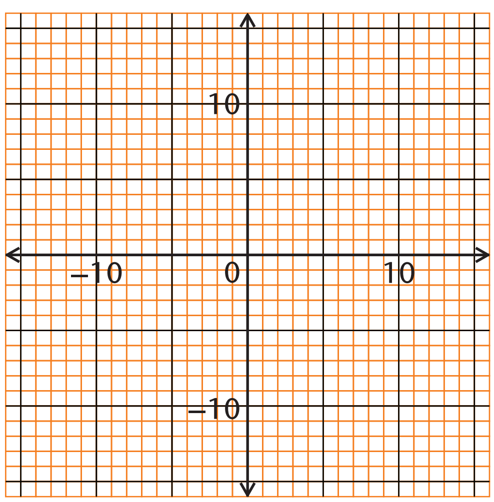
1. Complete the table.

x	0	1	2	3	4	5	6	7	8	9
Function A	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$						
Function B	4	5	6	7	8	9	10	11	12	13
Function C	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$						
Function D	–4	–2	0	2						

2. Represent each of the functions in question 1 with a graph by plotting the points on the grids below. You may join the points in each case and write down the constant difference between the function values.

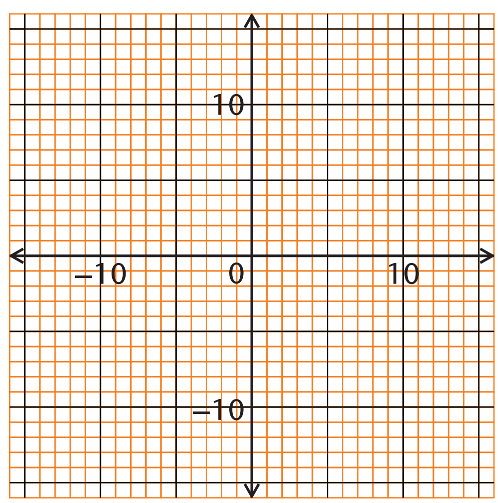
Function A

Constant difference = ...



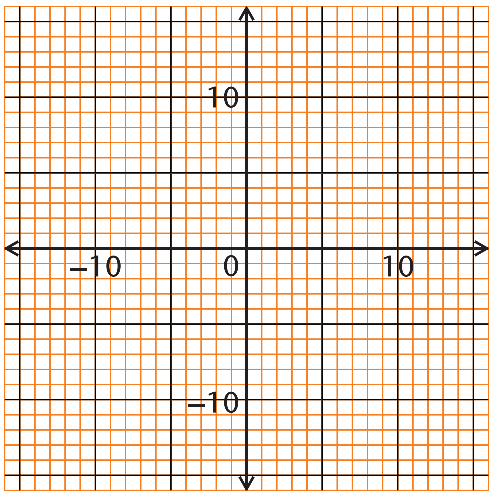
Function B

Constant difference = ...



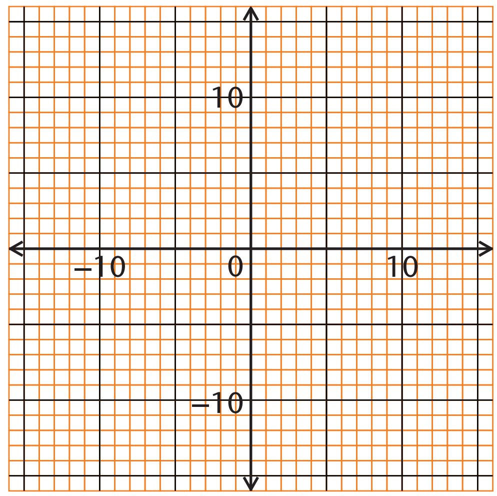
Function C

Constant difference = ...



Function D

Constant difference = ...



3. Some of the graphs you have drawn "go upwards" (or downwards) quickly, like a steep hill or mountain; others "go up" (or down) slowly.
   (a) Is there a link between the constant difference and the "steepness" of the graph?

   ....

   (b) Try to explain why this is the case.

   ....

   ....

4. (a) Complete the following tables.

x	1	2	3	4	5	6	7	8	9	10
$2x + 3$										
$5x + 4$										
$3x + 3$										

   (b) Determine the difference between consecutive terms in each of the above three number sequences. What do you notice about this difference?

   ....

   ....

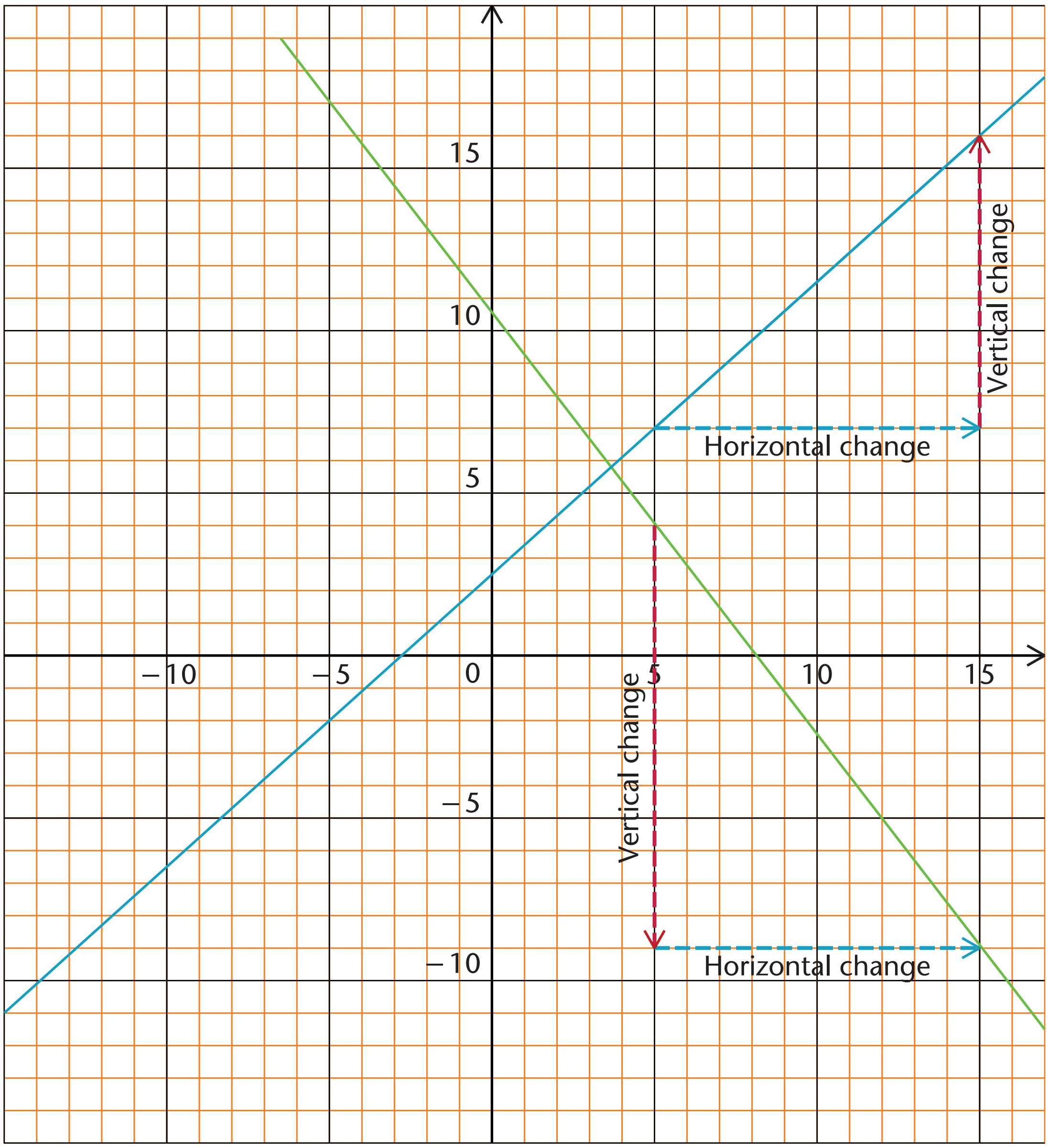
   (c) What difference between consecutive terms would you expect in the output numbers for $4x + 5$, if the input numbers are the natural numbers 1; 2; 3; .....?

   ....

4.4 Gradient

The "steepness" or **slope** of a line can be indicated by a number, as described below. This number is called the **gradient** of the line.

> The gradient is the vertical change divided by the horizontal change as you move from left to right on the line.
>
> $$\text{Gradient} = \frac{\text{vertical change}}{\text{horizontal change}}$$



The gradient of the blue line above is $\frac{9}{10} = 0{,}9$.

The gradient of the green line is $\frac{-13}{10} = -1{,}3$.

Note that the horizontal change is always taken to be positive (moving to the right), but the vertical change can be positive (if it is upwards) or negative (if it is downwards).

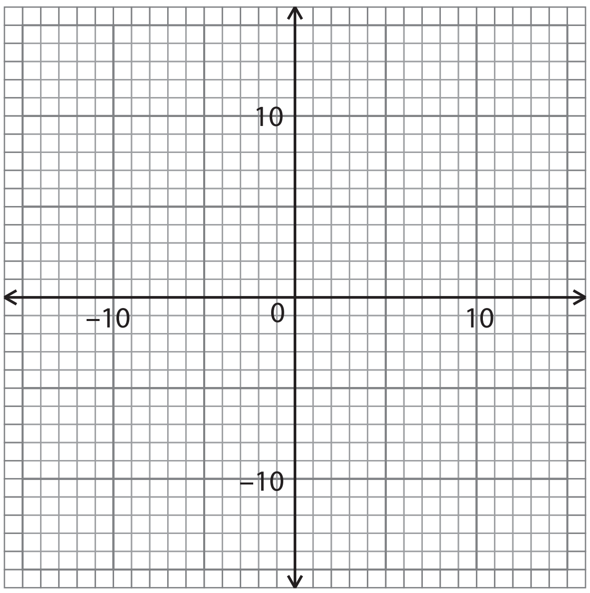
1. A certain line passes through the points (2; 3) and (8; 15). A straight line is drawn through the two points.
   (a) Try to think of a way in which you can work out the gradient of the line that passes through the two points.

   ....

   ....

   ....

   ....



   (b) Plot the two points on the graph sheet on the right.
   (c) What horizontal change and vertical change is needed to move from the point (2; 3) to the point (8; 15)? You may draw arrows on your graph to help you to think clearly about this.

   ....

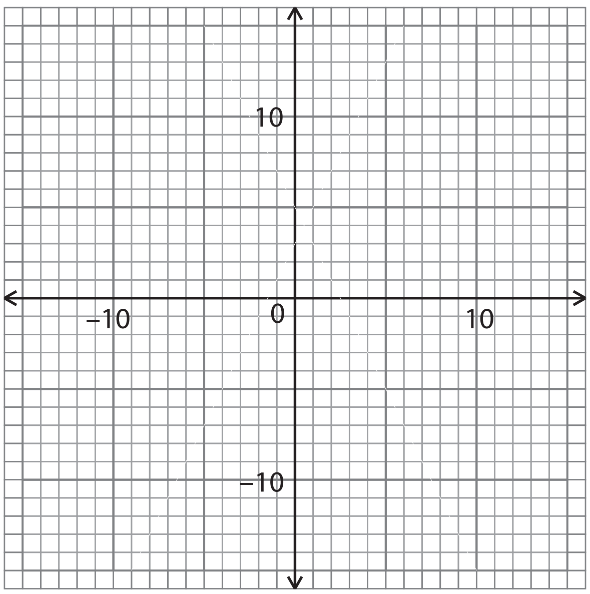
   ....

   (d) Work out the gradient of the line that passes through the two points.

   ....

2. Complete the table and plot graphs of $y = 2x + 3$ and $y = -2x + 5$ on the given graph sheet.

x	−3	1	3	5
$2x + 3$				
$-2x + 5$				



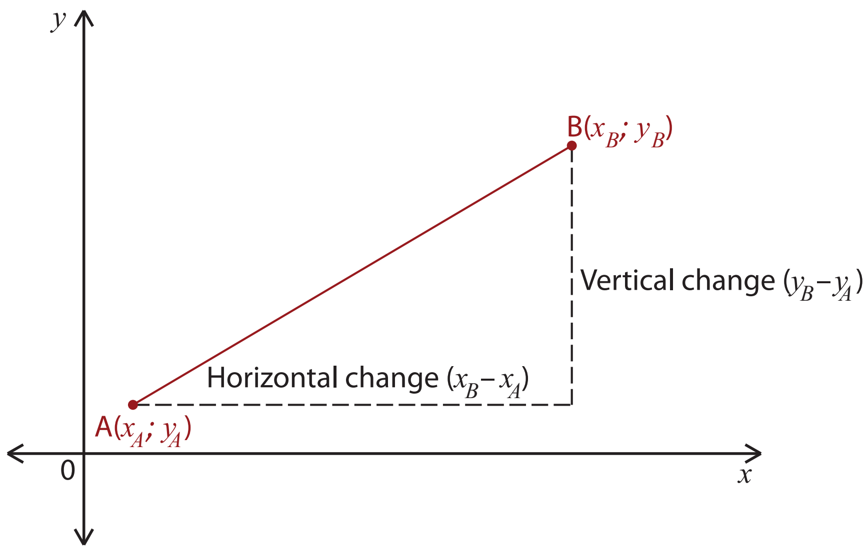
Maths2_Gr9_LB_Book.indb 64 2014/09/08 09:06:38 AM

3. Work out the gradients of the graphs of $y = 2x + 3$ and $y = -2x + 5$. You may use the coordinates of any of the points you have plotted.

....

....

Suppose the coordinates of point A are $(x_A; y_A)$ and the coordinates of B are $(x_B; y_B)$.



The gradient of line AB is: $m_{AB} = \frac{\textit{Vertical change}}{\textit{Horizontal change}} = \frac{y_A - y_B}{x_A - x_B}$

In summary:
If you have two points A $(x_A; y_A)$ and B $(x_B; y_B)$ then the formula for the gradient is: $m = \frac{y_A - y_B}{x_A - x_B}$

Examples of finding the gradient between two points:

Calculate the gradient of the line that goes through the points:

(a) A(2; 5) and B(4; 1)

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{5-1}{2-4} = \frac{4}{-2} = -2$$

(b) C(2; 2) and D(−6; 0)

$$m = \frac{y_C - y_D}{x_C - x_D} = \frac{2-0}{2-(-6)} = \frac{2}{8} = \frac{1}{4}$$

(c) A(0; −1) and B(1; 1)

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{-1-1}{0-1} = \frac{-2}{-1} = 2$$

The gradient of a straight line is the same everywhere, so it doesn't matter which 2 points you use to determine the gradient.

DETERMINE THE GRADIENT

Do the following task in your exercise book.

1. Determine the gradient of the lines that go through the following points:

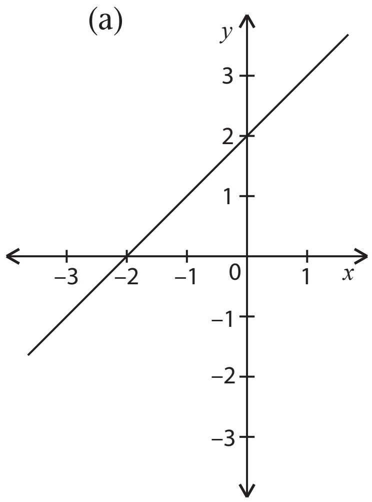
   (a) A(2; 10) and B(6; 12) (b) C(1; 3) and D(–2; –3) (c) E(0; 3) and F(4; –1)
   (d) G(5; 2), H(4; 4) and I(2; 8)

   ....

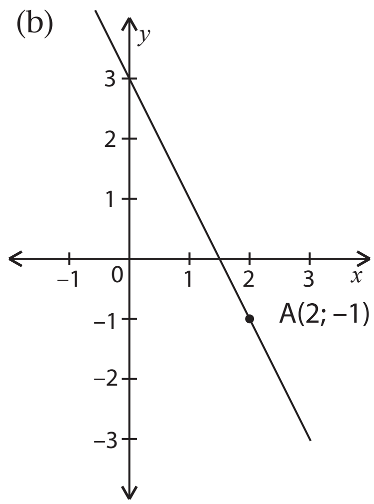
   ....

2. Determine the gradient of the following lines:

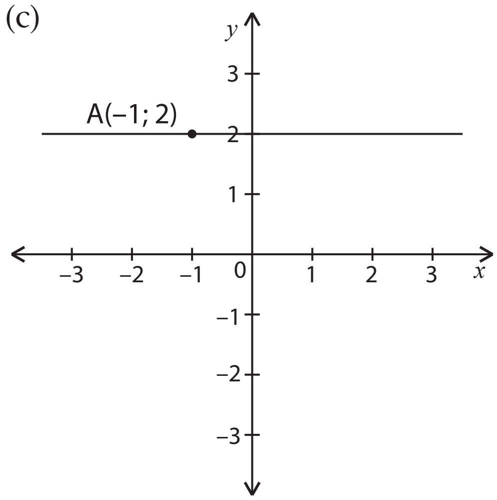
(a)



(b)



(c)



....

4.5 Finding the formula for a graph

TABLES AND FORMULAS

1. Each table on the next page shows values for a relationship represented by one of these rules:

$y = -2x + 3$	$y = 2x - 5$	$y = -3x + 5$	$y = -3(x + 2)$
$y = 3x + 2$	$y = 5(x - 2)$	$y = 2x + 3$	$y = 2x + 5$
$y = -3x + 6$	$y = 5x + 10$	$y = 5x - 10$	$y = -x + 3$

(a) Complete the tables below by extending the patterns in the output values.

A.

x	0	1	2	3	4	5	6	7
y	2	5	8					

B.

x	0	1	2	3	4	5	6	7
y	3	1	–1	–3				

C.

x	0	1	2	3	4	5	6	7
y	–10	–5	0	5				

D.

x	0	1	2	3	4	5	6	7
y	–5	–3	–1					

E.

x	0	1	2	3	4	5	6	7
y	6	3	0					

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

G.

x	0	1	2	3	4	5	6	7
y	3	5	7					

(b) For each table, describe what you did to produce more output values. Also write down the rule (formula) that corresponds to the table.

....

....

....

....

....

....

....

You may have noticed that the equations of straight lines look similar.

The equation of a straight line is: $y = mx + c$

m tells us the **gradient** of the line.

c tells us where the line crosses the y-axis.

This is called the ***y*-intercept** and it has the coordinates (0; c).

Gradient
Gradient means the steepness or slope of the line.

Intercept
The point where a line crosses one of the axes.

- the line $y = 3x + 4$ has a **gradient of 3** and the y-intercept is **(0; 4)**.
- the equation of a line with a **gradient of −2** and y-intercept of **(0;10)** is $y = -2x + 10$.
- the line $y = 2x$ has a **gradient of 2** and the y-intercept is **(0; 0)**.
- the line $y = 5$ has a **gradient of 0** and the y-intercept is **(0; 5)**.
- What is the gradient and y-intercept of the line $2y = 6x + 10$?

If you said $m = 6$ and $c = 10$ you would be wrong. The equation is not in standard form. The equation must be written in standard form before you can read off the values of the gradient and the y-intercept.

$$2y = 6x + 10 \quad \rightarrow \text{Divide both sides by 2}$$
$$y = 3x + 5$$

Therefore the **gradient is 3** and the y-intercept is **(0; 5)**.

Standard form
The standard form of a straight line graph is $\boldsymbol{y = mx + c}$.
On one side there should only be a "y" (with a coefficient of 1).

If $m > 0$ the line will be increasing.
If $m < 0$ the line will be decreasing.
If the line is horizontal $m = 0$.
If the line is vertical m is undefined.

2. Complete the following table:

Equation	Gradient	y-intercept
$y = 3x + 5$		
$y = \frac{x}{2} - 7$		
$y = 2 - 3x$		
$-y = 5x - 10$		
$y = 3$		
	1	(0; 0)
	−2	(0; −7)

3. Write each of the following equations in standard form and then determine the gradient and y-intercept.

(a) $2y + 4x = 10$ (b) $-3x = y + 4$ (c) $3x - 4 = y$

....

....

....

(d) $3y + 6 = x$

……………………

……………………

……………………

(e) $y = -3x + 4y - 12$

……………………

……………………

……………………

(f) $y = 3x - 2$

……………………

……………………

……………………

(g) $y = \frac{1}{4}x + 6$

……………………

……………………

……………………

(h) $y = -12$

……………………

……………………

……………………

(i) $x = 15$

……………………

……………………

……………………

DETERMINE THE EQUATION OF A STRAIGHT LINE

The equation of a straight line is $y = mx + c$. If you need to determine the equation of a straight line then all you need to know are the values of m and c.

If you know the values of two points on the graph then you can determine the gradient using the formula: $m = \frac{y_A - y_B}{x_A - x_B}$

Once you know the gradient you can calculate the value of the y-intercept using substitution.

Example 1: Determine the equation of the straight line that goes through (1; 1) and (5; 13).

Step 1: Calculate the gradient.

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{1 - 13}{1 - 5} = \frac{-12}{-4} = 3$$

Step 2: Since you now know $m = 3$ you can substitute it into the equation $y = mx + c$.
Therefore $y = 3x + c$.

Step 3: To determine c you need to substitute the coordinates of a point on the line into the equation. (It can be either of the points that were given, so choose the easier one.)

Substitute (5; 13) into $y = 3x + c$

$$(13) = 3(5) + c$$
$$13 = 15 + c$$
$$13 - 15 = c$$
$$-2 = c$$

Step 4: Write down the equation: $y = 3x - 2$

Example 2: Determine the equation of the line that passes through (4; –1) and (7; 5).

Information	m (Gradient)	c (y-intercept)	$y = mx + c$ (Equation)
(4; –1) **(7; 5)**	$m = \frac{y_A - y_B}{x_A - x_B}$ $= \frac{-1-5}{4-7}$ $= \frac{-6}{-3}$ $= 2$	Substitute $m = 2$ and (7; 5) $y = mx + c$ $y = 2x + c$ $(5) = 2(7) + c$ $5 = 14 + c$ $-9 = c$	$y = 2x - 9$

Example 3: Determine the equation of the line with a gradient of 4 passing through (2; 6).

Information	m (Gradient)	c (y-intercept)	$y = mx + c$ (Equation)
$\boldsymbol{m = 4}$ **(2; 6)**	$m = 4$	Substitute $m = 4$ and (2; 6) $y = mx + c$ $y = 4x + c$ $6 = 4(2) + c$ $-2 = c$	$y = 4x - 2$

You may want to set your work out as shown in Examples 2 and 3 above.

1. Determine the equation of the each of the straight lines passing through the points given.

   (a) (3; 10) and (2; 5) ....
   (b) (–4; 5) and (2; 5) ....
   (c) (0; 0) and (4; –8) ....

   (d) $(1\frac{1}{2}; 4)$ and $(-\frac{1}{2}; 12)$ ....
   (e) (3; 4) and (–7; –1) ....
   (f) (0; 3) and (–14; –4) ....

2. Determine the equation of the straight line with:

   (a) a gradient of 5 and passing through the point (1; –3)

   ....

   (b) a gradient of –2 passing through the point (0; 0)

   ....

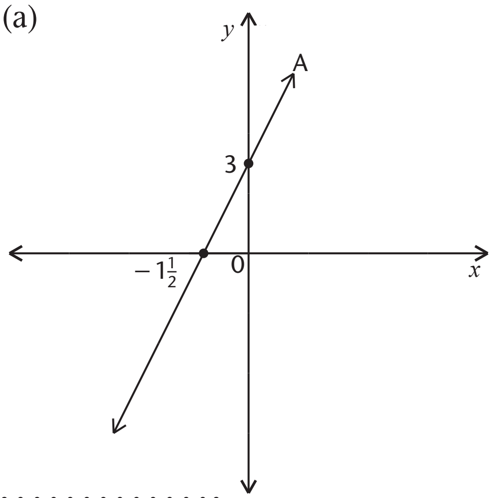
   (c) a y-intercept of 7 passing through the point (1; –3)

   ....

Maths2_Gr9_LB_Book.indb 70 2014/09/08 09:06:42 AM

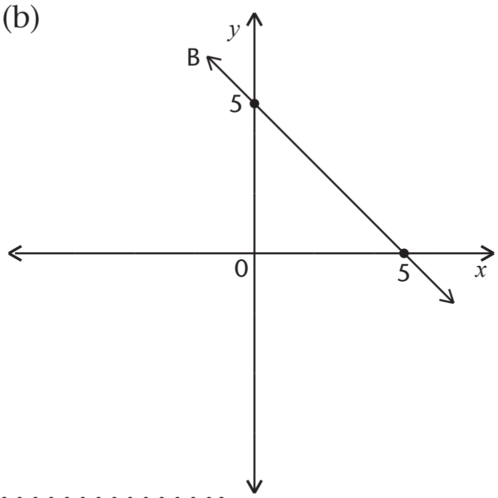
3. Determine the equations of the straight lines. Question (d) is a challenge.

(a)



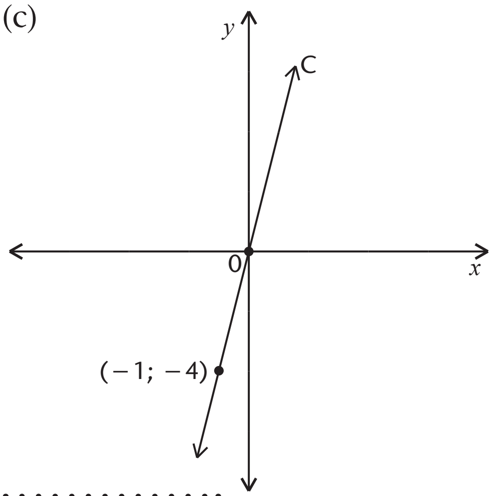
....

(b)



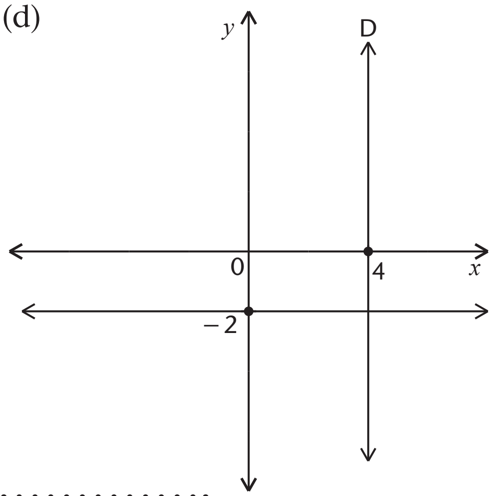
....

(c)



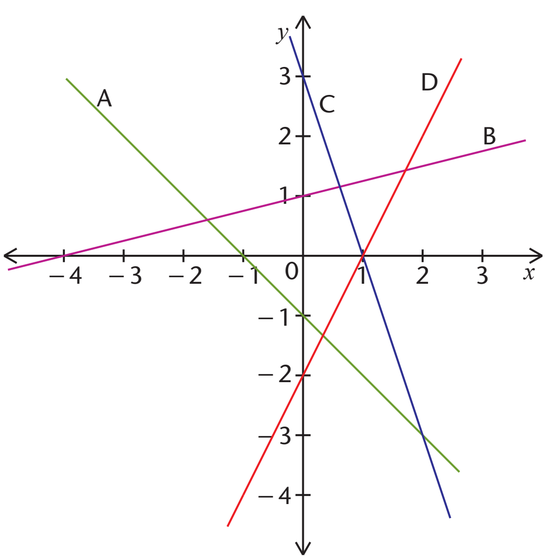
....

(d)



....

4.6 *x*- and *y*-intercepts



1. Write down the coordinates of the points where each line cuts the 2 axes:

	x-intercept	*y*-intercept
A		
B		
C		
D		

2. What do all the *x*-intercepts have in common?

....

3. What do all the y-intercepts have in common?

....

4. Determine the coordinates of the intercepts of the following straight line graphs.

(a) $y = 3x + 12$

(b) $y = x - 3$

(c) $y = -2x - 4$

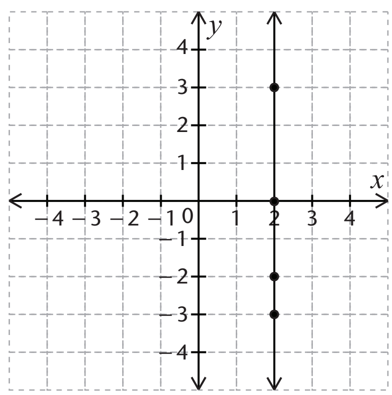
(d) $2y = 6x + 12$

(e) $4x + 2y = 20$

(f) $13 - y = -26x$

VERTICAL AND HORIZONTAL LINES

Some special lines are so easy that you don't need any fancy methods to draw them or get their equation; you can just look at them.



1. What do the following coordinate pairs have in common?
(2; 3), (2; –2), (2; 0) and (2; –3)

....

2. Write down two more points that have an x-coordinate of 2.

....

If you plot these points on a set of axes you will see that they form a **vertical line**. The equation of the line is $x = 2$.

3. Will the two extra points you wrote down (question 2) also be on the line?

....

4. Write down five coordinate pairs with $x = -1$.

....

4.7 Graphs of non-linear functions

Some of the following relationships are represented by graphs on the next page. Identify which of the relationships are represented by which set of points on the graph. You may use the tables below to help you to answer this question. For example, you may calculate some output numbers by using the formulas and record this in the tables.

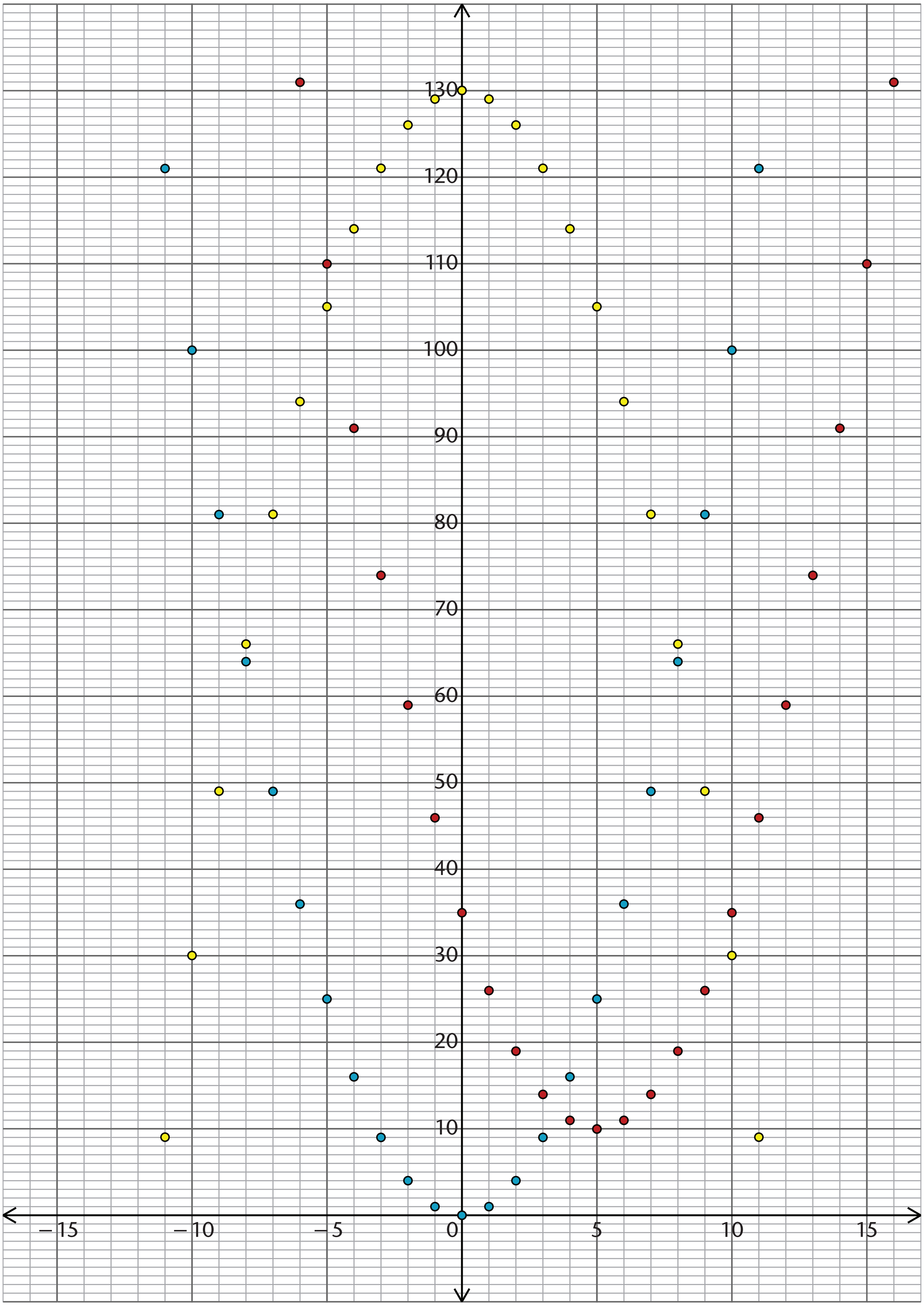
$y = -x^2$	$y = (-x)^2$	$y = x^2 + 130$	$y = (x - 5)^2 + 10$
$y = x^2$	$y = -x^2 + 130$	$y = 130 - x^2$	$y = x^2 - 10x + 35$

Write your answers here:

Set of points in yellow ....

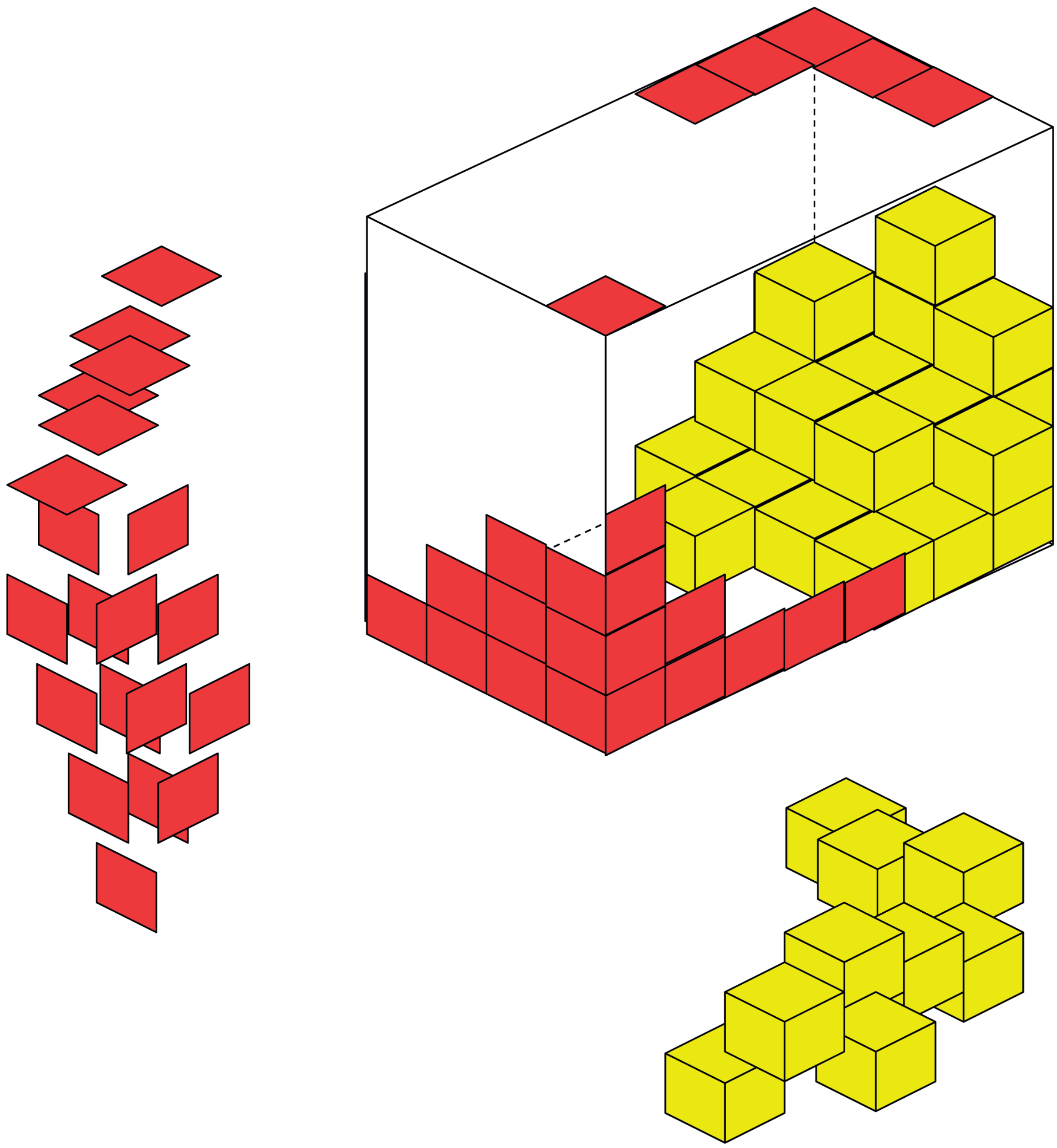
Set of points in blue ....

Set of points in red ....


130
120
110
100
90
80
70
60
50
40
30
20
10
– 15
– 10
– 5
0
5
10
15

Chapter 5
Surface area, volume and capacity of 3D objects

By now you should know how to calculate the surface area and volume of cubes, rectangular prisms and triangular prisms. In this chapter, you will revise how to do this, practise converting between equivalent units used for volume, and revise the difference between volume and capacity. You will investigate how to calculate the surface area and volume of cylinders, and explore how the volumes of a prism and cylinder are affected when one or more of their dimensions is doubled.



5 Surface area, volume and capacity of 3D objects

5.1 Surface area

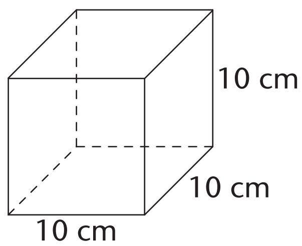
SURFACE AREA OF PRISMS

The **surface area** of an object is the total area of all of its faces added together. You learnt the following formula in previous grades:

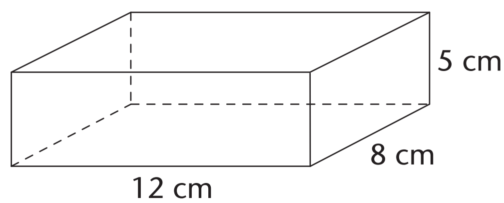
- Surface area of a prism = Sum of the areas of all its faces

Calculate the surface area of the following objects to revise what you should already know.

1.

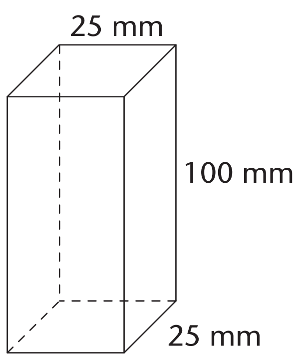


2.

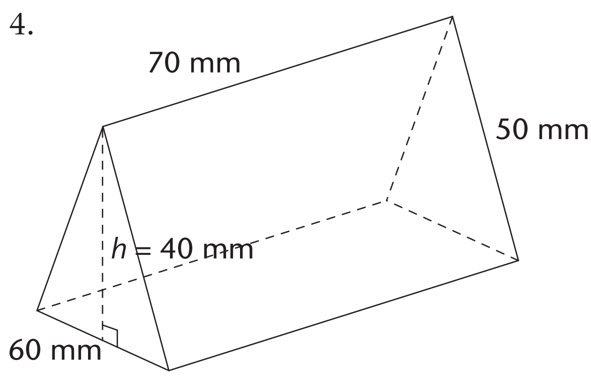


[We use *SA* for surface area.]

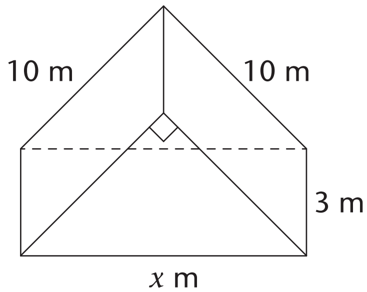
3.



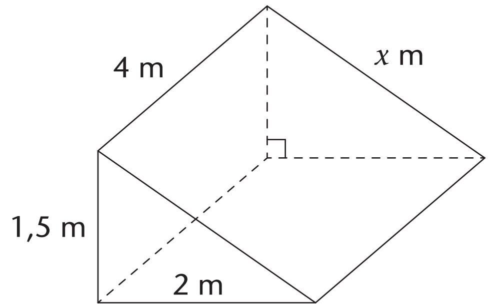
4.



5.



6.



....

....

....

....

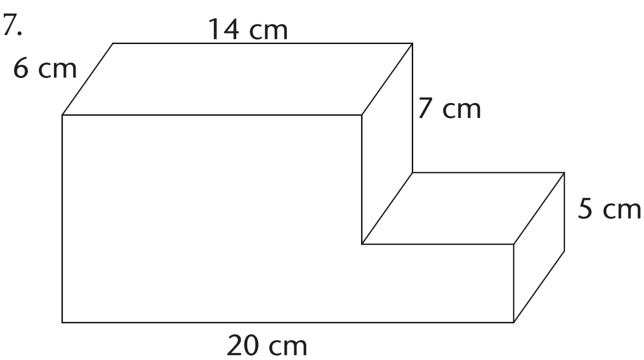
....

....

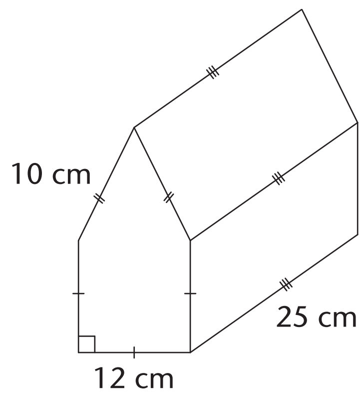
....

....

7.



8.



....

....

....

....

....

....

....

....

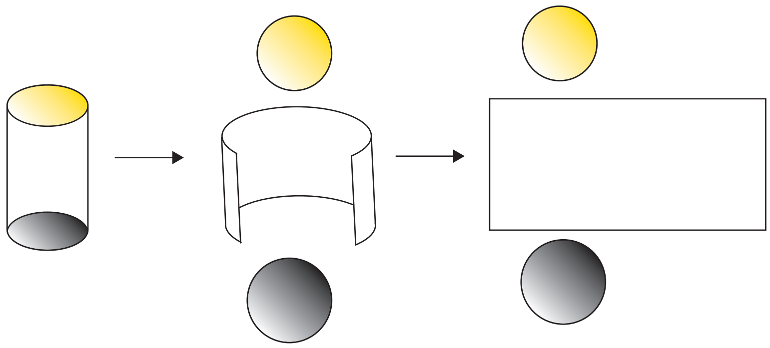
....

....

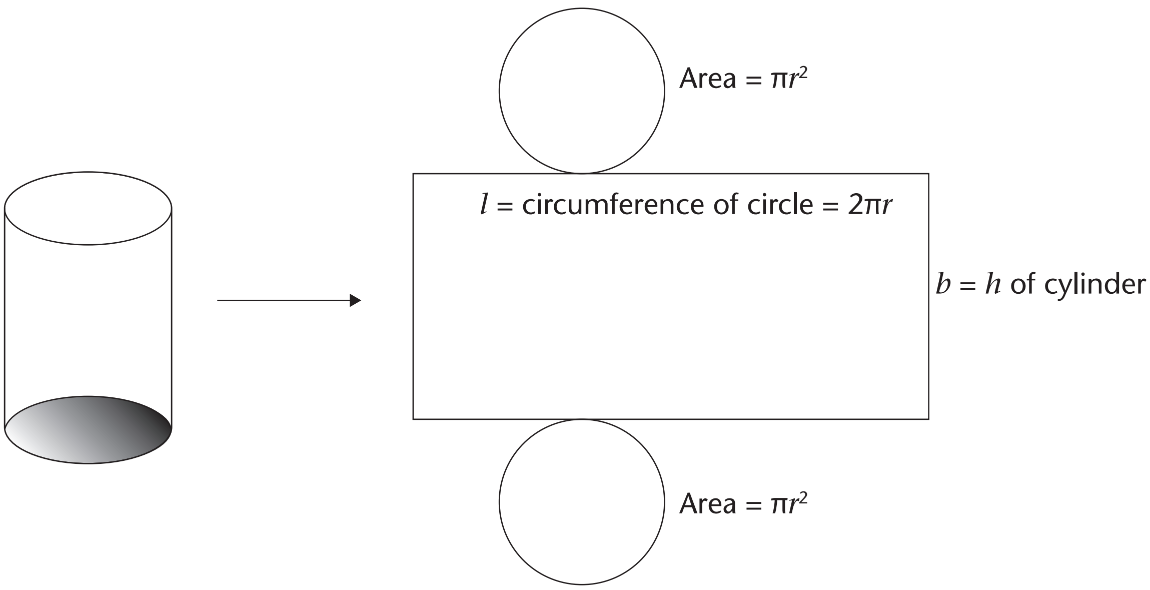
INVESTIGATING THE SURFACE AREA OF CYLINDERS

In order to calculate the surface area of a cylinder, you need to know what shape the surfaces of the cylinder are.

The surfaces of the top and base of a cylinder are made up of circles. The curved surface between the top and base of a cylinder can be unrolled to create a rectangle.



So the net of a cylinder looks like this:



Surface area of a cylinder = Area of all its surfaces

= Area of top + Area of base + Area of curved surface

= $\pi r^2 + \pi r^2 + (l \times b)$

= $2\pi r^2 + (2\pi r \times h)$

= $2\pi r(r + h)$

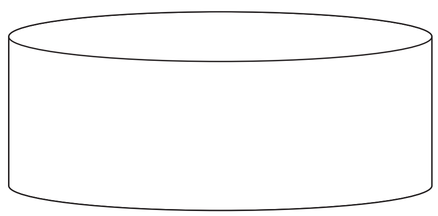
> Can you explain why the length of the rectangle is equal to the circumference of the top or base of the cylinder?

CALCULATING THE SURFACE AREA OF CYLINDERS

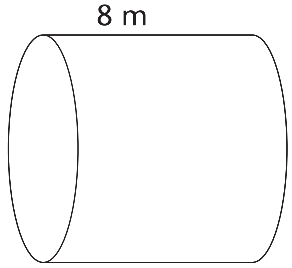
From the formula on the previous page, you can see that we need only know the radius (r) and the height (h) of a cylinder in order to work out its surface area.

1. Calculate the surface areas of the following objects. Use $\pi = 3{,}14$ and round off all your answers to two decimal places.

A.



B.



....

....

....

2. Calculate the surface area of a cylinder if its height is 60 cm and the circumference of its base is 25,12 cm.

....

....

....

....

3. Calculate the surface area of a cylinder if its height is 5 m and the circumference of its base is 12,56 m.

....

....

....

....

4. The outside of a cylindrical structure at a factory must be painted. Its radius is 3,5 m and its height is 8 m. How many litres of paint must be bought if 1 litre covers 10 m^2? (The bottom of the structure will not be painted.)

....

....

....

....

....

....

5.2 Volume

The **volume** of an object is the amount of space it occupies. We usually measure volume in cubic units, such as mm^3, cm^3 and m^3.

To convert between cubic units, remember:
$1\ cm^3 = 1\ 000\ mm^3$
$1\ m^3 = 1\ 000\ 000\ cm^3$

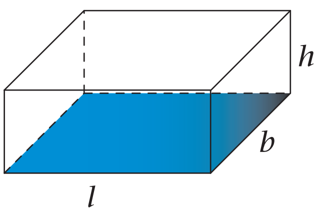
FORMULAS FOR VOLUME OF PRISMS

The general formula for the volume of a prism is:
Volume of a prism = Area of base × height.

In case of a triangular prism do not confuse the height of the base of the triangle (h_b) with the height of the prism (h_p).

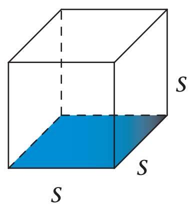
Therefore, the formulas to work out the volumes of the following prisms are:

Rectangular prism



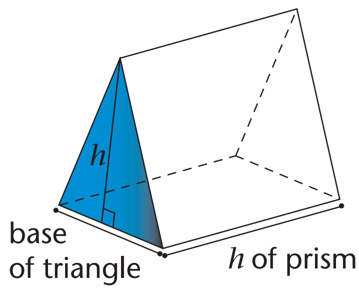
$V = (l \times b) \times h$

Cube



$V = (s \times s) \times s$
$= s^3$

Triangular prism

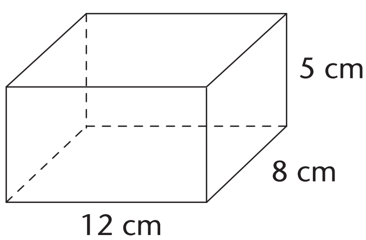


$V = (\frac{1}{2}\text{ base} \times h_b) \times h_p$

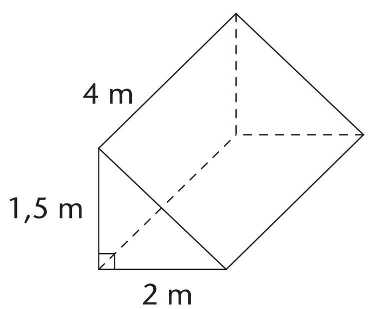
CALCULATING THE VOLUME OF PRISMS

1. Calculate the volumes of the following prisms.

A.



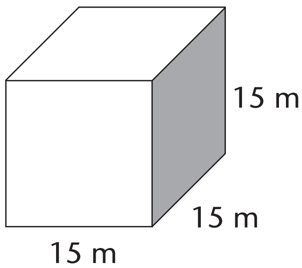
B.



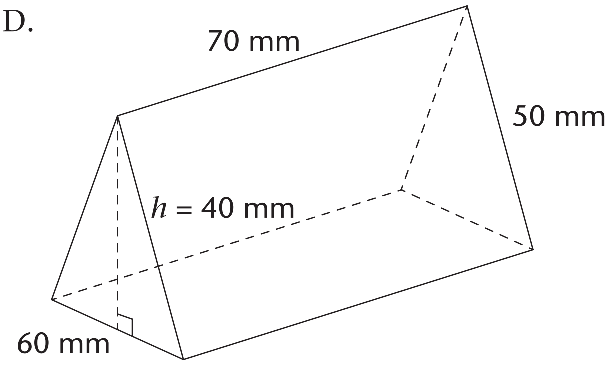
....

....

C.



D.



....

....

2. (a) The area of the base of a rectangular prism is 32 m^2 and its height is 12 m. What is its volume?

....

(b) The volume of a cube is 216 m^3. What is the length of one of its edges?

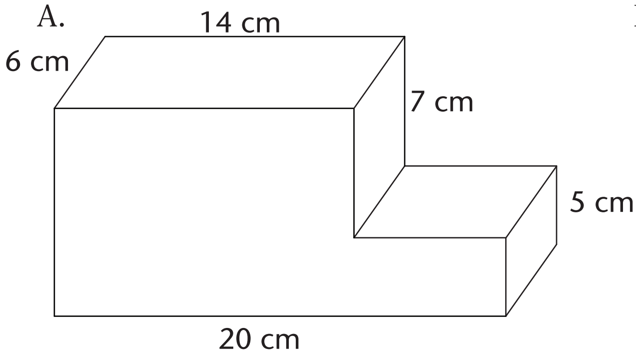
....

....

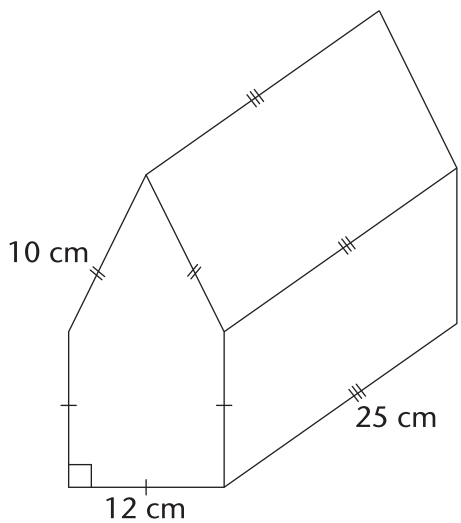
....

3. Calculate the volume of the following objects.

A.



B.



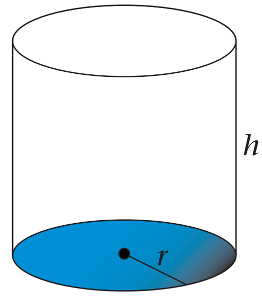
....

....

....

VOLUME OF CYLINDERS

You also calculate the volume of a cylinder by multiplying the area of the base by the height of the cylinder. The base of a cylinder is circular, therefore:

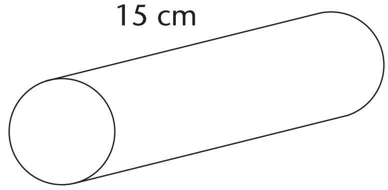


Area of circle = πr^2

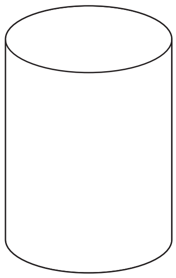
$$\text{Volume of a cylinder} = \text{Area of base} \times h$$
$$= \pi r^2 \times h$$

1. Calculate the volume of the following cylinders. Use $\pi = 3{,}14$ and round off all answers to two decimal places.

A.



B.

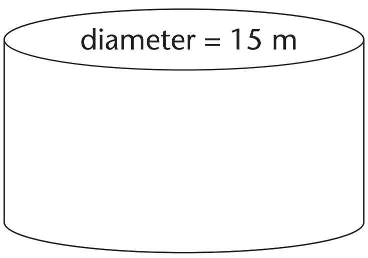


....

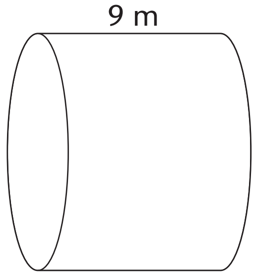
....

....

C.



D.



....

....

....

Maths2_Gr9_LB_Book.indb 83 2014/09/08 09:06:48 AM

2. Without using a calculator, calculate the volume of cylinders with the following measurements. Use $\pi = \frac{22}{7}$.

(a) $r = 14$ cm, $h = 20$ cm

....

....

....

(b) $r = 7$ cm, $h = 35$ cm

....

....

....

(c) diameter = 28 cm, $h = 50$ cm

....

....

....

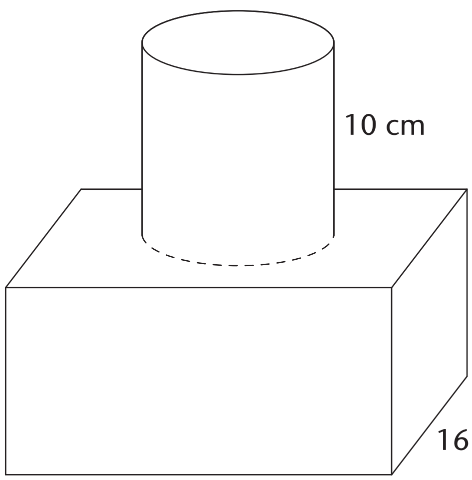
(d) diameter = 7 cm, $h = 10$ cm

....

....

....

3. Calculate the volume of the following object. Use a calculator and round off all answers to two decimal places.



....

....

....

....

....

....

....

....

....

....

....

....

....

5.3 Capacity

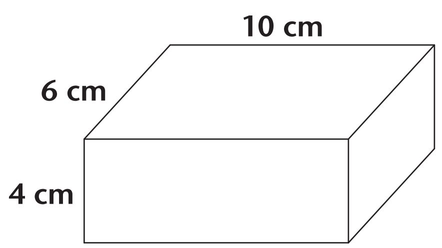
Remember that the **capacity** of an object is the amount of space *inside* the object. You can think of the capacity of an object as the amount of liquid that the object can hold.

The **volume** of an object is the amount of space that the object itself takes up.

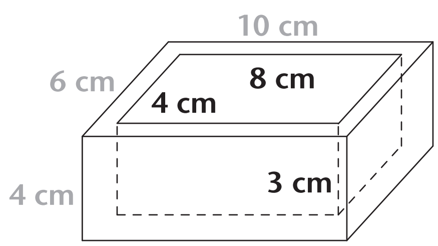
The volume of a solid block of wood is 10 cm × 6 cm × 4 cm = 240 cm^3.

The same block of wood is carved out to make a hollow container with inside measurements of 8 cm × 4 cm × 3 cm. (Its walls are 1 cm thick.) The amount of space inside the container must be calculated using the *inside* measurements. So the capacity of the container is 8 cm × 4 cm × 3 cm = 96 cm^3.

A. Solid block with outside measurements



B. Hollowed block with inside measurements



1. Write, in ml, the volume of water that would fill container B.

   ....

Remember:
1 cm^3 = 1 ml
1 m^3 = 1 kl

2. If the walls and bottom of container B were 0,5 cm thick, what would its capacity be? Write the answer in ml.

   ....

   ....

3. The inside measurements of a swimming pool are 9 m × 4 m × 2 m. What is the capacity of the pool in kl?

   ....

   ....

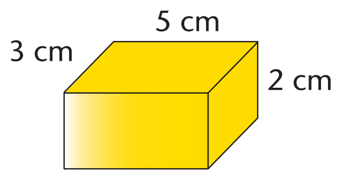
   ....

5.4 Doubling dimensions and the effect on volume

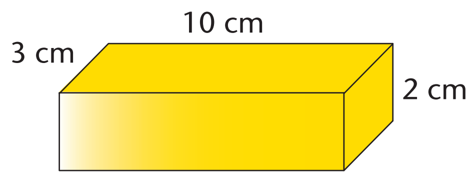
DOUBLING THE DIMENSIONS OF A PRISM

The first prism below measures 5 cm × 3 cm × 2 cm. The other diagrams show the prism with one or more of its dimensions doubled.

1. Work out the volume of each prism.



One dimension doubled

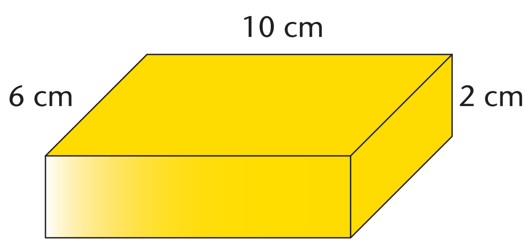


....

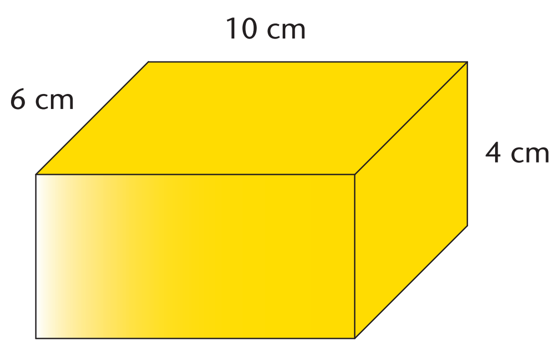
....

....

Two dimensions doubled



Three dimensions doubled



....

....

....

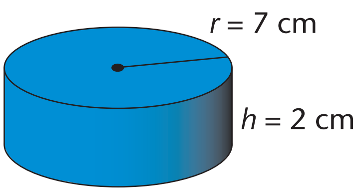
2. Complete the following:
   (a) When one dimension of a prism is doubled, the volume ....
   (b) When two dimensions of a prism are doubled, the volume increases by .... times.
   (c) When all three dimensions of a prism are doubled, the volume increases by .... times.

3. The volume of a prism is 80 cm³. What is its volume if:
   (a) its length is doubled? ....
   (b) its length and breadth are doubled? ....
   (c) its length, breadth and height are doubled? ....

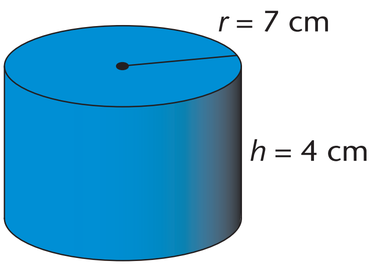
DOUBLING THE DIMENSIONS OF A CYLINDER

The first cylinder below has a radius of 7 cm and a height of 2 cm. The other diagrams show the cylinder with one or more of its dimensions doubled.

1. Work out the volume of each cylinder.



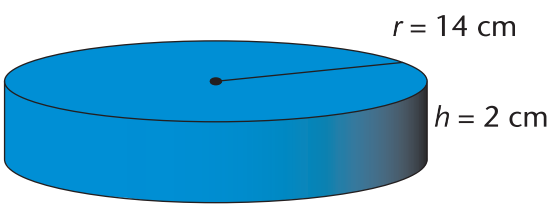
Only height doubled



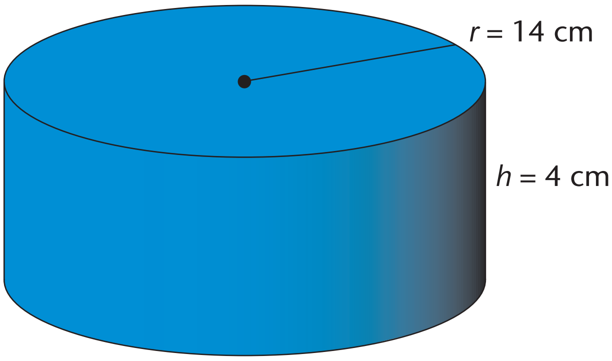
....

....

Only radius doubled



Radius and height doubled



....

....

2. Complete the following:
   (a) When the height of a cylinder is doubled, the volume ....
   (b) When the radius of a cylinder is doubled, the volume increases by .... times.
   (c) When height and radius of cylinder are doubled, the volume increases by .... times.

3. The volume of a cylinder is 462 cm^3. What is its volume if:
   (a) its height is doubled? ....
   (b) its radius is doubled? ....
   (c) its height and radius are doubled? ....

4. (a) Study the following tables. Without using the formulas to calculate volume, complete the last column in each table. (Hint: Identify which dimensions are doubled each time, then work out the volume accordingly.)

Rectangular prism			
Length (l) in m	**Breadth (b) in m**	**Height (h) in m**	**Volume (V) in m^3**
4	2	1	
4	4	1	
8	2	1	
8	2	2	
8	4	2	

Cylinder		
Radius (r) in m	**Height (h) in m**	**Volume (V) in m^3**
3,5	4	
7	4	
3,5	8	
7	8	

   (b) Explain how you worked out the answers in the tables.

....

....

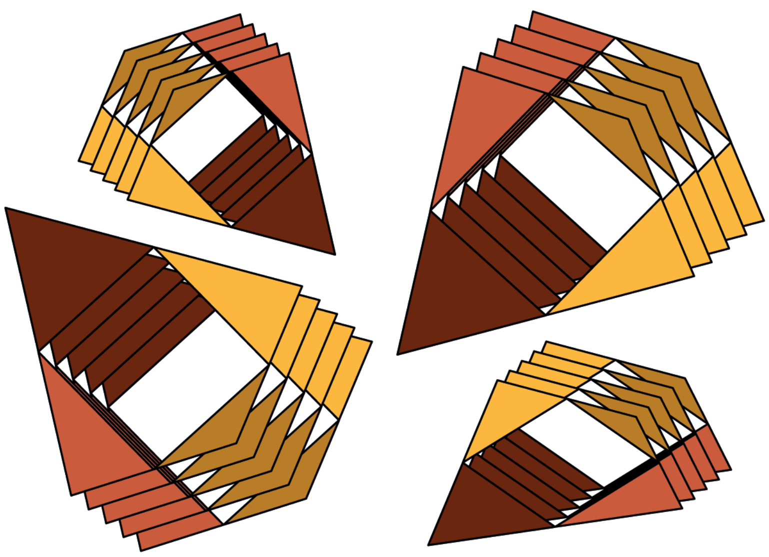
....

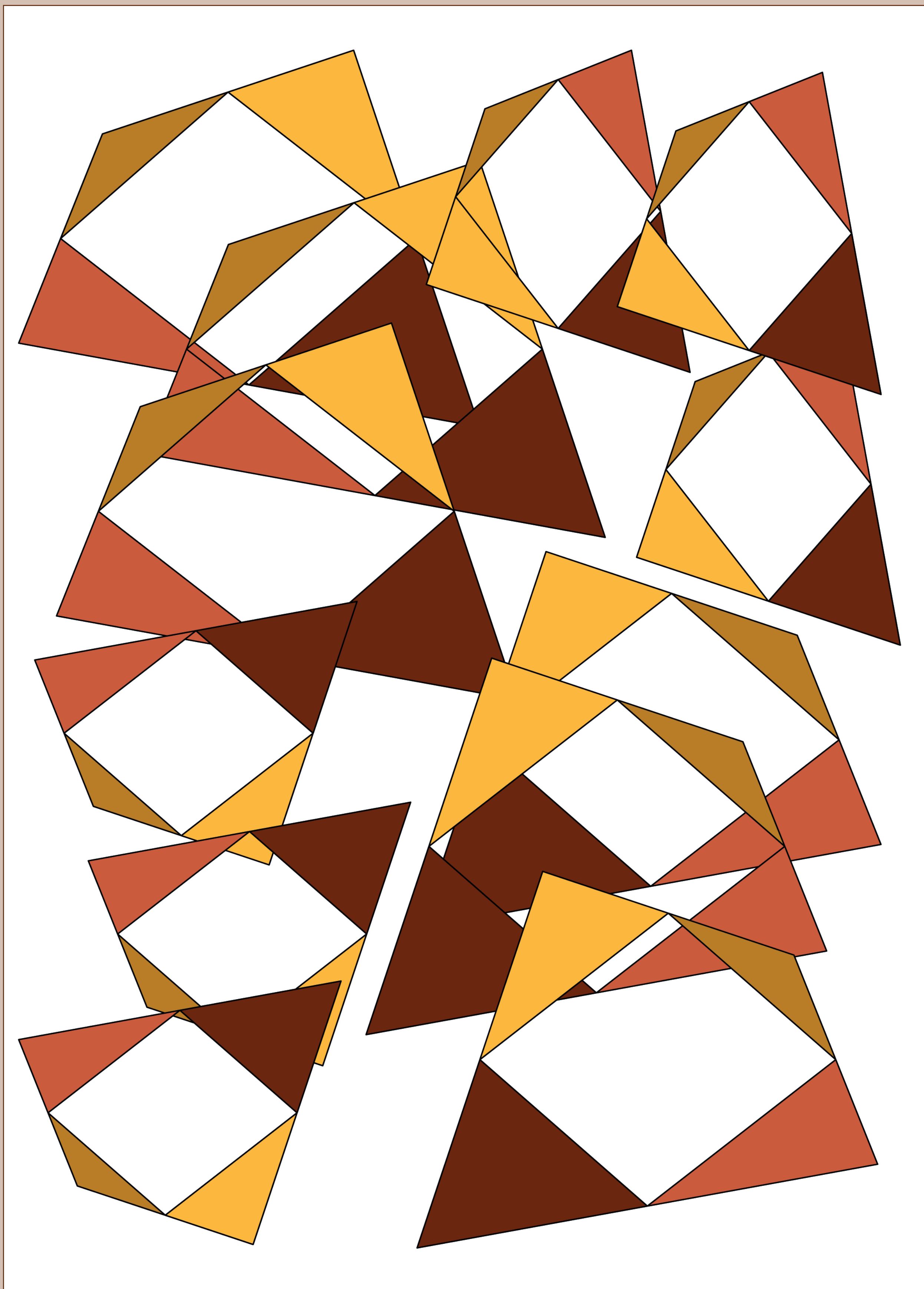
....

Chapter 6
Transformation geometry

In Grade 8, you learnt how to describe and perform translations, reflections and rotations on a coordinate system. In such transformations, the original figure and its image are always congruent. In this grade, you will explore in more detail the change in the coordinates of original figures and their images after different types of reflections and translations.

You will then revise how a figure is enlarged or reduced when its sides are multiplied by the same number, called a scale factor, and how the scale factor affects the area and perimeter of an image. In enlargements and reductions, the corresponding sides of the original figure and its image are in proportion, which makes the figures similar. You will also perform enlargement and reduction of figures on the coordinate system, and investigate the coordinates of the vertices of such figures.





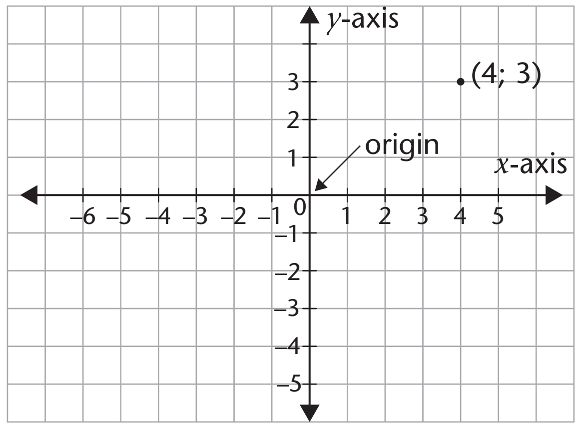
6 Transformation geometry

6.1 Points on a coordinate system

A rectangular coordinate system is also called a **Cartesian coordinate system**. It consists of a horizontal *x*-axis and a vertical *y*-axis.

The intersection of the axes is called the **origin**, and represents the point (0; 0).

Any point can be represented on a coordinate system using an *x*-value and a *y*-value. These numbers are called **coordinates**, and describe the position of the point with reference to the two axes.



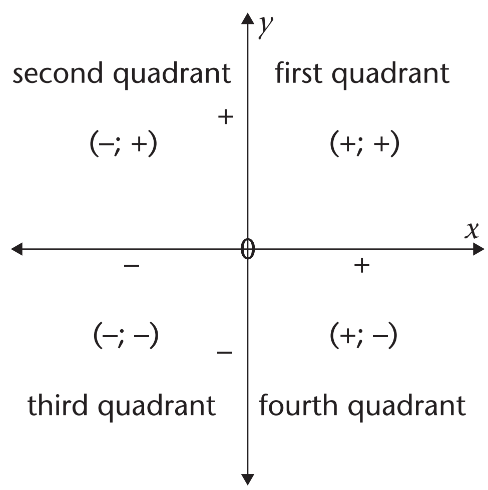
The coordinates of a point are always written in a certain order:

The horizontal distance from the origin (*x*-coordinate) is written first.

The vertical distance from the origin (*y*-coordinate) is written second.

These numbers, called an **ordered pair**, are separated by a semi-colon (;) and are placed between brackets. Here is an example of an ordered pair: (4; 3) (see on the coordinate system above).

The *x*-axis and *y*-axis divide the coordinate system into four sections called **quadrants**. The diagram alongside shows how the quadrants are numbered, and also whether the *x*- and *y*-coordinates are negative or positive in each quadrant.



1. In which quadrant will the following points be plotted?
   (a) (–4; 1) .... (b) (–1; –5) ....
   (c) (4; –3) .... (d) (5; 2) ....

2. Plot the points in question 1 on the coordinate system above.

When a point is translated to a different position on a coordinate system, the new position is called the image of the point. We use the prime symbol (') to indicate an image. For example, the image of A is indicated by A' (read as "A prime"). If the coordinates of A are labelled as (*x; y*), the coordinates of A' can be labelled as (*x'; y'*).

We write A → A' and (*x; y*) → (*x'; y'*) to indicate that A is mapped to A'.

6.2 Reflection (flip)

The **mirror image** or **reflection** of a point is on the opposite side of a **line of reflection**.

"Reflecting a point in the *x*-axis" means that the *x*-axis is the line of reflection.

The original point and its mirror image are the same distance away from the line of reflection, and the line that joins the point and its image is perpendicular to the line of reflection.

Any line on the coordinate system can be a line of reflection, including the *x*-axis, the *y*-axis and the line $y = x$.

REFLECTING POINTS IN THE *x*-AXIS, *y*-AXIS AND THE LINE $y = x$

1. The points A(5; 4) and B(–3; –2) are plotted on a coordinate system.
   (a) Reflect points A and B in the *x*-axis and write down the coordinates of the images.
   (b) Reflect points A and B in the *y*-axis and write down the coordinates of the images.
   (c) Compare the coordinates of the original points with those of its images. What do you notice?

....

....

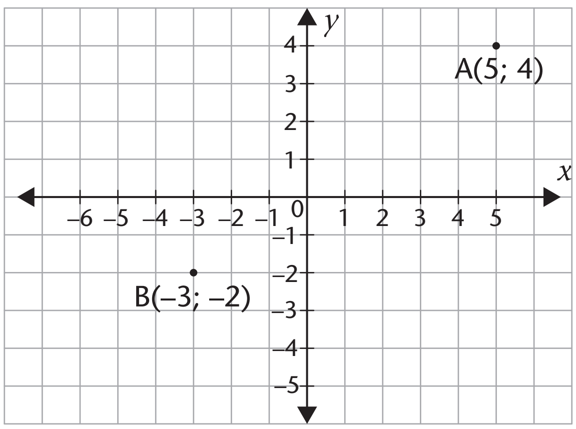
....

....

....

....

....



Maths2_Gr9_LB_Book.indb 92 2014/09/08 09:06:51 AM

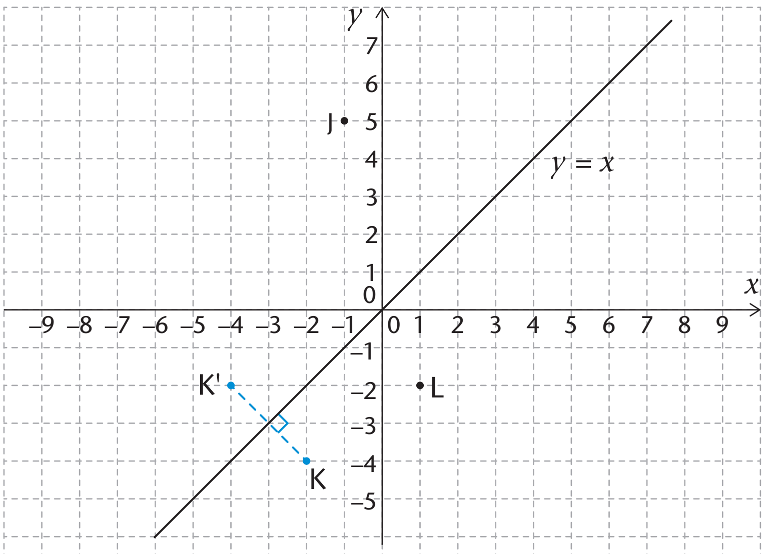
2. Write down the coordinates of the images of the following reflected points.

Point	Reflection in the x-axis	Reflection in the y-axis
(–131; 24)		
(–459; –795)		
$(x; y)$		

3. The points J(–1; 5), K(–2; –4) and L(1; –2) are plotted on the coordinate system. K' is the reflection of point K in the line $y = x$. This means that the line $y = x$ is the line of reflection.

(a) Reflect J and L in the line $y = x$.

(b) Write down the coordinates of the images of the points.



....

....

....

(c) What do you notice about the coordinates of the images of the points in (b) above?

....

....

(d) Use your observation in (c) above to complete this table.

Point	Coordinates of the image of the point reflected in $y = x$
(–1 001; –402)	
(459; –795)	
(–342; 31)	
(21; 67)	
$(x; y)$	

While doing the previous activity, you may have noticed the following.

- For a reflection in the y-axis, the sign of the x-coordinate changes and the y-coordinate stays the same: $(x; y) \rightarrow (-x; y)$ or $x' = -x$ and $y' = y$, for example: $(-3; 4) \rightarrow (3; 4)$
- For a reflection in the x-axis, the sign of the y-coordinate changes and the x-coordinate stays the same: $(x; y) \rightarrow (x; -y)$ or $x' = x$ and $y' = -y$, for example: $(-3; 4) \rightarrow (-3; -4)$
- For a reflection in the line $y = x$, the values of the x- and y-coordinates are interchanged: $(x; y) \rightarrow (y; x)$ or $x' = y$ and $y' = x$, for example: $(-3; 4) \rightarrow (4; -3)$.

4. Investigate the effect of reflection in the line $y = -x$ on the coordinates of a point.

....

....

5. A is the point $(5; -2)$. Write the coordinates of the mirror images of A if the point is reflected in:

(a) the y-axis ....

(b) the line $y = -x$ ....

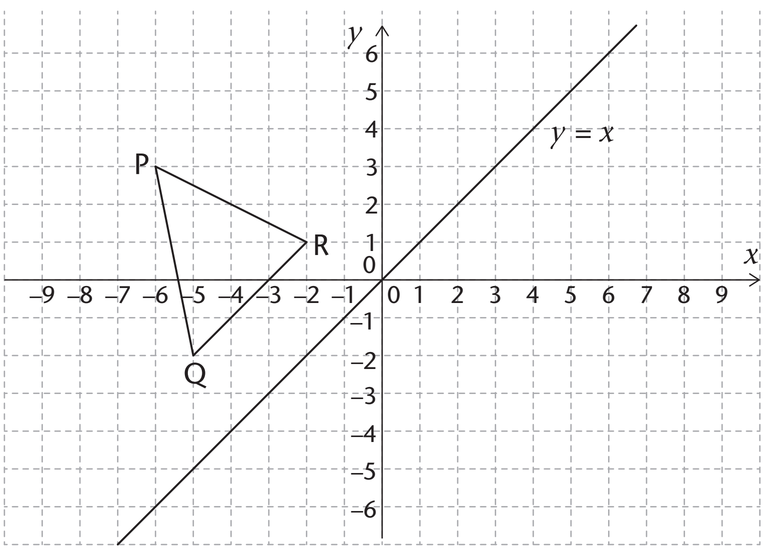
(c) the line $y = x$ ....

(d) the x-axis ....

REFLECTING GEOMETRIC FIGURES

The same principles as above apply when reflecting geometric figures.

1. (a) Reflect ΔPQR in the x-axis, in the y-axis and in the line $y = x$ in the coordinate system (first reflect the vertices and then join the reflected points).



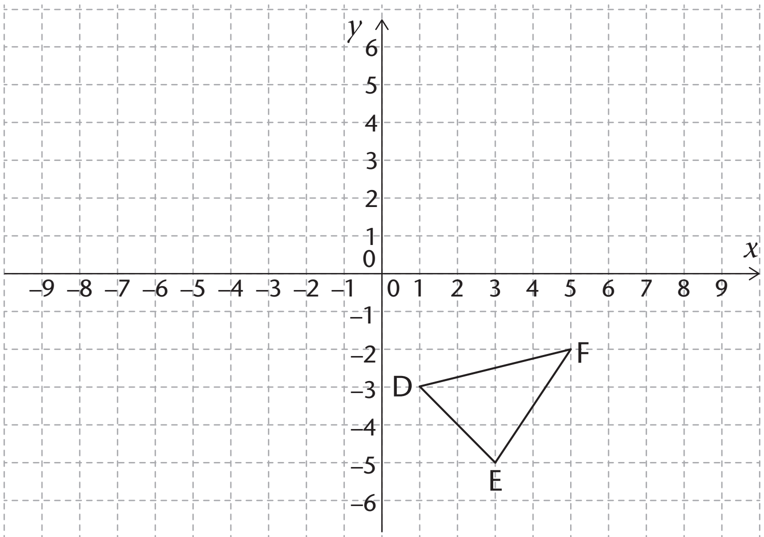
(b) Look at your completed reflections in question 1(a), and write down the coordinates of the image points in the following table.

Vertices of triangle	Reflection in the x-axis	Reflection in the y-axis	Reflection in the line $y = x$
P(–6; 3)			
Q(–5; –2)			
R(–2; 1)			

(c) What do you notice about ΔPQR, ΔP'Q'R', ΔP''Q''R'' and ΔP'''Q'''R'''?

....

2. Reflect ΔDEF in the x-axis, in the y-axis and in the line $y = x$.



3. A quadrilateral has the following vertices: A(1; 4), B(–6; 1), C(–2; –1) and D(7; 2). Without performing the actual reflections, write down the coordinates of the vertices of the image when the quadrilateral is:
   (a) reflected in the x-axis

   ....

   (b) reflected in the y-axis

   ....

   (c) reflected in the line $y = x$

   ....

4. In each case state around which line the point was reflected.
   (a) (–4; 5) → (–4; –5) ....
   (b) (2; –3) → (–2; –3) ....
   (c) (–13; –3) → (–3; –13) ....
   (d) (1; 16) → (16; 1) ....
   (e) (12; –8) → (–12; –8) ....
   (f) (–7; –5) → (–5; –7) ....
   (g) (2; –3) → (–2; –3) ....

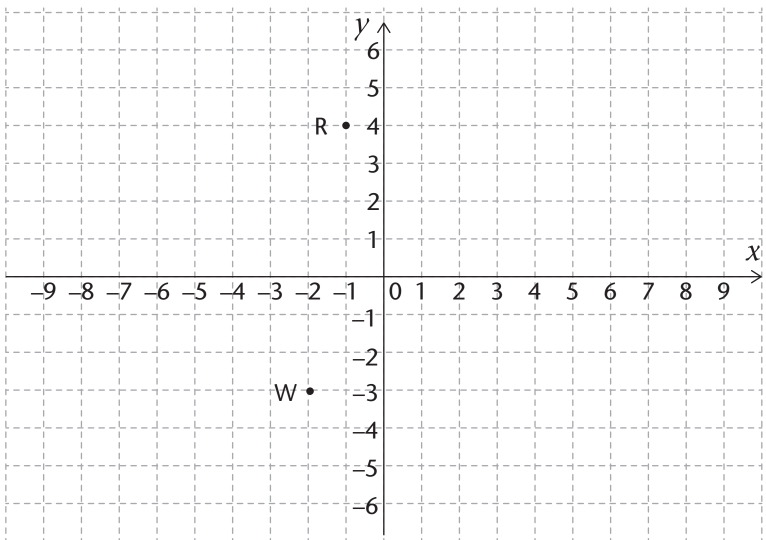
6.3 Translation (slide)

Remember: A translation of a point or geometric figure on a coordinate system means moving or sliding the point in a vertical direction, in a horizontal direction, or in both a vertical and horizontal direction.

TRANSLATING POINTS HORIZONTALLY OR VERTICALLY ON A COORDINATE SYSTEM

1. Points R and W are plotted on a coordinate system.
   (a) Plot the image of point R after a translation of:
   - 5 units to the right
   - 5 units to the left
   - 2 units up
   - 2 units down

   (b) Plot the image of point W after a translation of:
   - 4 units to the right
   - 4 units to the left
   - 3 units up
   - 3 units down



(c) Look at your completed translations in (a) and (b) on the previous page. Complete the following table by writing down the coordinates of the original points and their images after each translation.

Coordinates of original points	**R(–1; 4)**	**W(–2; –3)**
Coordinates of image after a translation to the **right**		
Coordinates of image after a translation to the **left**		
Coordinates of image after a translation **up**		
Coordinates of image after a translation **down**		

(d) Look at your completed table in (c) above. Choose the correct answers below to make each statement true:

- For translations to the **right or left**, the (x-value/y-value) changes and the (x-value/y-value) stays the same.
- For translations **up or down**, the (x-value/y-value) changes and the (x-value/y-value) stays the same.
- For translations to the **right**, (add/subtract) the number of translated units (to/from) the x-value.
- For translations to the **left**, (add/subtract) the number of translated units (to/from) the x-value.
- For translations **up**, (add/subtract) the number of translated units (to/from) the y-value.
- For translations **down**, (add/subtract) the number of translated units (to/from) the y-value.

2. Write down the coordinates of each image after the following translations.

Point	**3 units to the right**	**4 units to the left**	**2 units up**	**5 units down**
(3; 5)				
(–13; 42)				
(–59; –95)				
$(x; y)$				

3. Write down the coordinates of each image after the following translations:

Point	4 units to the right and 3 units up	2 units to the left and 1 unit up	1 unit to the right and 5 units down	6 units to the left and 2 units down
(4; 2)				
(–32; 21)				
(–68; –57)				
$(x; y)$				

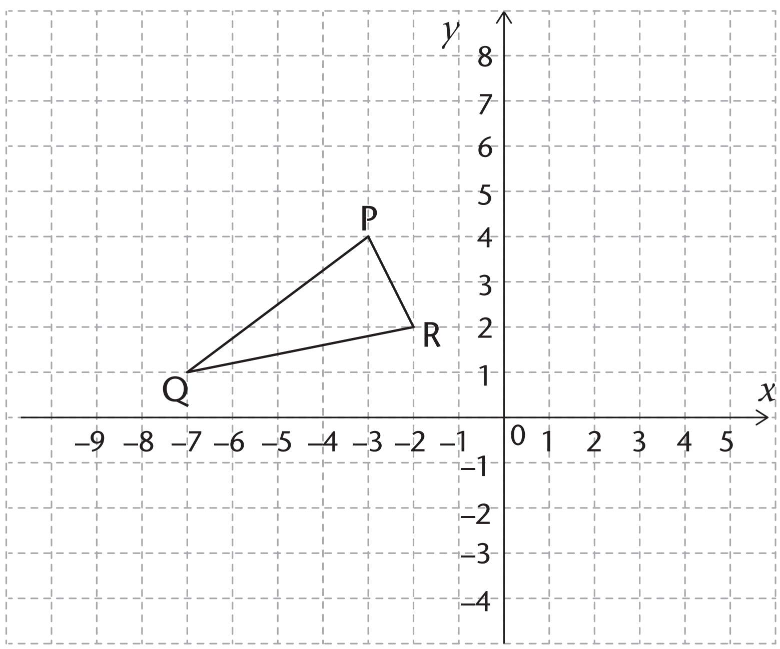
While doing the previous activity, you may have noticed the following:

- For a horizontal translation through the distance p, the x-coordinate increases by the distance p if the slide is to the right, and decreases by the distance p if the slide is to the left. We may write $x' = x + p$, with $p > 0$ for a translation to the right, and $p < 0$ for a translation to the left. The y-coordinate remains the same, so $(x; y) \rightarrow (x + p; y)$.
- For a vertical translation through the distance q, the y-coordinate increases by the distance q if the slide is upwards, and decreases by the distance q if the slide is downwards. We may write $y' = y + q$, with $q > 0$ for a translation vertically upwards, and $q < 0$ for a translation vertically downwards. The x-coordinate remains the same, so $(x; y) \rightarrow (x; y + q)$.

TRANSLATION OF GEOMETRIC FIGURES ON A COORDINATE SYSTEM

1. (a) Translate ΔPQR 5 units to the right and 3 units down.
   (b) Translate ΔPQR 2 units to the left and 3 units up.
   (c) Are all the triangles congruent?

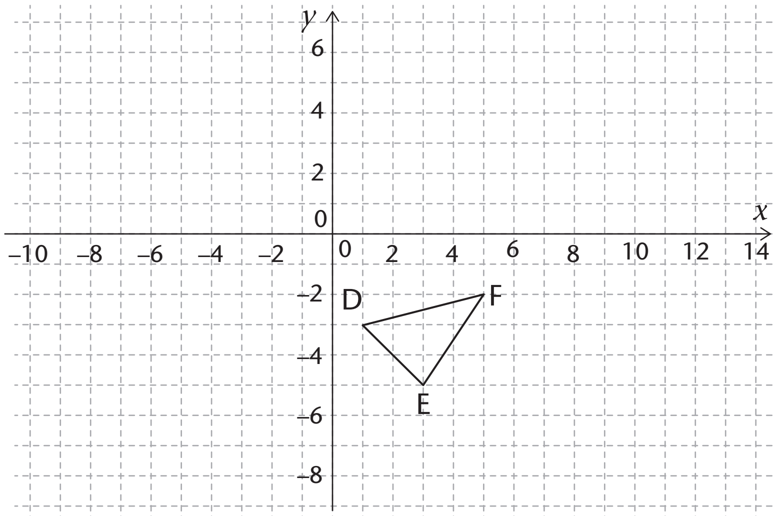
....



2. (a) Translate ΔDEF 4 units to the left and 2 units down.

   (b) Translate ΔDEF 3 units to the right and 4 units up.

   (c) Are all the triangles congruent? ....



3. The vertices of a quadrilateral have the following coordinates: K(–5; 2), L(–4; –2), M(1; –3) and N(4; 3). Write down the coordinates of the image of the quadrilateral after the following translations.

   (a) 7 units to the right and 2 units up

   ....

   (b) 5 units to the right and 2 units down

   ....

   (c) 4 units to the left and 3 units down

   ....

   (d) 2 units to the left and 7 units up

   ....

4. Describe the translation if the coordinates of the original point and the image point are:

   (a) (–2; –3) → (–2; –5) ....

   (b) (4; –7) → (–6; 0) ....

   (c) (3; 11) → (16; 20) ....

   (d) (–1; –2) → (5; –4) ....

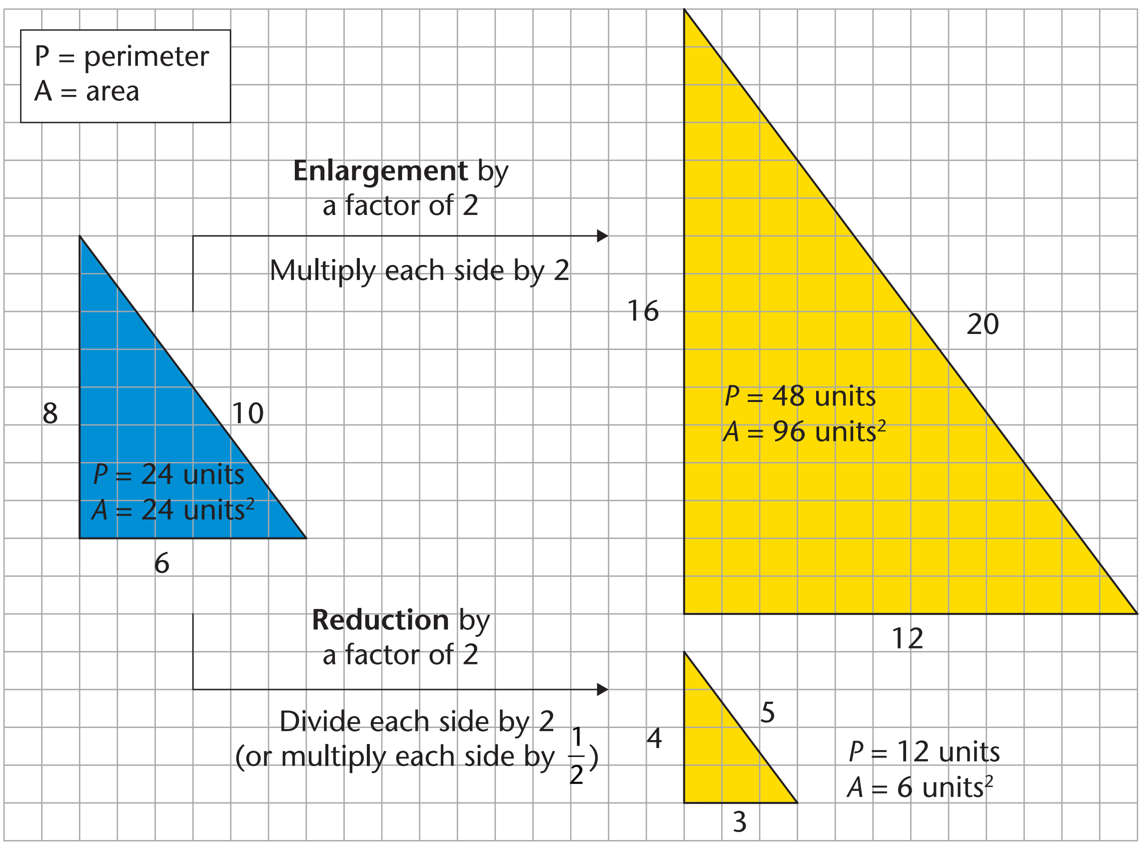
   (e) (8; –11) → (–2; –3) ....

6.4 Enlargement (expansion) and reduction (shrinking)

WHAT ARE ENLARGEMENTS AND REDUCTIONS?

You will remember the following from Grade 8.

- An image is an enlargement or reduction of the original figure only if all the corresponding sides between the two figures are in **proportion**. This means that *all the sides* of the original figure are multiplied by the same number (the **scale factor**) to produce the image.
- Scale factor = $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}}$
  - If the scale factor is > 1, the image is an enlargement.
  - If the scale factor is < 1, the image is a reduction.
- The original figure and its enlarged or reduced image are **similar.**
- Perimeter of image = Perimeter of original figure × scale factor
- Area of image = Area of original figure × (scale factor)^2



Sometimes the terminology used for enlargements and reductions can be confusing. Make sure you understand the following examples. Refer to the diagram on the previous page.

"**Enlarge** a figure by a scale factor of 2" means:

- $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}} = 2$
- Each side of the original figure must be *multiplied* by 2.
- Each side of the image will be 2 times *longer* than its corresponding side in the original figure.
- The perimeter of the image will be 2 times *longer* than the perimeter of the original figure.
- The area of the image will be 2^2 times ($2 \times 2 = 4$ times) *bigger* than the area of the original figure.

"**Reduce** a figure by a scale factor of 2" means:

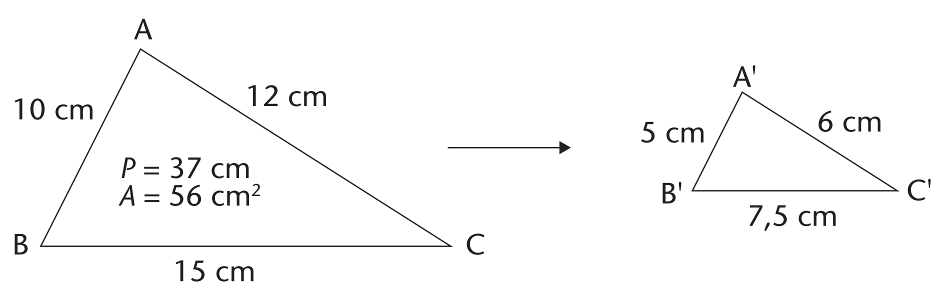
- $\frac{\text{side length of image}}{\text{length of corresponding side of original figure}} = 0{,}5$
- Each side of the original figure must be *multiplied* by $\frac{1}{2}$ (or *divided* by 2).
- Each side of the image will be 2 times *shorter* than its corresponding side in the original figure.
- The perimeter of the image will be 2 times *shorter* than the perimeter of the original figure.
- The area of the image will be 2^2 times ($2 \times 2 = 4$ times) *smaller* than the area of the original figure. (Or area of image $= (\frac{1}{2})^2 = \frac{1}{4}$ of the area of the original figure.)

> Note that the multiplicative inverse of 2 is $\frac{1}{2}$.

PRACTISE WORKING WITH ENLARGEMENTS AND REDUCTIONS

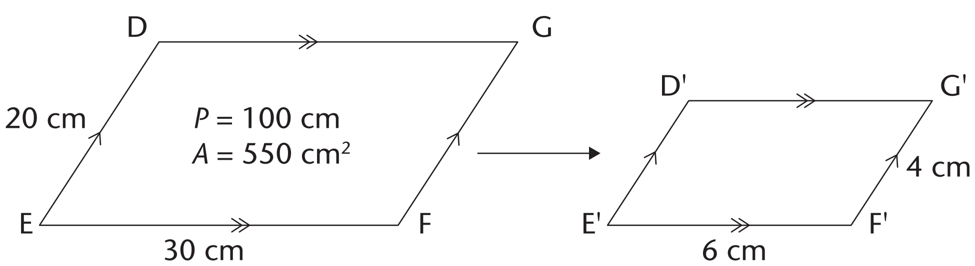
1. Work out the scale factor of each original figure and its image.

   (a)



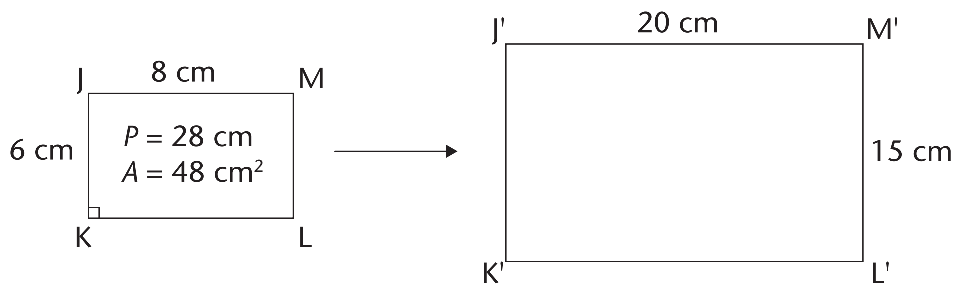
....

(b)



....

(c)



....

2. For each set of figures in question 1, write down by how many times the *perimeter* of each image is longer or shorter than the perimeter of the original image. Also write down the perimeter of each image.

   (a) ....

   (b) ....

   (c) ....

3. For each set of figures in question 1, write down by how many times the *area* of each image is bigger or smaller than the area of the original image. Also write down the area of each image.

   (a) ....

   (b) ....

   (c) ....

4. The perimeter of rectangle DEFG = 20 cm. Write down the perimeter of the rectangle D'E'F'G' if the scale factor is 3. ....

5. The perimeter of quadrilateral PQRS = 30 cm and its area is 50 cm^2.

   (a) Find the perimeter of P'Q'R'S' if the scale factor is $\frac{1}{5}$. ....

   (b) Determine the area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$. ....

6. The perimeter of ΔDEF = 17 cm and the perimeter of $\Delta D'E'F'$ = 25,5 cm.
   (a) What is the scale factor of enlargement? ....
   (b) What is the area of $\Delta D'E'F'$ if the area of $\Delta DEF = 14\ cm^2$? ....

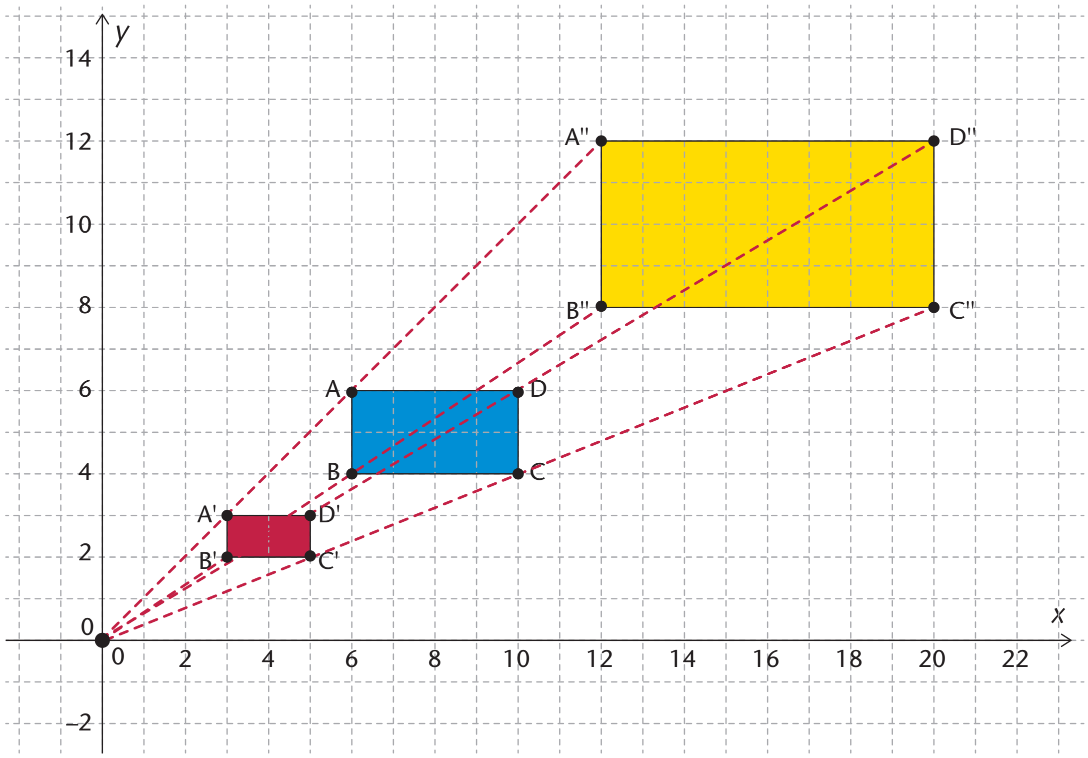
7. The area of $\Delta ABC = 20\ cm^2$ and the area of $\Delta A'B'C' = 5\ cm^2$.
   (a) What is the scale factor of reduction? ....
   (b) What is the perimeter of the image if the perimeter of ΔABC = 22 cm? ....

INVESTIGATING ENLARGEMENT AND REDUCTION

When we do enlargements or reductions on a coordinate system, we use one point from which to perform the enlargement or reduction. This point is known as the **centre of enlargement or reduction.**

The centre of enlargement or reduction can be any point on the coordinate system. In this chapter, we will always use the **origin** as the centre of enlargement or reduction.

Rectangle ABCD, rectangle A'B'C'D' and rectangle A"B"C"D" are plotted on a coordinate system as shown below.



1. (a) Is rectangle A"B"C"D" an enlargement of rectangle ABCD? Explain your answer.

   ....

(b) Is rectangle A'B'C'D' a reduction of rectangle ABCD? Explain your answer.

....

2. (a) The origin is the centre of enlargement and reduction. Draw four line segments to join the origin with A'', B'', C'' and D''.
   (b) What do you notice about these line segments?

....

....

3. (a) List the coordinates of the images to complete the following table

Vertices of ABCD	Vertices of A'B'C'D'	Vertices of A''B''C''D''
A(6; 6)		
B(6; 4)		
C(10; 4)		
D(10; 6)		

   (b) What do you notice about the coordinates of the vertices of the original rectangle and the coordinates of the vertices of the image?

....

....

From the previous activity, you should have found the following:

On a coordinate system, the line that joins the centre of an enlargement or reduction to a vertex of the original figure also passes through the corresponding vertex of the enlarged or reduced image.

The coordinates of a vertex of the enlarged or reduced image are equal to the scale factor × the coordinates of the corresponding vertex of the original figure.

For example:

B(6; 4) → B' (3; 2): The coordinates of B' are $\frac{1}{2}$ the coordinates of B. Note that the scale factor is $\frac{1}{2}$.

B(6; 4) → B'' (12; 8): The coordinates of B'' are 2 times the coordinates of B. Note that the scale factor is 2.

In general, we therefore use the following notation to describe the enlargement or reduction with respect to the origin:

$(x; y) \rightarrow (kx; ky)$ or $(x'; y') = (kx; ky)$ where k is the scale factor.

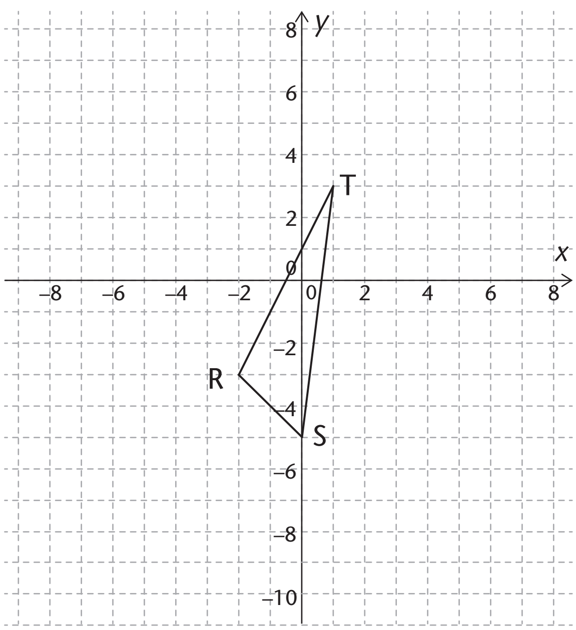
If $0 < k < 1$, the image is a reduction.

If $k > 1$, the image is an enlargement.

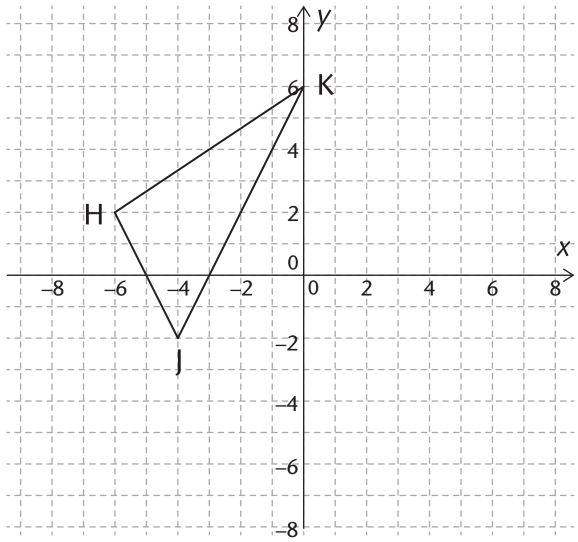
PRACTISE

1. Draw the enlarged or reduced images of the following figures according to the scale factor given. In each case, use the **origin as the centre of enlargement or reduction.**

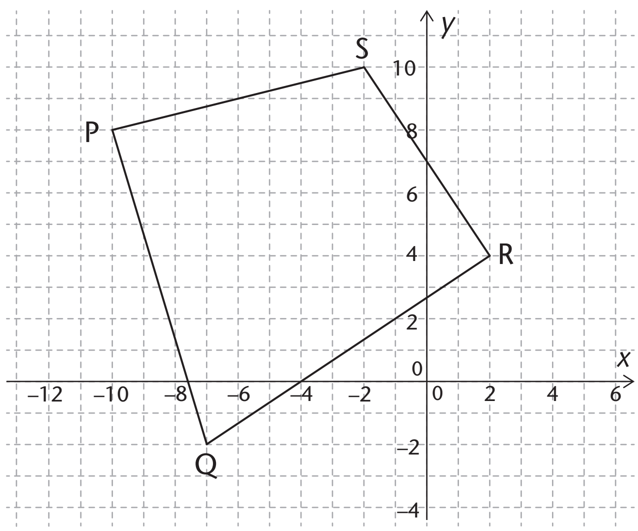
   (a) Scale factor = 2



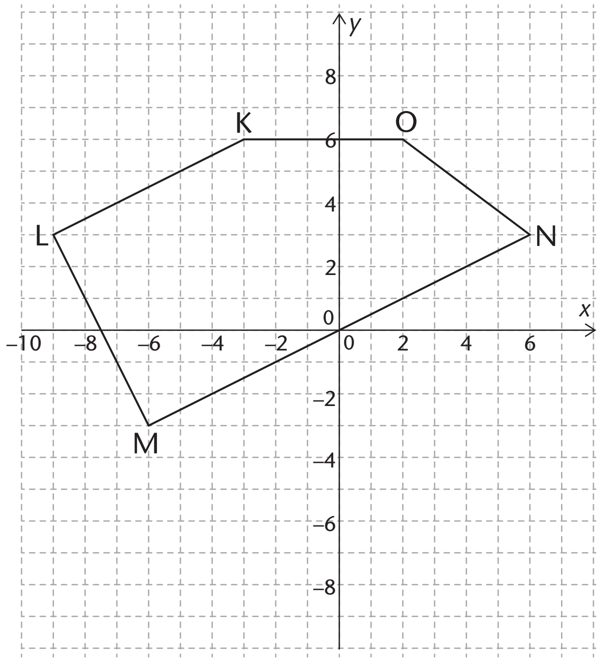
(b) Scale factor = $\frac{1}{2}$



(c) Scale factor = $\frac{1}{2}$



(d) Scale factor = $\frac{1}{3}$



2. A quadrilateral has the following vertices: A(–2; 4), B(–4; –2), C(4; –3) and D(2; 1). Determine the coordinates of the enlarged image if the scale factor = 2.

....

3. A quadrilateral has the following vertices: P(–4; 0), Q(2,5; 4,5), R(6; –2,25) and S(2; –4). Determine the coordinates of the enlarged image if the scale factor = 4.

....

4. A quadrilateral has the following vertices: D(6; –4), E(4; –6), F(–4; 2) and G(–2; –2). Determine the coordinates of the reduced image if the scale factor = $\frac{1}{2}$.

....

5. A quadrilateral has the following vertices: K(8; –2), L(4; –6), M(–8; –4) and N(–6; 10). Determine the coordinates of the reduced image if the scale factor = $\frac{1}{4}$.

....

6. Describe the following transformations:
   (a) A(7; –5) → A'(9; 0)

....

(b) A(–4; 6) → A'(4; 6)

....

(c) A(–3; –2) → A'(–2; –3)

....

(d) A(8; 1) → A'(8; –1)

....

(e) A(4; –2) → A'(8; –4)

....

(f) A(12; –16) → A'(3; –4)

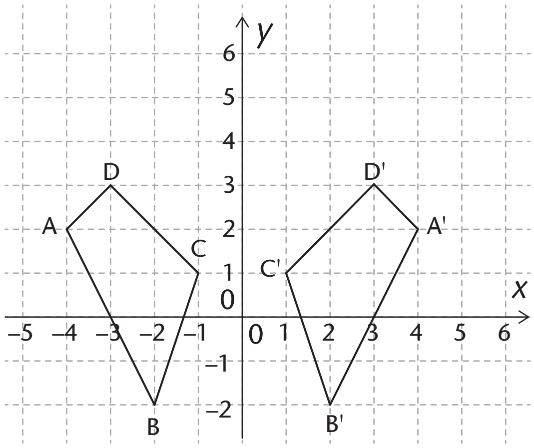
....

(g) A(2; –1) → A'(–3; –5)

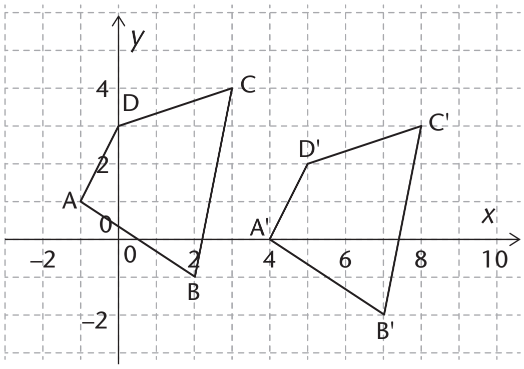
....

7. Describe each of the following transformations.

(a)



(b)



....

....

Chapter 7
Geometry of 3D objects

In this chapter, you will revise the properties of prisms and pyramids, which you investigated in previous grades. This includes using nets to construct models of these objects as a further means of consolidating your understanding of polyhedra. You will also revise the properties and definitions of the five Platonic solids, which you first learnt about in Grade 8, as well as how Euler's formula describes a relation between the number of vertices, faces and edges of any polyhedron.

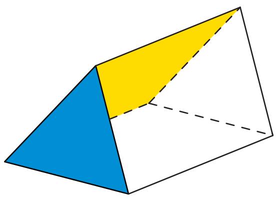
New to this grade are investigations of the properties of cylinders and spheres. Although you should be able to recognise these 3D objects by now, you will examine some of their properties in more detail, and learn how to construct a net and model of a cylinder.

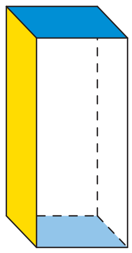
7 Geometry of 3D objects

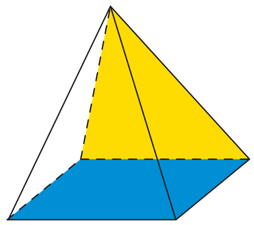
7.1 Classifying 3D objects

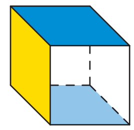
3D objects with flat faces which are called **polyhedra**. Prisms and pyramids are two types of polyhedra.

A polyhedron is a 3D object with only flat faces.

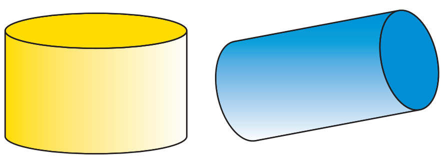

Triangular prism

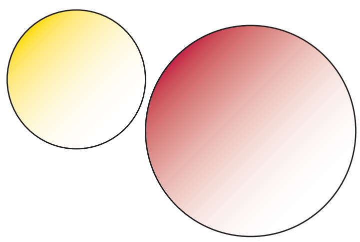

Rectangular prism

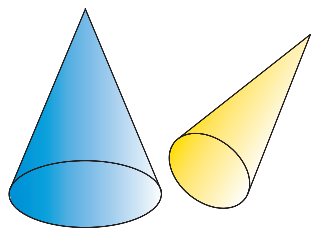

Rectangular-based pyramid


Cube

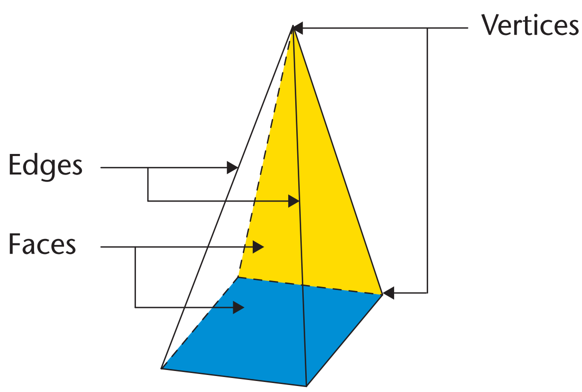
Examples of 3D objects that have at least one curved surface are **cylinders, spheres** and **cones.**


Cylinders


Spheres


Cones

When we study the properties of a 3D object, we investigate the shapes of its faces, its number of faces, its number of vertices and its number of edges. For example, the pyramid alongside has 1 square face and 4 triangular faces, 5 vertices and 8 edges.



Square-based pyramid

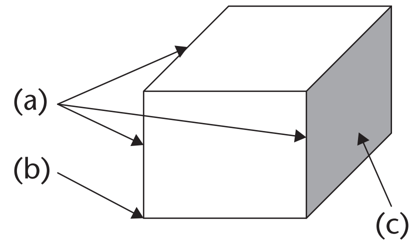
CLASSIFYING AND DESCRIBING 3D OBJECTS

1. Label parts (a) to (c) on the prism correctly.

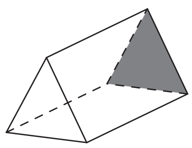
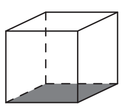
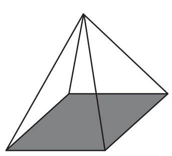
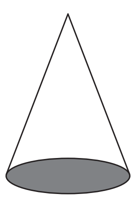
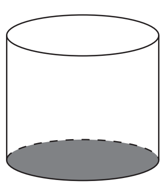
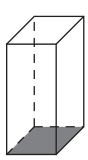
   (a) ....

   (b) ....

   (c) ....



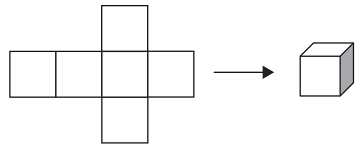
2. Complete the table.

	3D object	Name of the object	Number of faces and shape of faces	Number of vertices
(a)		Triangular prism	2 triangles and 3 rectangles	6
(b)				
(c)			6 squares	8
(d)			1 rectangle and 4 triangles	5
(e)				
(f)				
(g)				

3. Say whether each statement below is true or false.
   (a) A cylinder is a polyhedron. ....
   (b) A triangular-based pyramid has 4 triangular faces. ....
   (c) A cube is also known as a hexahedron. ....
   (d) A triangular-based pyramid has 6 vertices. ....
   (e) A pyramid is a 3D object. ....

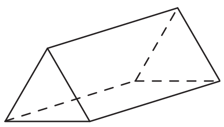
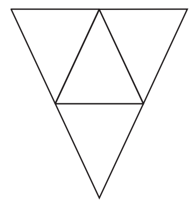
7.2 Nets and models of prisms and pyramids

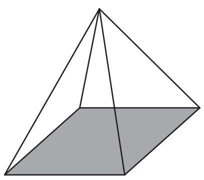
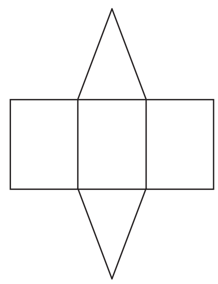
A **net** is a flat pattern that can be used to represent a 3D object. The net can be folded up to create a model of the 3D object.

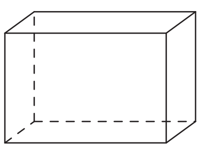
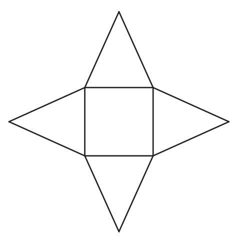


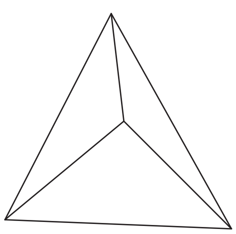
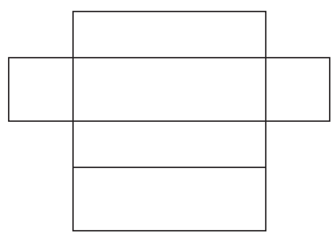
Net of a cube

1. Name each object below and draw an arrow to match it with its net.

(a)  .... 

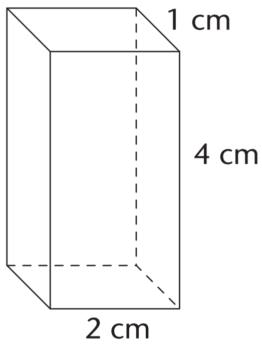
(b)  .... 

(c)  .... 

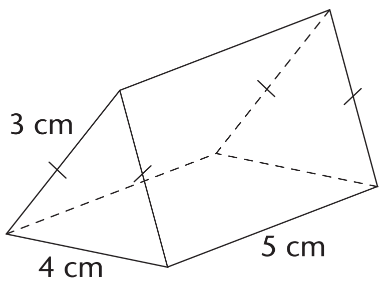
(d)  .... 

2. Construct an accurate net for each of the following 3D objects.

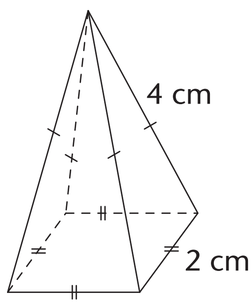
(a)



(b)



(c)

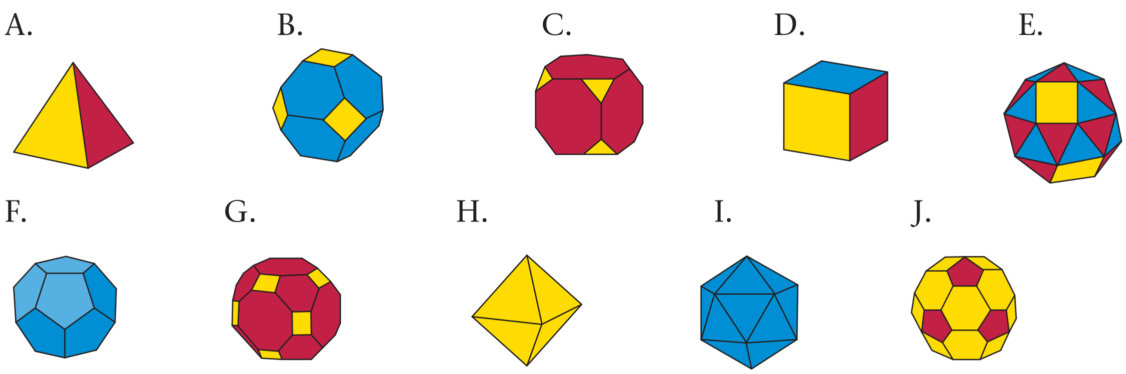


3. Construct models of the objects in question 2 but double all the measurements.

7.3 Platonic solids

A **Platonic solid** is a 3D object which has identical faces, and all of the faces are identical regular polygons. This means that all its faces are the same shape and size and all the vertices are identical.

1. Which of the following objects are Platonic solids?

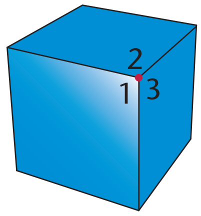


....

2. How many Platonic solids are there in question 1? ....

ONLY FIVE PLATONIC SOLIDS?

You can use your knowledge about angles to prove that the five Platonic solids are the only 3D objects that can be made from identical regular polygons. Keep the following facts in mind:



- A 3D object has *at least* three faces that meet at each vertex.
- The sum of the angles that meet at a vertex must be less than 360°. If it is equal to 360°, it will form a flat surface. If it is greater than 360°, the faces will overlap.
- Each Platonic solid is made up of one type of regular polygon only.

What 3D objects can you make from equilateral triangles?

We use the following reasoning:

size of each interior angle = 180° ÷ 3 = 60°

$\therefore$ 3 triangles = 3 × 60° = 180° [< 360°]

4 triangles = 4 × 60° = 240° [< 360°]

5 triangles = 5 × 60° = 300° [< 360°]

6 triangles = 6 × 60° = 360°

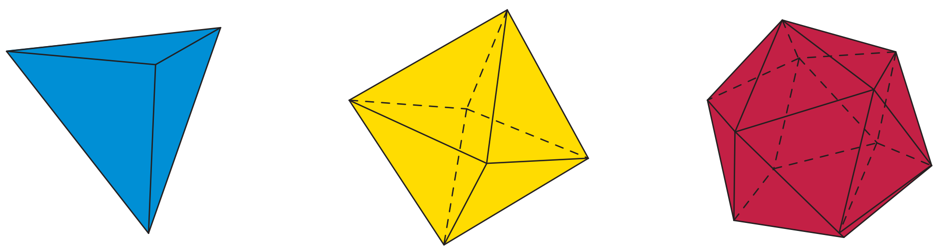
Any more than 5 triangles will be equal to or more than 360° and will therefore form a flat surface or overlap.

This means that we can make three 3D objects from equilateral triangles:

If 3 triangles are at each vertex, it will form a **tetrahedron**.

If 4 triangles are at each vertex, it will form an **octahedron**.

If 5 triangles are at each vertex, it will form an **icosahedron**.



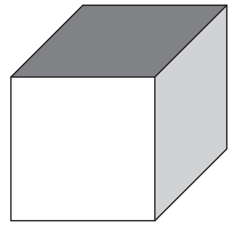
Tetrahedron Octahedron Icosahedron

What 3D objects can you make from squares?

Complete the statements: size of each interior angle ....

∴ 3 squares = 3 × ....

4 squares = 4 × ....



Hexahedron (cube)

Therefore we can make only one 3D object using squares. This 3D object is called a **hexahedron (or cube).**

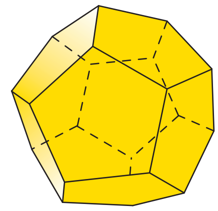
What 3D objects can you make from regular pentagons?

Complete the statements:

Size of each interior angle ....

∴ 3 pentagons = ...

4 pentagons = ...



Dodecahedron

Therefore we can make only one 3D object using regular pentagons. This 3D object is called a **dodecahedron.**

What 3D objects can you make from regular hexagons?

Complete the statements:

Size of each interior angle ....

∴ 3 hexagons = ...

Three hexagons will already form a flat surface. Therefore it is impossible to make a 3D object from regular hexagons.

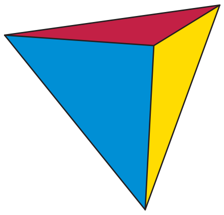
Also, the interior angles of polygons with more than 6 sides are bigger than those of a hexagon, so it is not possible to make 3D objects from any other regular polygons.

> Therefore the five Platonic solids already mentioned (tetrahedron, octahedron, icosahedron, hexahedron and dodecahedron) are the only ones that can be made of identical regular polygons. Each of these solids is named after the number of faces it has.

PROPERTIES OF THE PLATONIC SOLIDS

Complete the information about each of the following Platonic solids.

1.



Name: ....

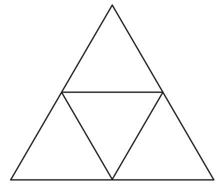
....

Shape of the faces: ....

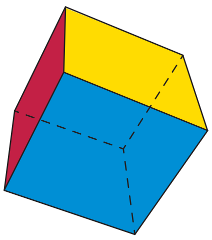
Number of faces: ....

Number of edges: ....

Number of vertices: ....



2.



Name: ....

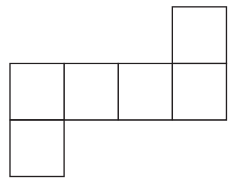
....

Shape of the faces: ....

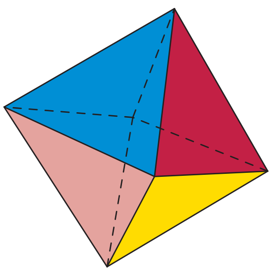
Number of faces: ....

Number of edges: ....

Number of vertices: ....



3.



Name: ....

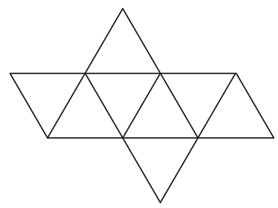
....

Shape of the faces: ....

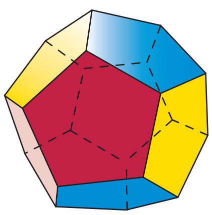
Number of faces: ....

Number of edges: ....

Number of vertices: ....



4.



Name: ....

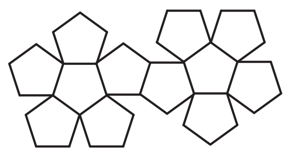
....

Shape of the faces: ....

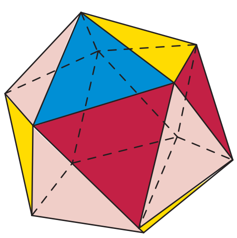
Number of faces: ....

Edges: ....

Vertices: ....



5.



Name: ....

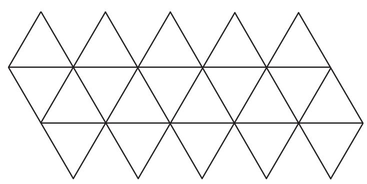
....

Shape of the faces: ....

Number of faces: ....

Edges: ....

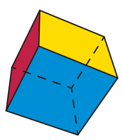
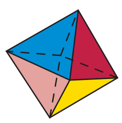
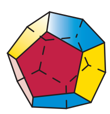
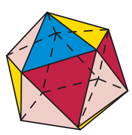
Vertices: ....



7.4 Euler's formula

EULER'S FORMULA AND PLATONIC SOLIDS

1. You learnt about Euler's formula in Grade 8. Complete the following table to investigate whether or not Euler's formula holds true for Platonic solids.

	Name	Shape of faces	No. of faces (F)	No. of vertices (V)	No. of edges (E)	F + V – E
						
						
						
						
						

2. Complete Euler's formula for polyhedra:

   F + ...

3. Apply Euler's formula to each of the following:
   (a) A polyhedron has 25 faces and 13 vertices. How many edges will it have?

   ....

   (b) A polyhedron has 11 vertices and 23 edges. How many faces does it have?

   ....

   (c) A polyhedron has 8 faces and 12 edges. How many vertices does it have?

   ....

EULER'S FORMULA AND OTHER POLYHEDRA

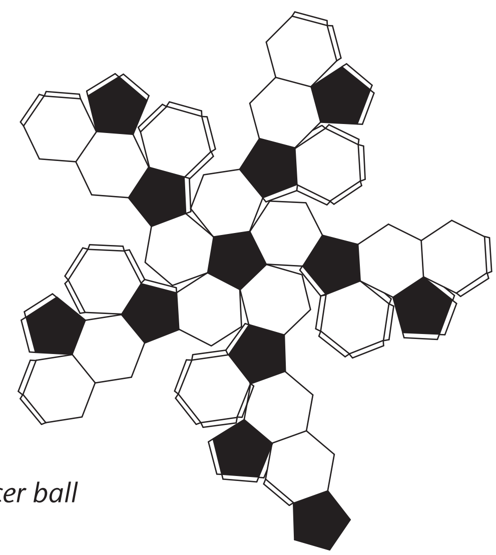
1. Is each of the following statements true or false?
   (a) A polyhedron with 10 vertices and 15 edges must have 7 faces. ....
   (b) A polyhedron will always have more edges than either faces or vertices. ....
   (c) A polyhedron with 5 faces must have 6 edges. ....
   (d) A pyramid will always have the same number of faces and vertices. ....

2. Complete the following table.

	No. of faces (F)	No. of vertices (V)	No. of edges (E)	Name of polyhedron	Shapes of faces
(a)	6		12		Rectangles
(b)		7		Hexagonal pyramid	
(c)	4	4			
(d)	5	6	9		Triangles and rectangles

3. A soccer ball consists of pentagons and hexagons.
   (a) How many pentagons does it consist of? ....
   (b) How many hexagons does it consist of? ....
   (c) How many edges does it have? ....
   (d) How many vertices does it have? ....
   (e) Does Euler's formula apply to soccer balls too?

   ....

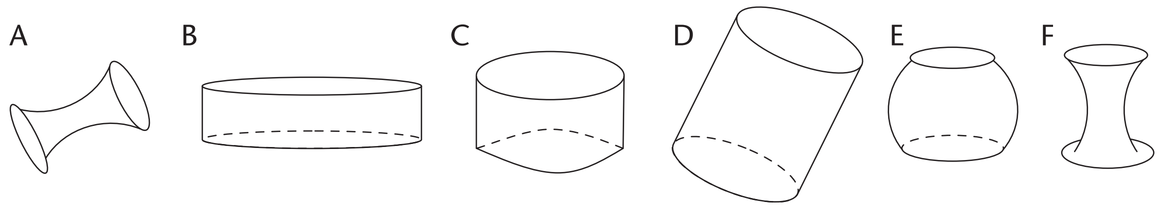


Net of a soccer ball

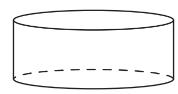
7.5 Cylinders

PROPERTIES OF CYLINDERS

1. Which of the following 3D objects are cylinders? ....

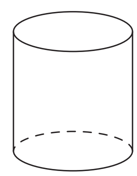


2. Tick the statement or statements below that are true only for cylinders and not for the other objects shown in question 1:



- ☐ It is a 3D object.
- ☐ It has a curved surface.
- ☐ It has two circular bases that are parallel to each other.
- ☐ It has two flat circular bases and a curved surface.
- ☐ The radius of its curved surface is equal from the top to the bottom between the bases.
- ☐ It has two circular bases opposite each other, joined by a curved surface whose radius is equal from the top to the bottom between the bases.

3. Look at the cylinder alongside and complete the following:



   (a) Number and shape of faces: ....

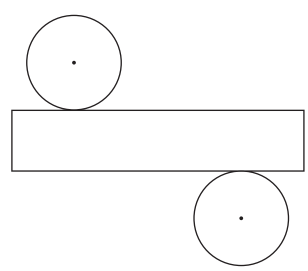
   (b) Number of vertices: ....

   (c) Number of edges: ....

NETS OF CYLINDERS

In Chapter 5, you learnt about the net of a cylinder. If you cut the curved surface of a cylinder vertically and flatten it, it will be the shape of a rectangle.

1. Explain why the length of the rectangular face is equal to the circumference of the base.



....

....

....

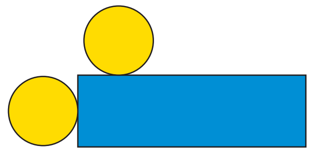
....

2. Will each of the following nets form a cylinder?

A.

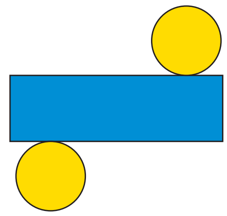
……………………

B.



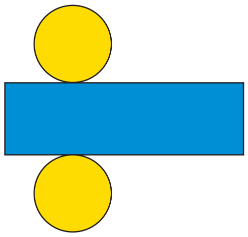
……………………

C.



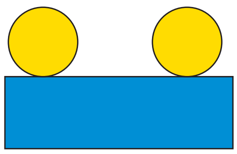
……………………

D.



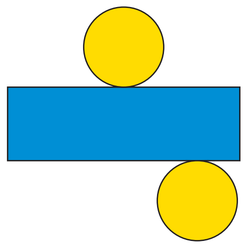
……………………

E.



……………………

F.



……………………

3. In each of the following questions, use $\pi = \frac{22}{7}$ and round off your answer to two decimal places to do the calculations.

(a) If the radius of a cylinder is 3 cm, what is the length of the rectangular surface of the cylinder?

…………………………………………………………………………

(b) If the radius of a cylinder is 5 cm, what is the length of the rectangular surface of the cylinder?

…………………………………………………………………………

(c) If the diameter of a cylinder is 8 cm, what is the length of the rectangular surface of the cylinder?

…………………………………………………………………………

(d) If the diameter of a cylinder is 9 cm, what is the length of the rectangular surface of the cylinder?

…………………………………………………………………………

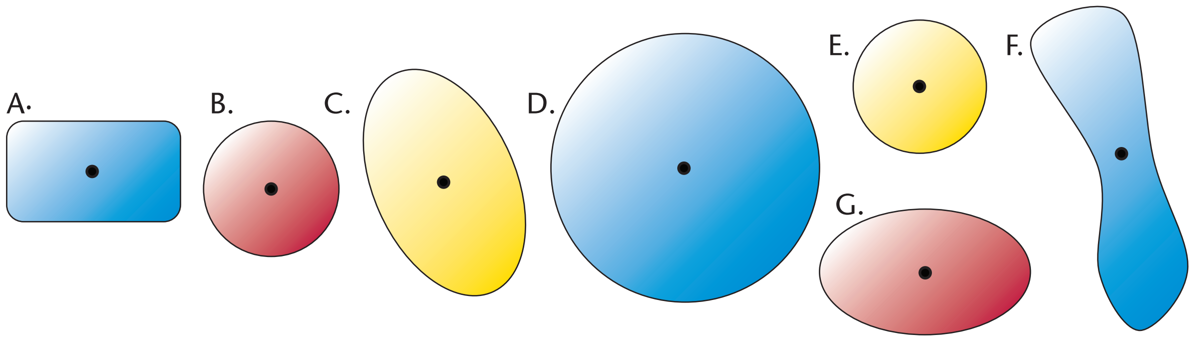
4. Use a ruler and a set of compasses to construct the following nets as accurately as possible. Show the measurements on each net.
   (a) Net of a cylinder with a radius of 1 cm and a height of 4 cm.

   (b) Net of a cylinder with a radius of 1,5 cm and a height of 3 cm

5. Construct models of the cylinders in question 5 but double the measurements.

7.6 Spheres

1. Which of the following 3D objects are spheres?

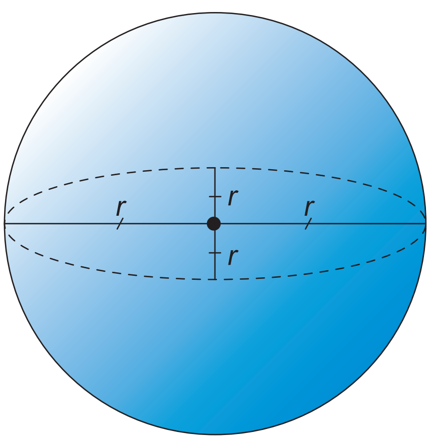


. . . .

2. Tick the property or properties below that are true for spheres only and not for the other objects shown in question 1.
   - ☐ It is a 3D object.
   - ☐ It has one curved surface.
   - ☐ It has no bases.
   - ☐ It has no vertices.
   - ☐ It has no edges.
   - ☐ The distance from its centre to any point on its surface is always equal.

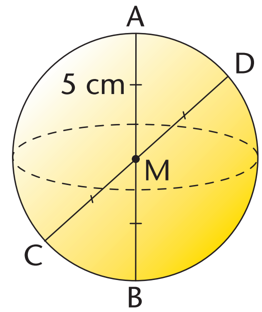
3. Complete the following information for a sphere:
   (a) Number and shape of faces: . . . .
   (b) Number of vertices: . . . .
   (c) Number of edges: . . . .

From your study of spheres in the above activity, you should have found the following:



A **sphere** is a round 3D object with only one curved surface and the distance from its centre to any point on its surface is always equal. It has no vertices or edges.

4. In the sphere alongside, write down the length of:
   (a) the radius: ....
   (b) the diameter: ....
   (c) MD: ....
   (d) CD: ....



5. The drawing alongside shows part of a sphere with a diameter of 100 km. Imagine that you are at point M, at the centre inside the sphere. People A, B and C are all at different places on the surface of the sphere.
   (a) Which of the people – A, B or C – is closest to you?

   ....

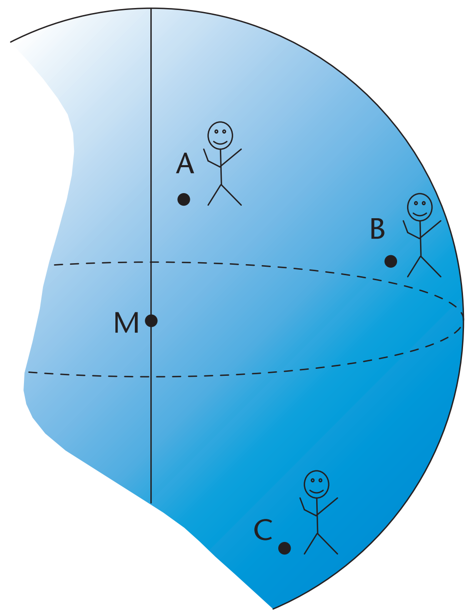
   ....

   ....

   (b) How far away is person C from you?

   ....

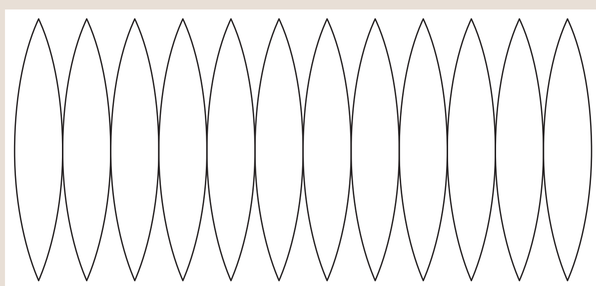
   ....



NET OF A SPHERE

It is impossible to make a perfect sphere (ball or globe) from a flat sheet of paper. Paper can curve in one direction, but cannot curve in two directions at the same time. So all spheres made from paper or card will be approximations. This is the best net we can make of a sphere.

Can you make your own paper model of a sphere?



WORKSHEET

1. Grade 9 learners were asked to represent a 3D object and give the class clues as to which polyhedron they represent. Name their objects:

   (a) Amy: I have 6 faces and they are all the same size. ....

   (b) John: I have 6 faces and 12 edges. I am not a cube. ....

   (c) Onke: I have 3 faces. I also have two edges. ....

   (d) Tessa: I have 8 edges and I have 5 vertices. ....

   (e) Mandlakazi: I have 6 edges and 4 vertices. ....

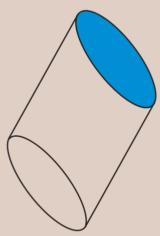
   (f) Chiquita: I have 8 faces and am a Platonic solid. ....

   (g) Seni: I do not have any edges. ....

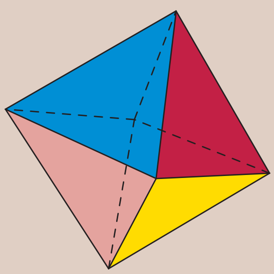
   (h) Mpu: My faces are made only of regular pentagons. ....

2. Write down the required information about each object below.

   A.



   B.



	Object A	Object B
Name		
Number of faces		
Shape/s of faces		
Number of edges		
Number of vertices		
Does Euler's formula work?		
Is it a Platonic solid?		

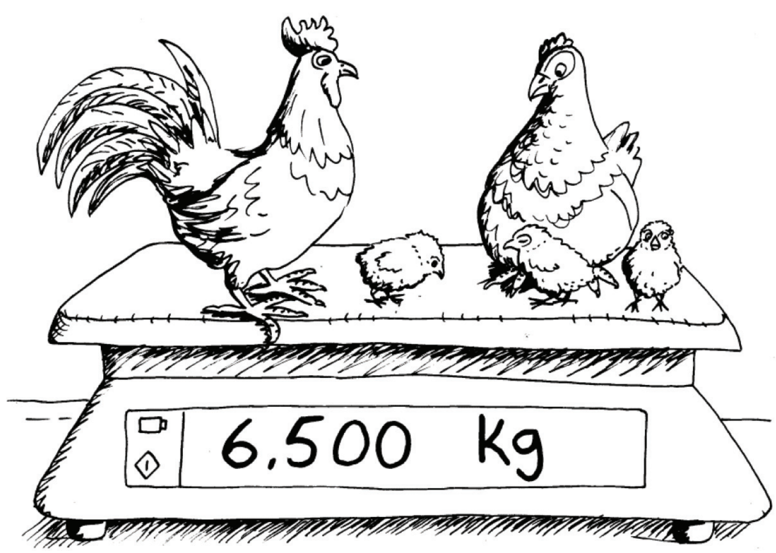
3. (a) On a separate sheet of paper, construct a net of a cylinder with a diameter of 7 cm and a height of 10 cm.

   (b) Fold your net to make a model of the cylinder.

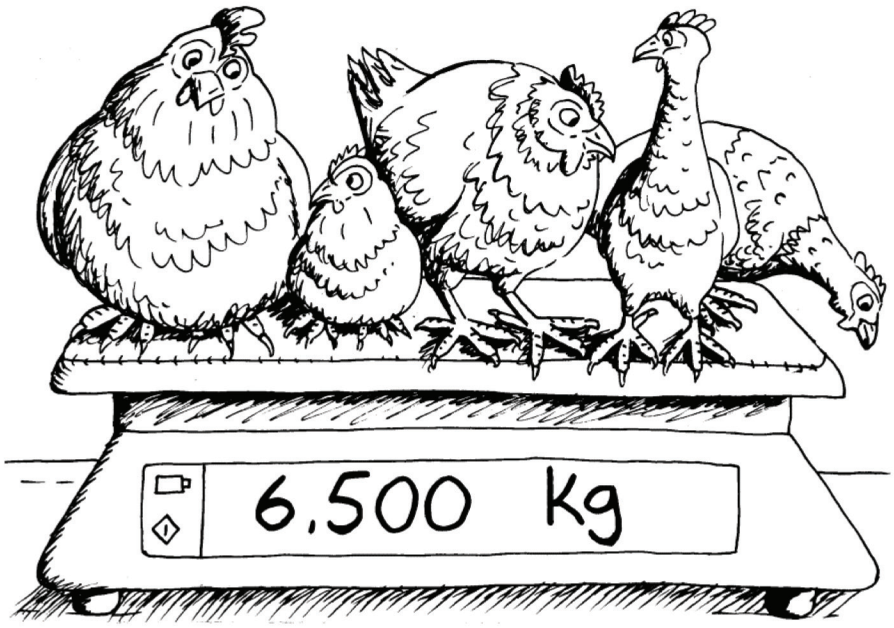
Maths2_Gr9_LB_Book.indb 126 2014/09/08 09:07:05 AM

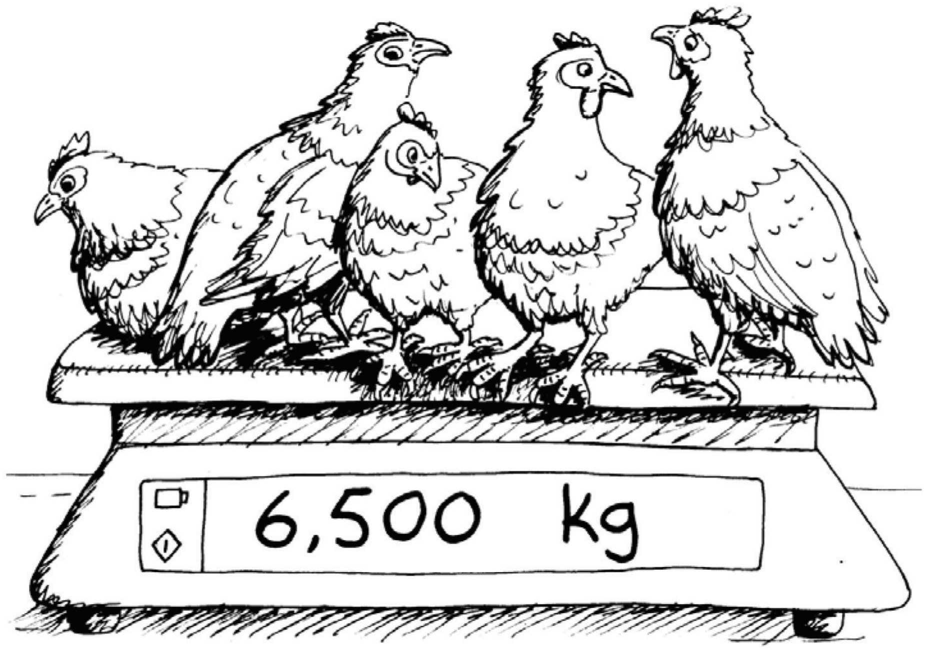
Chapter 8
Collect, organise and summarise data

You have learnt how to collect, organise and summarise data in previous grades. In Grade 9, you need to decide which methods are best in certain situations and you need to be able to justify your choices.



Maths2_Gr9_LB_Book.indb 127 2014/09/08 09:07:05 AM


6,500 Kg


6,500 kg

8 Collect, organise and summarise data

8.1 Collecting data

Avoiding bias when selecting a sample

The methods that we use to collect data must help us to make sure that the data is reliable. This means that it is data that we can trust.

Data cannot be trusted unless it has been collected in a way that makes sure that every member of the population had the same chance of being selected in the sample.

It is not practical to taste all the oranges on a tree to know whether the oranges are sweet. Only a small number of oranges can be tested, otherwise the farmer would have too few oranges to sell. The oranges that are tested are called a **sample**, and all the oranges harvested from the tree are called the **population**.

Sample bias occurs when the particular section of the population from which the sample is drawn does not represent that population. The way to avoid sample bias is to take a **random** sample. A sample is random if **every member of the population has the same chance** of being selected. A random sample of the orange trees means that every tree should have a chance of being selected for the sample. Every person in the country should have a chance of being selected for the housing survey in a random sample.

An example of sample bias would be to survey only the people in Limpopo about their views on housing provision when you want to know the views of the whole country. For the sample to provide information on the population as a whole, each person in the country should have the same chance of being part of the survey.

Data can be collected through questionnaires, through observation and through access to databases.

How to develop a good questionnaire

The questionnaire also has an important role in making sure that the information you collect is reliable. You should aim to get a high number of respondents and accurate information. If not enough people fill in the questionnaire, then you don't know whether the information you get reflects the real situation. Sampling techniques and rules developed by statisticians determine the numbers needed.

There are some important points to consider when designing a questionnaire. Two of the most important points are that the questions are **clear and accurate** and that people find the questionnaire relatively **easy to complete**.

1. Keep in mind the length of the questionnaire and the time that it takes to complete. Your participants will more likely complete a short questionnaire that is quick and easy to complete. Exclude information that is not needed.

2. Write down a selection of questions that you think will provide the information that you want.

3. Check the wording for each question.

4. Order the items so that they are in a logical sequence. It might make sense to have the easiest questions first but in some cases the more general questions should come first and the more specific questions towards the end of the questionnaire.

5. Then try the questionnaire out on a partner. Ask the following questions:
   - Is this question necessary? What information will be provided by the answer?
   - How easy will it be for the respondent to answer this question? How much time will it take to answer the question?
   - Do the questions ask for sensitive information? Will people want to answer the question? Will the respondent answer the question honestly?
   - Can the question be answered quickly?

6. Decide how the answers should be provided. Questions may require **open-ended** responses or **closed-ended** responses, as described blow.

In an **open-ended** question, the person responds in his or her own words. Through his, or her, own words important information can be gained; the person is free to write what they like. A disadvantage is that you might not get the information you want and that it might take a long time to answer.

In a **closed-ended** question the respondents are given some options to choose from. They tick the box which most closely represents their response. These options can be constructed in categories. For example age may be categorised as follows:

Under 10 ☐ From 10 to 14 ☐ From 15 to 19 ☐ 20 and older ☐

THINK ABOUT DATA COLLECTION AND DEVELOP A QUESTIONNAIRE

1. Which method for collecting data would be most appropriate for each of the cases below? Give reasons for your choice.
   (a) The number of learners who bring lunch to schools. What are the contents of the school lunch?

   ....

   ....

   ....

   (b) Whether the tellers at a supermarket chain are happy with their conditions of work.

   ....

   ....

   ....

   (c) Whether the clients of a clinic are satisfied with the professional conduct of the medical staff.

   ....

   ....

   ....

   (d) The types of activities preschool children choose during their free time.

   ....

   ....

   ....

   (e) The number of Grade 9 learners in the Gauteng North district.

   ....

   ....

2. You are doing some market research for a new fast food shop near the high school. The owners of the shop want to find out what kind of food and music the target market likes. The target market is learners from the high school. Develop a questionnaire to collect this information, on the next page.

8.2 Organising data

There is a difference between **data** and **information**. Data is unorganised facts. When data is organised and analysed so that people can make decisions, it may be called information. Data can be organised in many different ways. Some methods are described below.

Data can be organised by making a **tally table**. Here is an example of a tally table showing the numbers of learners in a class that participate in different sports.

Sport	Tally marks
Soccer	//// //// //// //// ////
Athletics	//// ///
Netball	//// //// //// //// /
Chess	//// /

The above data can also be organized in a **frequency table**:

Sport	Frequency
Soccer	25
Athletics	8
Netball	21
Chess	6

Numerical data sets with many items are often grouped into equal **class intervals** and represented in a table of frequencies for the different class intervals. This is very useful since it makes it easy to see how the data is spread out.

Here is an example of grouped data showing the heights of all the learners in a school. **To make a frequency table for numerical data, the data has to be arranged from smallest to biggest first.**

Height in m	Number of learners (Frequency)
< 1,20 m	13
1,20 m – 1,30 m	28
1,30 m – 1,40 m	57
1,40 m – 1,50 m	164
1,50 m – 1,60 m	274
1,60 m – 1,70 m	198
1,70 m – 1,80 m	73
> 1,80 m	13

A value equal to the **lower boundary** of a class interval is counted in that interval. For example a length of 1,60 m is counted in the interval 1,60 – 1,70, and not in the interval 1,50 – 1,60 m.

However, 1,599 m is less than 1,60 m, so it belongs in the interval 1,50 m – 1,60 m.

A **stem-and-leaf display** is a useful way to organise numerical data. It also shows you what the "shape" of the data is like. Here is an example of a stem-and-leaf display.

Key: 35 | 4 means 354

34	0 4
35	4 8 8
36	0 1 6 8
37	1 3 5 8 8 8 9
38	2 4 9
39	0 3 4 4 5 6 9
40	0 3 7
41	1

The above stem-and-leaf display represents the following data about the masses in grams of the chickens in a sample of 6-week-old chickens on a chicken farm.

399	378	382	360	396	389	344	411	378	394
394	354	375	378	400	371	379	358	366	403
358	395	390	340	393	384	361	407	373	368

To make a stem-and-leaf display, it helps to first arrange the data from smallest to largest, as shown here for the above data set.

340	344	354	358	358	360	361	366	368	371
373	375	378	378	378	379	382	384	389	390
393	394	394	395	396	399	400	403	407	411

The same data set is displayed in two slightly different ways below.

			379		399		
			378		396		
			378		395		
		368	378		394		
	358	366	375	389	394	407	
344	358	361	373	384	393	403	
340	354	360	371	382	390	400	411

In this display the width of each class interval is 10, as in the stem-and-leaf display above.

		384		
		382	399	
		379	396	
	368	378	395	
	366	378	394	
	361	378	394	411
354	360	375	393	407
344	358	373	390	403
340	358	371	389	400

In this display the width of each class interval is 15.

WORKING WITH GROUPED DATA

1. An organisation called Auto Rescue recorded the following numbers of calls from motorists each day for roadside service during March 2014.

28	122	217	130	120	86	80	90	120	140
70	40	145	187	113	90	68	174	194	170
100	75	104	97	75	123	100	82	109	120
81									

Set up a tally and frequency table for this set of data values, in intervals of width 40.

2. When geologists go out into the field they make sure they have their rulers and measurement instruments in their bags. They also have their "inbuilt rulers", for example their handspans. A handspan is the distance from the tip of the thumb to the tip of the fifth finger on an outstretched hand. Measure your handspan against the ruler! This frequency table shows the handspans of different Grade 9 learners, in cm.

Handspan of Grade 9 learners in cm	Frequency
15–18	7
18–21	9
21–24	10
24 and greater	4

(a) How many learner handspans were measured altogether?

....

(b) How many learner handspans are less than 21cm wide?

....

(c) How many handspans are 18 cm or wider?

....

(d) In which interval would you place a handspan of 18 cm?

....

8.3 Summarising data

The mean, median, mode and range are single numbers that provide some information about a data set, without listing all the data values.

The **mode** is the value that occurs most frequently. To find the mode, look for the number or category that is listed in the data set most often. Some data sets have more than one mode, and some may have none.

The **median** is the number that separates the set of values into an upper half and a lower half. The median can be found by arranging the values from small to big or big to small. If the data set consists of an even number of items, the median is the sum of the two middle values divided by 2.

The **mean** (average) of a set of numerical data is the sum of the values divided by the number of values in the data set.

Mean = the sum of the values ÷ the number of values.

The **range** is a number that tells us how spread out the data values are. It is the difference between the largest and smallest values.

The mean, median and mode don't work equally well for all sets of data. It depends on the kind of data, and also on whether the data is evenly spread out or not.

ORGANISE, SUMMARISE AND COMPARE SOME DATA

1. A researcher analyses data about the people who are suffering from three different types of the flu virus: A, B and C. The ages of the people in the different groups are:

   Type A: 60, 80, 75, 87, 88, 49, 94, 84, 59, 86, 82, 62, 79, 89 and 78.

   Type B: 27, 39, 43, 29, 36, 70, 56, 25, 54, 36, 66, 45, 33, 46, 14 and 41.

   Type C: 33, 48, 64, 15, 31, 20, 70, 21, 18, 49, 21, 19, 57, 23, 29 and 20.

For each group:

- Draw a stem-and-leaf plot.
- Calculate the range, mean and median of the ages.
- Look at the shape of the stem-and-leaf displays as well as the summary measures. Discuss the spread of the data in each case, and compare the three different groups.

Work and report on your work below and on the next page.

....

....

Type A:

....

....

....

....

Type B:

....

....

....

....

Type C:

....

....

....

....

....

2. Fill in the statistic (mode, mean or median) that would best summarise each data set, and indicate the central tendency of the data.

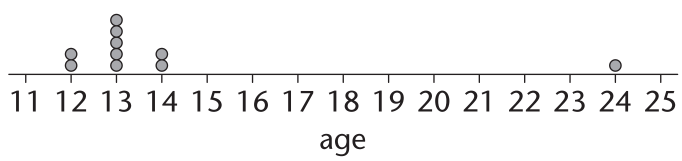
Data set	Best measure of central tendency
The shoe sizes of boys in Grade 9	
An evenly-spread set of measurement values, such as the heights of learners in a class	
A set of measurement values with a few very low values and mostly high values	
The number of siblings each person in your class has	
The sizes of properties in a town, where most people live in small apartments or RDP houses, and a few live on large properties	

EXTREME VALUES AND OUTLIERS

An **extreme value or outlier** is a data value that lies an abnormal distance from other values in a random sample from a population. Sometimes there are reasons why this data value is so different to the others. It is important to comment on the possible reasons.

When you are summarising data (and also when you analyse data), you need to decide whether an outlier makes sense in the context you are looking at.

It is possible that an outlier does not make sense, as it lies too far away and is an unreasonable measurement. Then you need to think about the fact that this data value may be an error. For example:



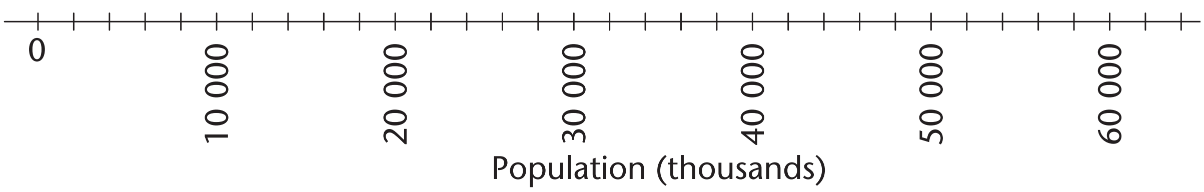
In this case, the value of 24 years old could be an unreasonable value. This depends on the context of the survey.

You will learn more about extreme values and outliers in Chapter 10.

Use this information about 14 countries to answer the questions that follow.

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Angola	18 498	4 830	4,6
Botswana	1 950	13 310	10,3
DRC	66 020	280	2,0
Lesotho	2 067	1 970	8,2
Malawi	15 263	810	6,2
Mauritius	1 288	12 580	5,7
Mozambique	22 894	770	5,7
Namibia	2 171	6 250	5,9
Seychelles	84	19 650	4,0
South Africa	50 110	9 790	8,5
Swaziland	1 185	5 000	6,3
Tanzania	43 739	1 260	5,1
Zambia	12 935	1 230	4,8
Zimbabwe	12 523	170	Not available

1. Look at the total population for each country.
   (a) Calculate the mean of the data.
   (b) Draw a dot plot on the number line below to show the data.



   (c) Find the median of the data.

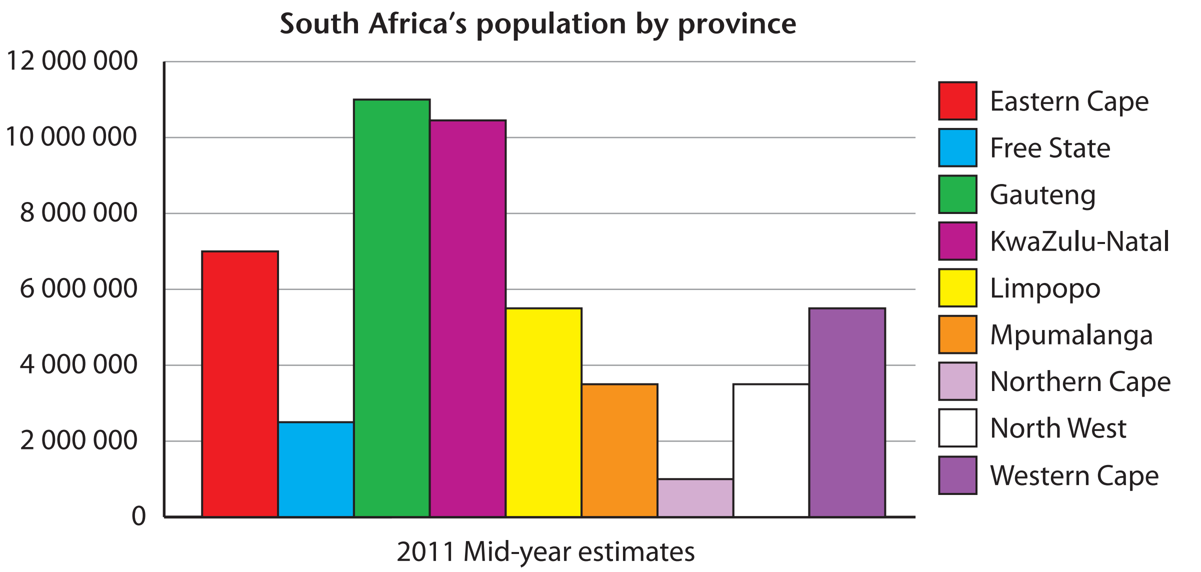
   (d) What is the range of the data?

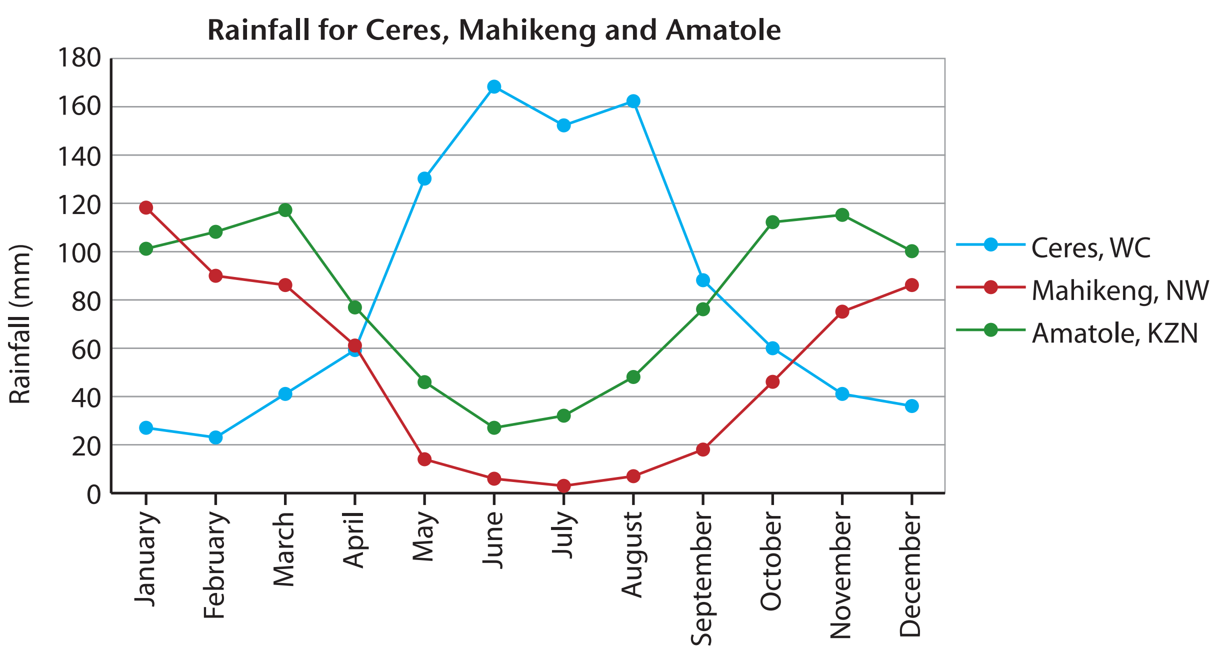
   (e) Which measure of central tendency do you think represents the data more accurately? Explain.

2. Look at the *Total annual national income per person in US dollars*. Comment on the spread of the data.

Chapter 9
Representing data

In the previous chapter, you focused on methods of collecting, organising and summarising data. Now we focus on representing data in bar graphs, double bar graphs, histograms, pie charts and broken-line graphs. You will practise drawing these graphs. You will also decide why a certain kind of graph is useful in a particular context.


South Africa's population by province
12 000 000
10 000 000
8 000 000
6 000 000
4 000 000
2 000 000
0
2011 Mid-year estimates
Eastern Cape
Free State
Gauteng
KwaZulu-Natal
Limpopo
Mpumalanga
Northern Cape
North West
Western Cape

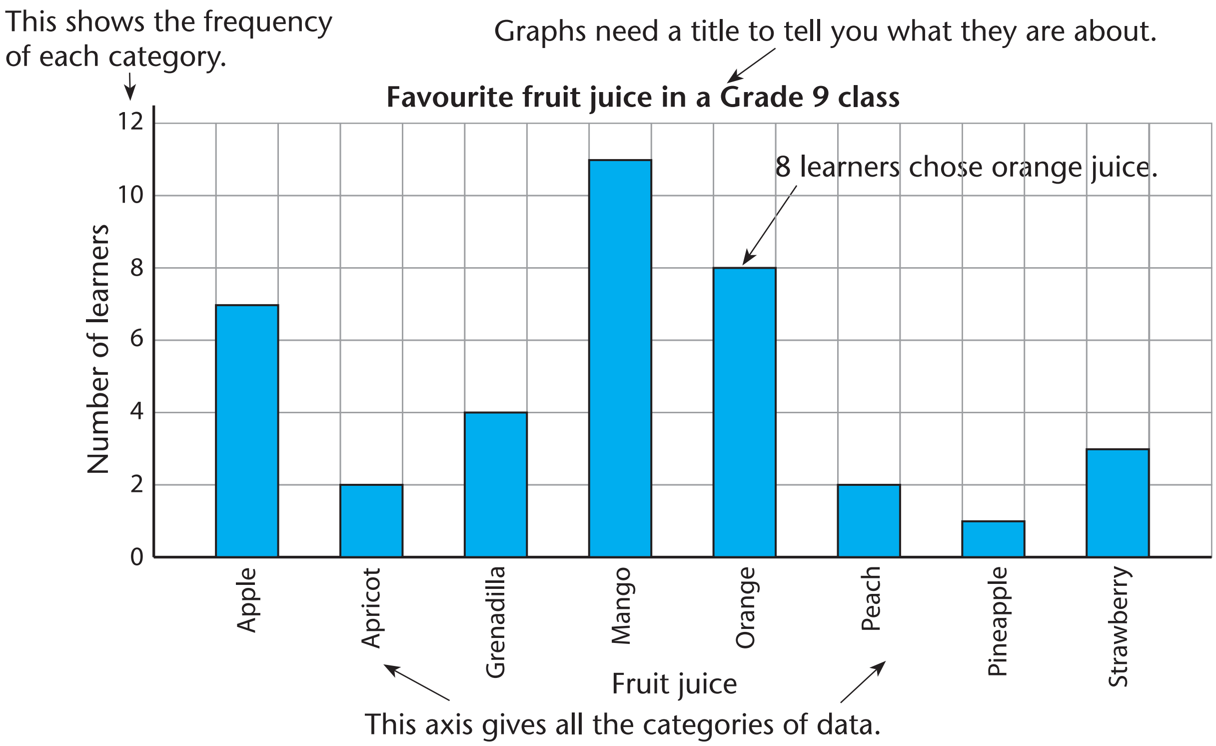

Rainfall for Ceres, Mahikeng and Amatole
Rainfall (mm)
180
160
140
120
100
80
60
40
20
0
January
February
March
April
May
June
July
August
September
October
November
December
Ceres, WC
Mahikeng, NW
Amatole, KZN

9 Representing data

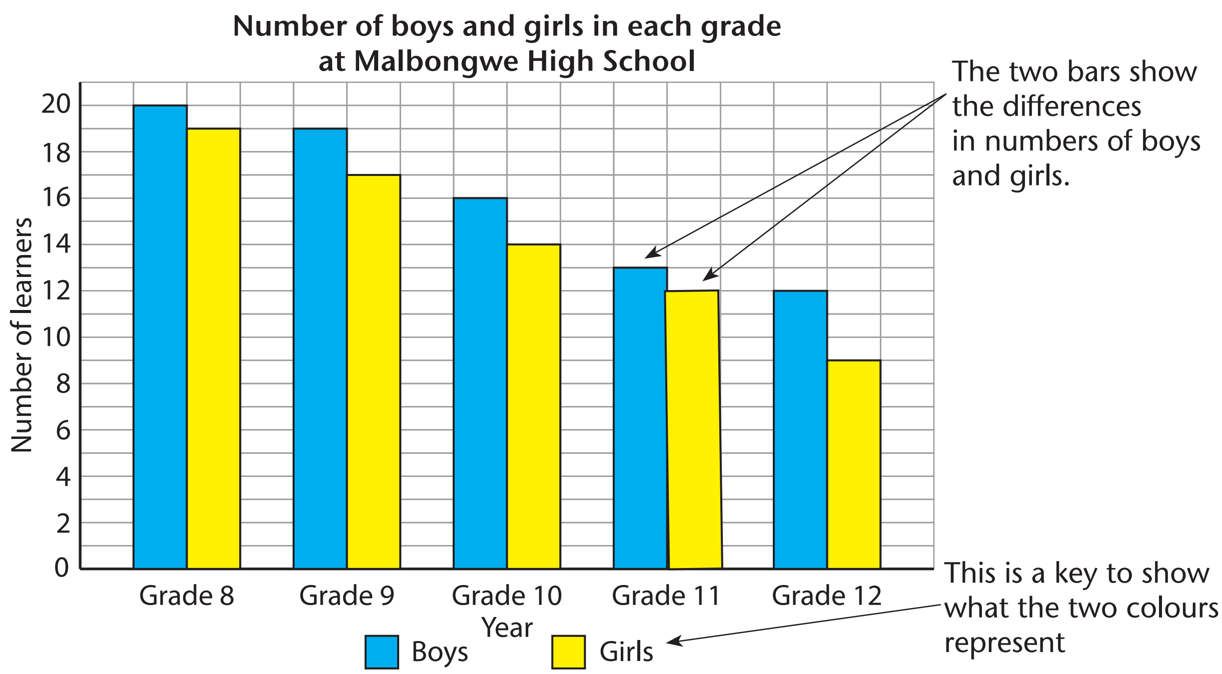
9.1 Bar graphs and double bar graphs

REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

A **bar graph** shows categories of data along the horizontal axis, and the frequency of each category along the vertical axis. An example is given below.



A **double bar graph** shows two sets of data in the same categories on the same set of axes. This is useful when we need to show two groups within each category.



DRAWING BAR GRAPHS AND DOUBLE BAR GRAPHS

1. Obese (very overweight) people have many health problems. It is a concern all around the world. Health researchers analysed the change over 28 years in the numbers of people who are overweight and obese in different areas of the world. This table summarises some of the data.

Percentage of population that is overweight and obese

	1980	2008
Sub-Saharan Africa	12%	23%
North Africa and Middle East	33%	58%
Latin America	30%	57%
East Asia (low income countries)	13%	25%
Europe	45%	58%
North America (high income countries)	43%	70%

(a) The table summarises "some" of the data. What would some other important data be? Think of as many things as you can.

....

....

....

....

(b) Which data stands out the most for you in the table above? Give your personal opinion.

....

....

(c) Plot a double bar graph to compare the data for the areas, and for the two years. Use the grid on the next page. Remember to give your graph a key.

(d) Look carefully at the comparisons that the graph makes. Has your opinion of the most interesting differences changed, now that you see the double bar graph? Explain.

(e) In some countries the obesity problem has been labelled "Obesity in the face of poverty". Write a short report on the data and your double bar graph to support this argument.

9.2 Histograms

REVISING HISTOGRAMS

A histogram is a graph of the frequencies of data in different **class intervals**, as demonstrated in the example below. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

A histogram looks somewhat like a bar graph, but is normally drawn without gaps between the bars.

REPRESENTING DATA IN HISTOGRAMS

1. (a) A fruit farmer wants to know which of his trees are producing good plums, and which trees need to be replaced.
He collects 100 plums each from two trees and measures their masses.
The data below gives the mass of plums from the first tree.

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	6	18	34	30	12

Represent the data in a histogram on the grid below.

(b) Now draw another histogram to represent the following data giving the mass of the same type of plums from another tree in the orchard.

Mass of plums (g)	20–29	30–39	40–49	50–59	60–69
Frequency	3	14	26	36	21

(c) Study the two histograms and then comment on the number of plums produced by the two trees.

2. (a) Draw a histogram to represent the data in the table below. Group the data in intervals of 0,5 kg.

Birth weights (kg) of 28 babies at a clinic

3,3	1,34	2,88	2,54	1,87	2,06	2,72
1,89	0,85	1,99	2,43	1,66	2,45	1,62
1,91	1,20	2,45	1,38	0,9	2,65	2,88
1,75	2,11	3,2	1,74	0,6	3,1	1,86

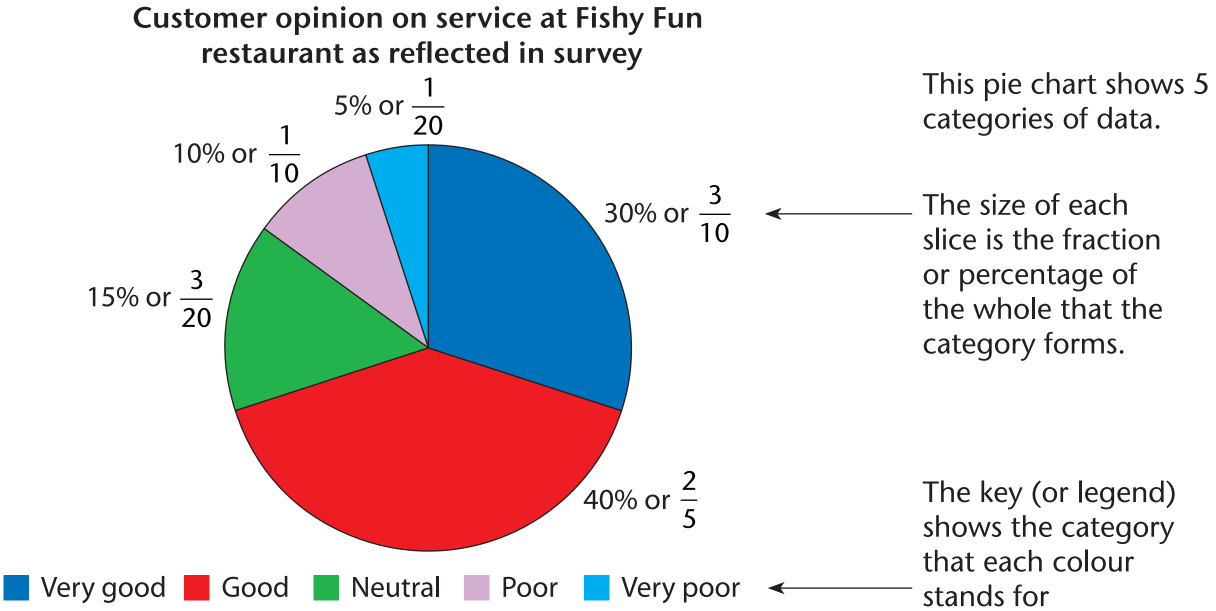
(b) Calculate the mean and median of the data.

(c) Records from the whole country show that the birth weight of babies ranges from 0,5 kg to 4,5 kg, and the mean birth weight is 3,18 kg. Use the graph and the mean and median to write a short report on the data from the clinic.

9.3 Pie charts

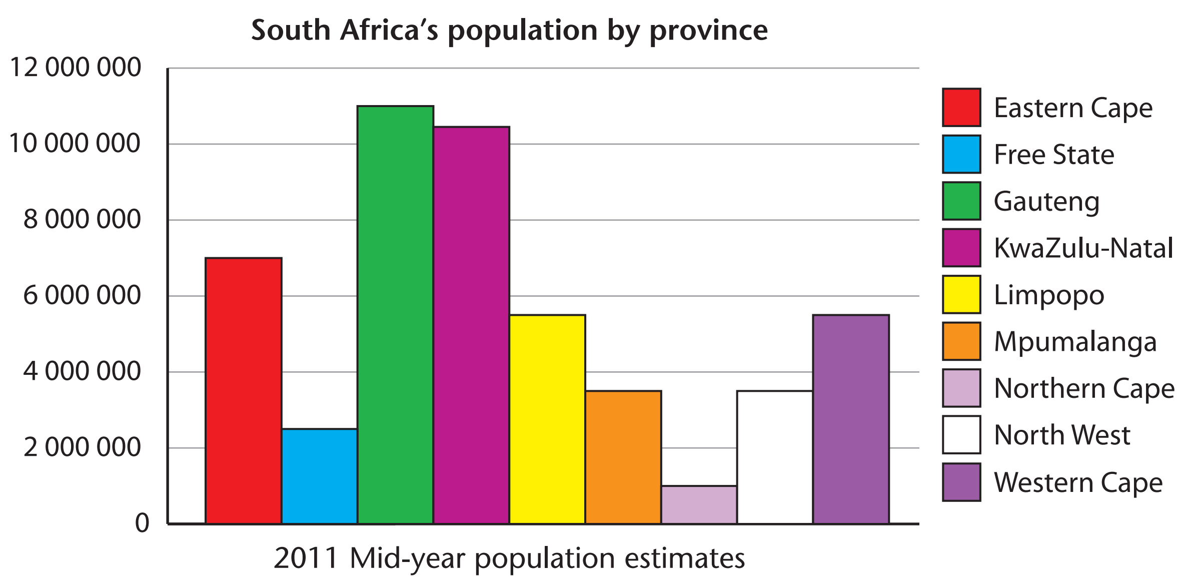
A **pie chart** consists of a circle divided into sectors (slices). Each sector shows one category of data. Bigger categories of data have bigger slices of the circle.

Here is an example of a pie chart:



DRAWING PIE CHARTS

1. The following bar graph shows the population of South Africa by province.



(a) Write the figures in the graph correct to the nearest 500 000.

Province	E Cape	FS	Gau	KZN	Lim	Mpum	NC	NW	WC
Population (× 1 000)									

(b) What is the total of the rounded off numbers? ....

(c) Work out the percentage of the whole for each province.

Province	E Cape	FS	Gau	KZN	Lim	Mpum	NC	NW	WC
Percentage of total									

(d) Draw a pie chart showing the data in the completed table. (Estimate the sizes of the slices.)

....

....

(e) Write a short report explaining the difference in the way the data is represented in the pie chart and the bar graph. Which do you think is a better method to show this data?

....

....

....

....

....

....

....

....

....

9.4 Broken-line graphs

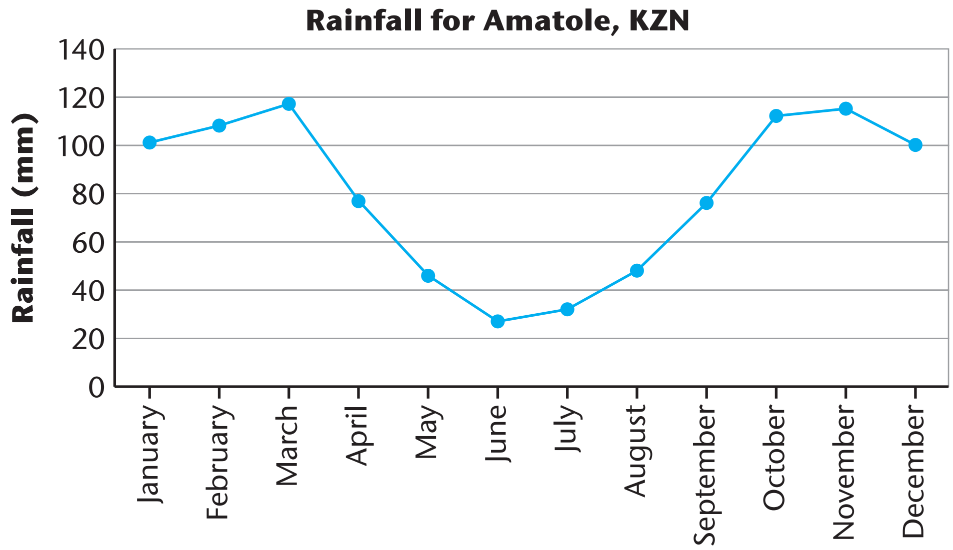
BROKEN-LINE GRAPHS

Broken-line graphs are used to represent data that changes continuously over time. For example, the rainfall for a whole month is captured as one data point, even though the rain is spread out over the month, and it rains on some days and not on others. Broken line graphs are useful to identify and display trends.

Here is some data that can be represented with broken-line graphs.

Rainfall at three locations in South Africa in 2012			
	Amatole, KZN	Mahikeng, NW	Ceres, WC
	Rainfall (mm)	Rainfall (mm)	Rainfall (mm)
January	101	118	27
February	108	90	23
March	117	86	41
April	77	61	60
May	46	14	130
June	27	6	168
July	32	3	152
August	48	7	162
September	76	18	88
October	112	46	60
November	115	75	41
December	100	86	36

Here is a broken line graph for the Amatole rainfall data.



1. During which four months does Amatole have the least rain?

....

2. During which six months does Amatole have the most rain?

....

3. During which months would you plan a hike if you were only considering the rainfall patterns?

....

4. What other factors should you consider when planning a hike in this region?

....

5. Make a broken-line graph for the Mahikeng rainfall data on the grid below.

Maths2_Gr9_LB_Book.indb 152 2014/09/08 09:07:08 AM

6. Make a broken-line graph for the Ceres-rainfall data on the grid below.

7. Write a few lines on the difference in rainfall patterns between Ceres and Mahikeng.

8. Draw a combined broken-line graph with the information from all three regions on one graph.

9.4 Scatter plots

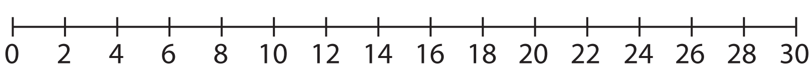
UNDERSTANDING AND CONSTRUCTING SCATTER PLOTS

Scatter plots show how two sets of numerical data are related. Matching pairs of numbers are treated as coordinates and are plotted as a single point. All the points, made up of two data items each, show a scattering across the graph.

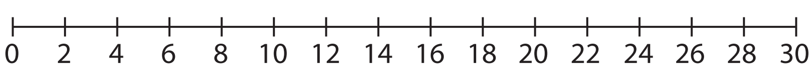
1. This table shows a data set with **two** variables. Study the information in the table.
2. Make a dot for each learner's mark for each subject on the number lines below.

Learners	Maths marks	Natural Science marks
Zinzi	**25**	**26**
John	23	25
Palesa	**22**	**25**
Siza	21	23
Eric	20	23
Chokocha	19	21
Gabriel	17	20
Simon	16	19
Miriam	15	18
Frederik	15	16
Sibusiso	**12**	**15**
Meshack	11	13
Duma	11	12
Samuel	10	12
Lola	10	11
Thandile	9	10
Jabulani	**8**	**10**
Manare	7	9
Marlene	7	7
Mary	5	7

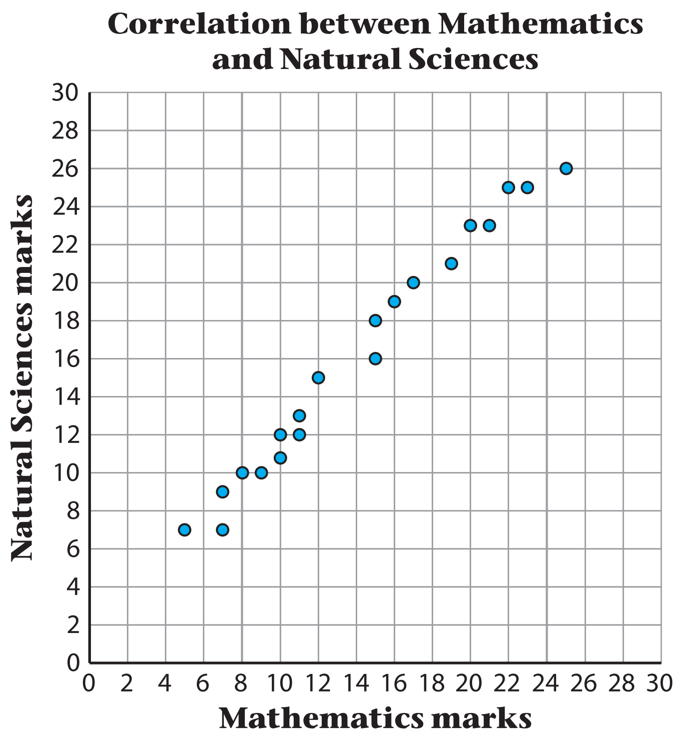
Natural Sciences marks



Mathematics marks



3. What if you were to show both sets of marks on the same graph, instead of a separate number line for each set? The graph below shows a scatter plot that represents both sets of data. Each dot represents one learner.



The scatter plot shows the **relationship** between the Natural Sciences mark and the Mathematics mark.

4. Find the dot for Sibusiso in the data set. He obtained a mark of 12 for the Mathematics test and a mark of 15 for Natural Sciences. Find 12 on the horizontal axis. Follow the vertical line up until you reach a blue dot. Find 15 on the vertical axis. Follow the line horizontally until you reach the same blue dot. This blue dot represents the two marks that belong to Sibusiso. Circle the blue dot and label it S.

5. Find the data points for Zinzi, Palesa, Jabulani and Mary. Circle them and label them Z, P, J and M.

In the above example, a higher Mathematics mark corresponds to a higher Science mark. We say there is a **positive correlation** between the Mathematics marks and the Science marks.

6. Study this data set and the scatter plot of the data given on the next page.

Learner	Maths marks	Art marks
Zinzi	**25**	**7**
John	23	7
Jabulani	22	9
Siza	21	10
Eric	**20**	**10**
Chokocha	19	11
Gabriel	17	12
Simon	16	12
Miriam	**15**	**15**
Frederik	**15**	**15**
Sibusiso	12	16
Mishack	11	17
Duma	11	19
Samuel	**10**	**20**
Lola	10	21
Thandile	9	23
Palesa	8	23
Manare	7	25
Marlene	7	25
Mary	**5**	**26**

7. Find Eric in the table. Note his marks for Mathematics and Art. Find the dot that represents his marks on the scatter plot. Encircle it and label it E.

8. Find Samuel in the table. Note his marks for Mathematics and Art. Find the dot that represents his marks. Encircle it and label it S.

9. Compare the two sets of marks for Eric and for Samuel. What do you notice about the marks?

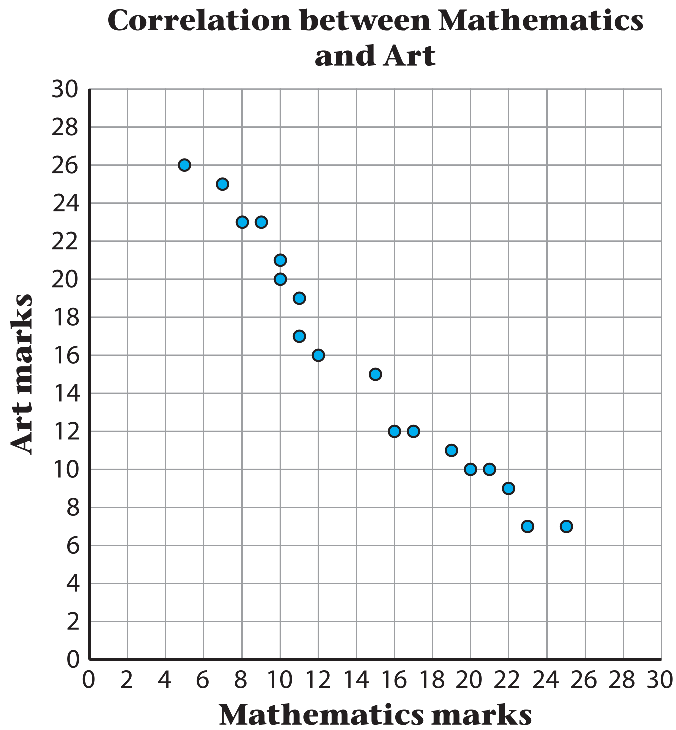
....

....

10. Find the data points on the scatter plot for Zinzi, Eric, Miriam, Frederik, Samuel and Mary. Circle the points and label them Z, E, M, F, S and Ma

11. What do you notice about the pattern of marks in Mathematics and Art for this data set?

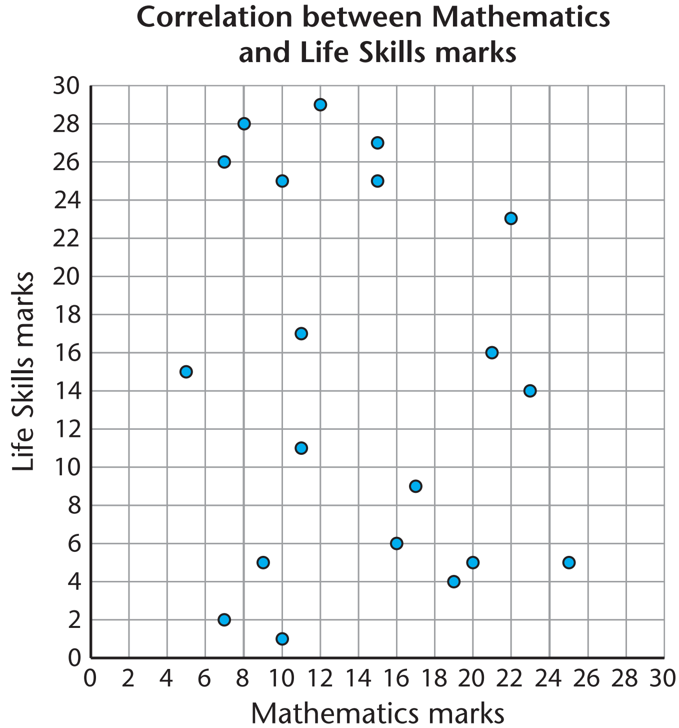
....



A **negative correlation** is a correlation in which an increase in the value of one piece of data tends to be matched by the decrease in the other set of data. Learners who obtain a high mark for Mathematics appear to obtain a low mark for Art. We say there is a negative correlation between the Mathematics and the Art scores for this data set.

A correlation is an assessment of how strongly two sets of data appear to be connected. Two sets of data may be correlated or may show **no correlation.**

Here is the scatter plot for the Mathematics and Life Skills marks of the same group of learners. The table for this data is given on the next page.



12. Study the scatter plot and the data table on the next page.

13. Find the data points on the scatter plot for Zinzi, Eric, Miriam, Lola, and Mary. Circle the points and label them Z, E, M, L and Ma.

14. What do you notice about the pattern of marks in Mathematics and Life Skills for this data set?

....

....

Learner	Maths	Life Skills
Zinzi	25	5
John	23	14
Jabulani	22	23
Siza	21	16
Eric	20	5
Chokocha	19	4
Gabriel	17	9
Simon	16	6
Miriam	15	25
Frederik	15	27
Sibusiso	12	29
Meshack	11	17
Duma	11	11
Samuel	10	1
Lola	10	25
Thandile	9	5
Palesa	8	28
Manare	7	26
Marlene	7	2
Mary	5	15

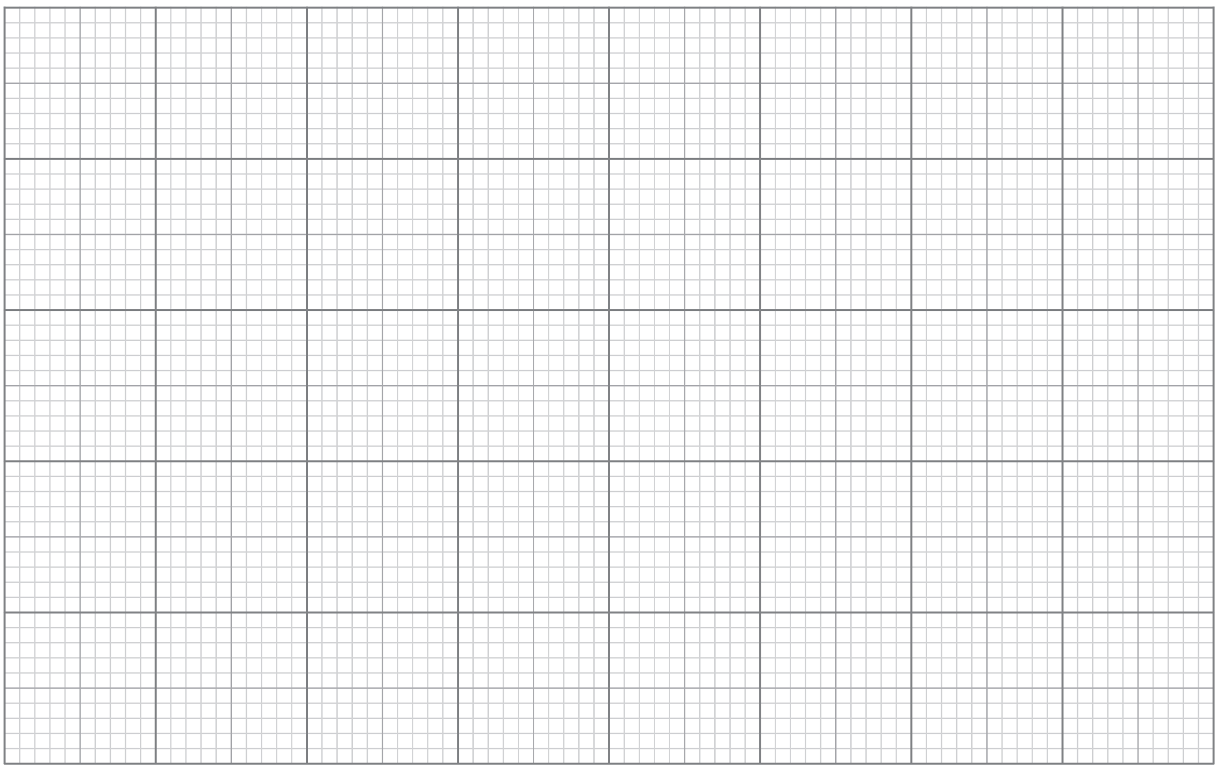
THE RELATIONSHIP BETWEEN ARM SPAN AND HEIGHT

The idea that a person's arm span (the distance from the tip of the middle finger on one hand to the tip of the middle finger on the other hand when the arms are stretched out sideways) is the same as one's height has been explored many times.

A data set for 13 people is given on the next page.

1. Make a scatter plot of this data on the given grid.
   For example, take Cilla's arm span. Find 156 on the horizontal axis. Follow a vertical line up. Then on the vertical axis find 162. Follow a horizontal line across. Where the two points meet, draw a dot.

Person	Arm span	Height
Cilla	156	162
Meshack	159	162
Tony	161	160
Ellen	162	170
Karin	170	170
Sibongile	173	185
Gabriel	177	173
Alpheus	178	178
Mfiki	188	188
Nathi	188	182
Manare	188	192
Khanyi	196	184



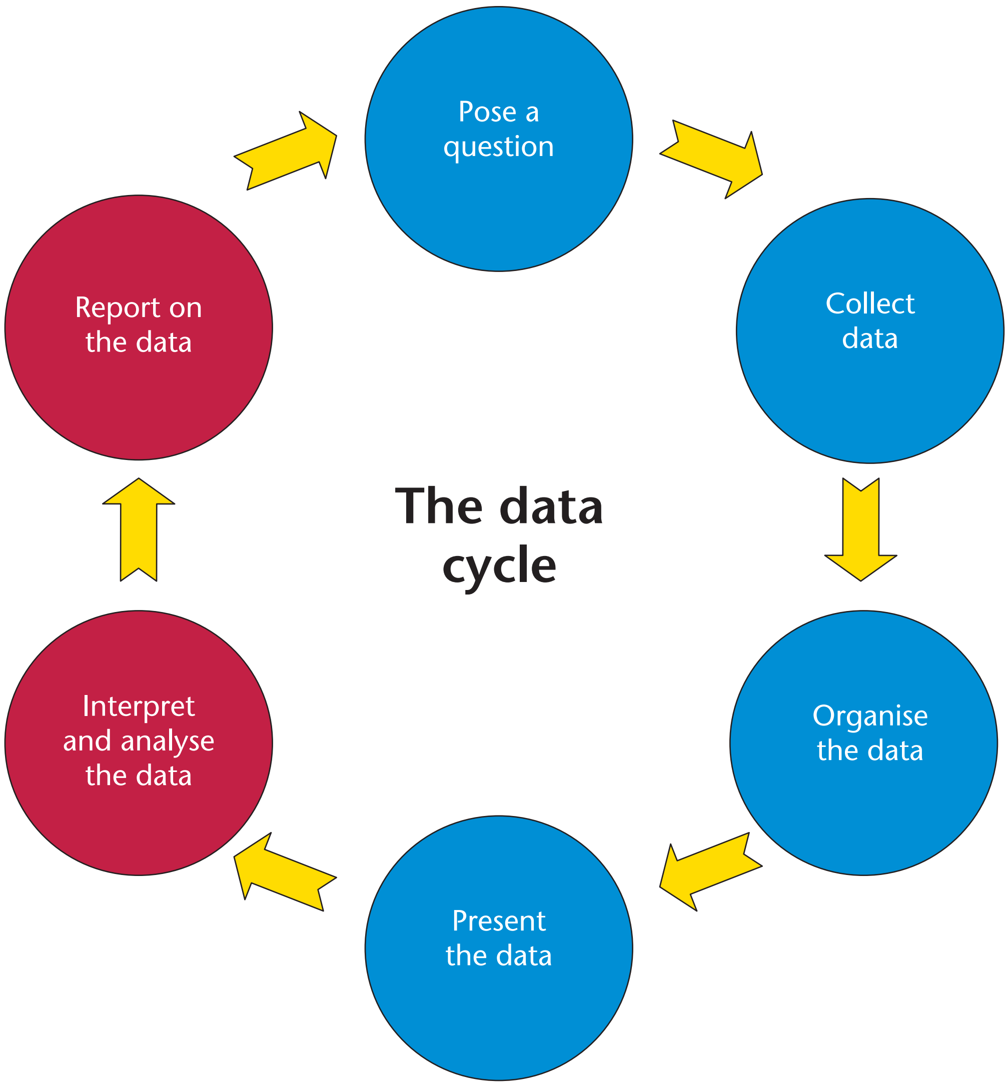
2. What would you say about the correlation between the arm span and the height?

....

Maths2_Gr9_LB_Book.indb 160 2014/09/08 09:07:10 AM

Chapter 10
Interpret, analyse and report on data

In this chapter, you will develop and practise some critical data analysis skills. This means looking at reported data and analysing the whole data handling cycle for this data. You need to decide which way of representing data is best in a given situation. In summarising data, some measures are more appropriate for different types of data. You also need to recognise some ways in which bias can appear in data, including methods of collecting, representing and summarising data.


Pose a question
Collect data
Organise the data
Present the data
Interpret and analyse the data
Report on the data
The data cycle

10 Interpret, analyse and report on data

10.1 Which graph is best?

You have learnt that certain types of graphs are best for displaying certain kinds of information. The type of graph depends mostly on the type of data that needs to be represented. Here is a summary of the advantages of different types of graphs:

Tables show more information than graphs but the patterns are not as easy to see. They do not give a visual impression of particular trends.

Pie charts show a whole divided into parts. They show how the parts relate to each other and how the parts relate to a whole. They do not show the quantities involved.

Bar graphs show the amounts or quantities involved but do not show the relationship as effectively as pie charts. They are useful for showing **quantitative** data. Bar charts allow us to compare the quantities of different categories, for example, the sales of different items.

A **double-bar graph** is used to compare two or more things for each category. For example, we could use a double-bar graph to compare the differences between males and females.

Histograms are used to represent numerical data that is grouped into equal class intervals. Histograms are useful to show the way the data is spread out.

Broken-line graphs show trends or changes in quantities over time.

CHOOSE THE BEST REPRESENTATION

1. Which kind of graph is best to represent each of the following? Explain your answers.
   (a) Showing the value of the rand against the US dollar over several years

   ....

   (b) Comparing the monthly sales of six different makes of car in 2014 and 2015

   ....

   (c) The proportion of people of different age groups in a town

   ....

   ....

   (d) The quantities of different crops produced on a farm

   ....

   ....

   (e) The percentages of different goods sold to make up the total sales for a shop

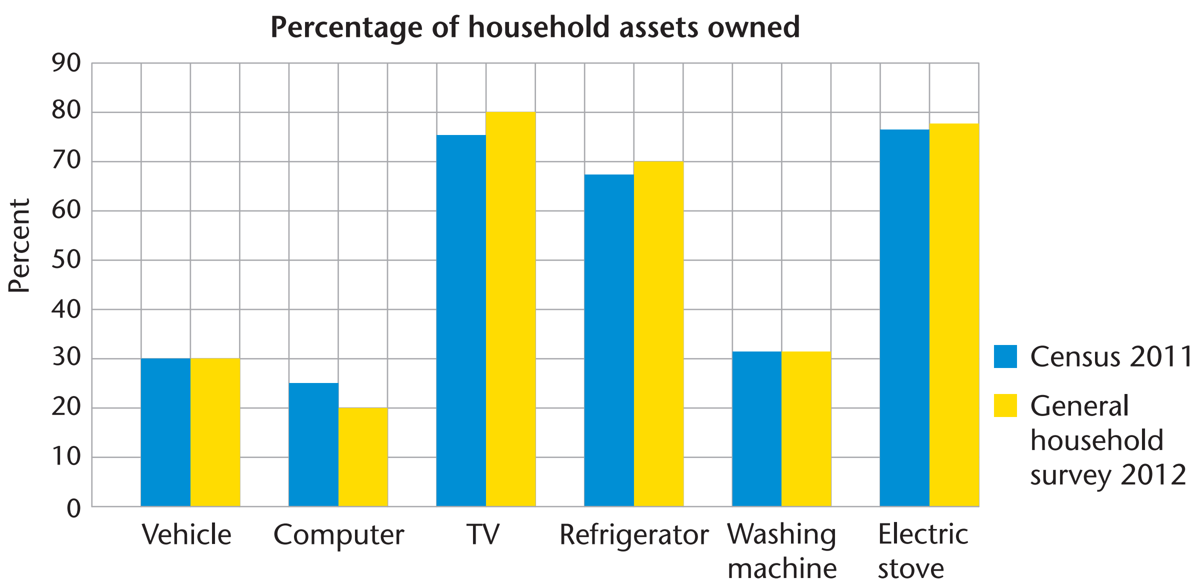
   ....

   ....

   (f) The change in HIV infection rates over time

   ....

2. This graph was published by Statistics South Africa to show the assets owned by South Africans. The blue bar shows the Census 2011 results and the yellow bar shows the General Household Survey 2012 results.



Give reasons for your answers to the questions below.

(a) Is it useful to show the differences in the results of Census 2011 and the General Household Survey 2012?

....

....

(b) Is it useful to collect data on assets that people own?

....

....

(c) Is it useful to show that lower percentages of people own certain assets?

....

....

....

(d) The different coloured bars represent the two different surveys. Draw up a table to show the data in table form. (Read the percentages as accurately as you can from the graph and round off the data to the nearest whole number for the table.)

(e) Does the table show the data as effectively as the double bar chart? Give your own opinion.

....

....

3. The table below shows the employment status of people ages 15–64 years in South Africa. Discuss some ways of representing the data (e.g. graphs). Justify your answers.

	Jul–Sept 2012	Apr–June 2013	Jul–Sep 2013
	Number of people (thousands)		
Population 15–64 years old	33 017	33 352	33 464
Labour force	18 313	18 444	18 638
Employed	**13 645**	**13 720**	**14 028**
Formal sector (non-agricultural)	9 663	9 694	10 008
Informal sector (non-agricultural)	2 197	2 221	2 182
Agriculture	661	712	706
Private households	1 124	1 093	1 132
Unemployed	**4 668**	**4 723**	**4 609**
Not economically active	**14 705**	**14 908**	**14 826**
Discouraged work-seekers	2 170	2 365	2 240
Other (not economically active)	12 535	12 543	12 586
Unemployment rate (%)	25,5	25,6	24,7

(a) The percentages of the employed, unemployed, and not economically active people in July–September 2013.

....

(b) The change in the employment **rates** over three time periods

....

(c) The proportions of employed people who work in the formal sector, informal sector, agriculture and private households.

....

(d) The numbers of the employed and unemployed over the three time periods.

....

....

10.2 The effects of summary statistics on how data is reported

Information articles often use averages to report information. The articles might not use the exact terms for average that you have learnt about: the mean, median and mode. Instead, they may use terms such as 'most'. However, it is important to be sure which kind of average a report refers to, because they give us different information.

- Remember that the **mean** is useful for describing a set of measurement values, but can also be used for other numerical data sets. The word 'average' usually refers to the 'mean' if it is not explained further. The mean is not reliable if a data set is too spread out.
- The **median** is the value in the middle of a data set when it is arranged in order. Half the values in the data set are lower than the median and half of them are higher than the median. The median is often the average used when data values are not uniformly distributed, because the mean is affected by extreme values in the data set, while the median is not. For example, house prices vary widely, so the median would be a better description of the data than the mean. When the median is given in a report, the writer should state that they are using the median or middle value.
- The **mode** is the number that occurs most often in a set of data. For example, if we collect data about people's favourite colours, the data set would be a list of colours, and the mode would be the colour that comes up most often. The mode can also be used for numbers. Not all data sets have a mode, because sometimes none of the numbers occurs more than once.

Example

The standard way of reporting house prices in South Africa and internationally is the median house price, which is used by economists in financial reports. The median is regarded as more useful than the mean house price because the sale of a few expensive houses would increase the mean, but would not affect the median.

If a bank gives bonds for eight houses to the value of R100 000, and for two houses to the value of R1 million, then the mean would be R280 000. This does not seem to be an accurate reflection of the value of the houses, because it is distorted by the higher values. The median house price would be R100 000, which is an accurate reflection of the prices.

Remember that the median is the middle point, and half of the values fall below the median, and half above. If the median is lower than the mean, this shows us that there are high values that are distorting the mean.

USING DIFFERENT SUMMARY STATISTICS

1. What kind of average is used in each of these statements?
   (a) The average family has 2,6 children. ....
   (b) Most families have 3 children. ....
   (c) Most people prefer red cars. ....
   (d) The average height for women is 1,62 m. ....
   (e) More people shop after work than at any other time during the day. ....

2. The mean monthly salary of all the staff at company ABC is R8 000 per month, but the median salary is R5 000.
   (a) Explain why the two summary statistics are so different.

   ....

   ....

   ....

   (b) Which summary statistic gives a better idea of the salaries at the company? Give reasons for your answer.

   ....

   ....

10.3 Misleading graphs

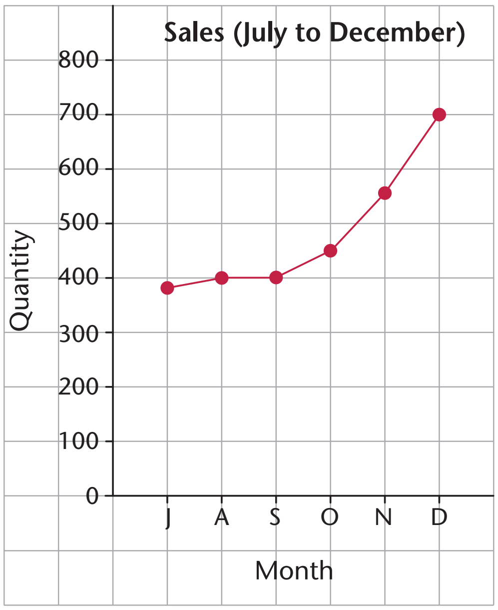
The media (newspapers, magazines, television), regularly use graphs to show information. Unfortunately, the information is often manipulated to emphasise a particular result. This may be because the writer simply wants to make his or her argument more obvious to the reader.

Changing the scale of the axis

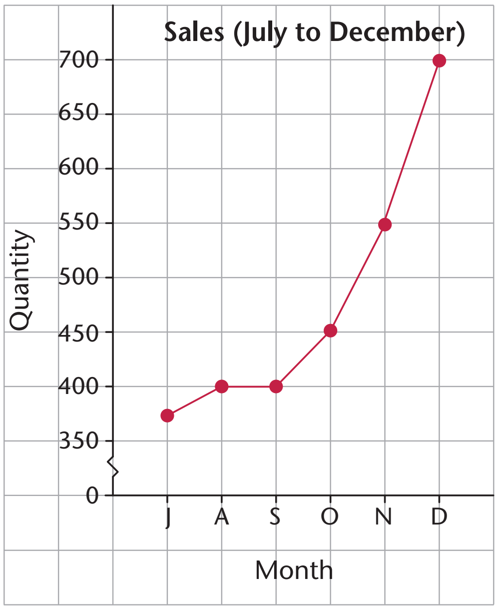
If you change the scale of the vertical axis on bar graphs and line graphs, you will change the way the graphs look. For a bar graph, the larger the spaces between the numbers on the vertical axis, the bigger the difference between the bars. The smaller the spaces between the numbers on the axis, the smaller the difference in the height of the bars. The same is true for a line graph which will either have sharp points or be much flatter depending on how you have changed the scale.

Example

The two broken-line graphs below show the same sales data for a business over a period of six months. Which graph gives the more accurate impression?



Graph A



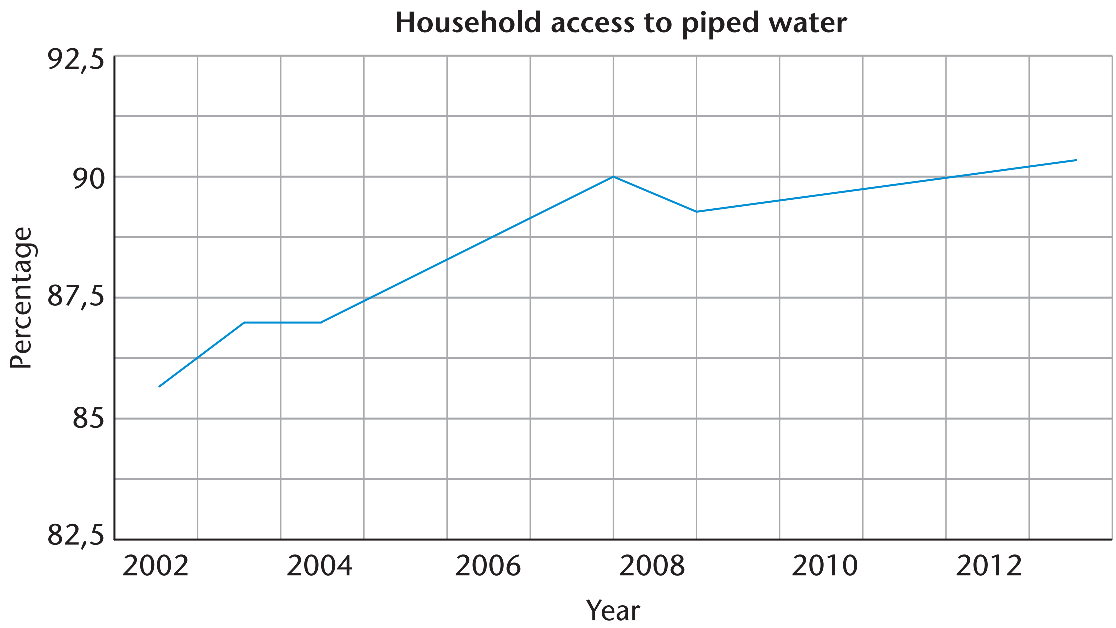
Graph B

Graph B has a different scale on the vertical axis. The vertical axis does not start at 0 and so **two** blocks on the vertical axis represent 100 items instead of only **one** block, as in Graph A. This makes it look as if the sales increased rapidly over the six months.

Note that it is not necessarily wrong to change the scale on the axes or not to start at 0. For example, graphs showing stock exchange fluctuations rarely show the origin on the graph and stockbrokers are taught to interpret the graphs in that form. Sometimes small changes in data values have important effects and in these cases, it may be valid to change the scale to show these.

ANALYSING GRAPHS

1. This graph from Statistics South Africa shows the increase in the percentage of households that had access to piped water over a ten-year period.



(a) Comment on the scale used on the vertical axis. Is this a misleading graph?

....

....

....

....

....

....

(b) How could you redraw the graph so that the differences on the graph are more noticeable?

....

....

(c) How could you draw the graph so that the differences are less noticeable?

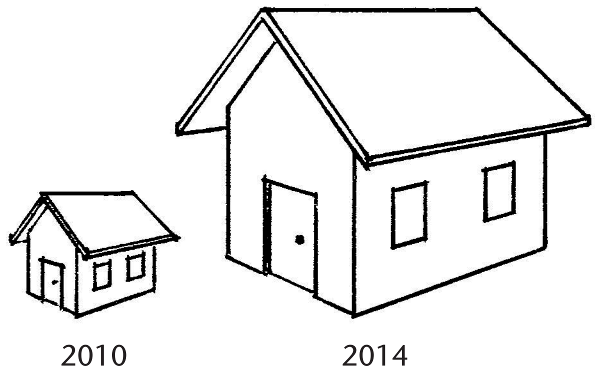
....

....

Maths2_Gr9_LB_Book.indb 170 2014/09/08 09:07:11 AM

2. In this graph the height of the houses represents the number of sales.

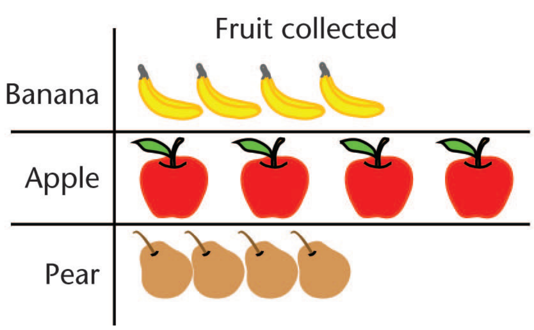
Home Sales



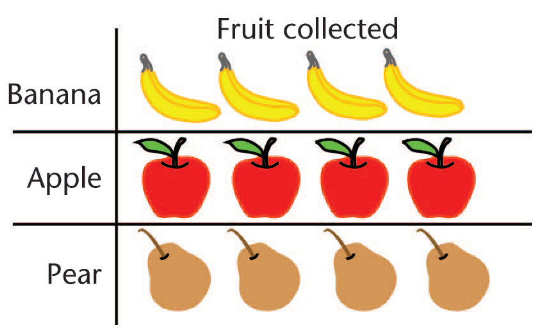
Do you think that this graph is misleading? Give reason(s) for your answer.

3. Look at the two graphs below:

Graph A



Graph B

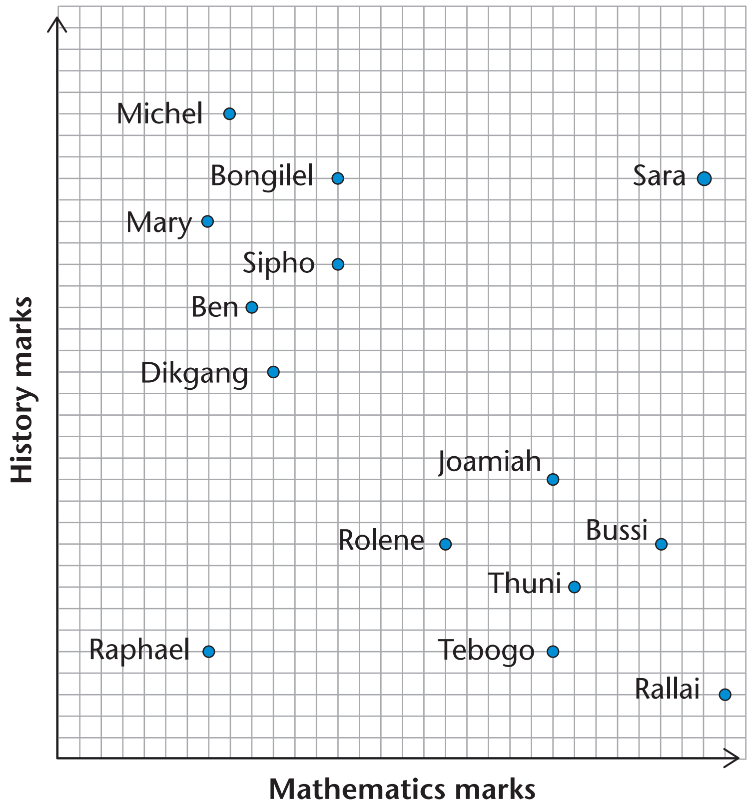


Which graph do you think is drawn correctly? Explain your answer.

10.4 Analysing extreme values and outliers

A data item that is very different from all (or most) of the other items in a data set is called an **outlier**.

It is sometimes difficult to notice outliers in numerical data. However, outliers often become clearly noticeable when data is displayed with graphs.



1. The above scatter plot shows the performance of a group of learners in Mathematics and History. Which of the points on the scatter plot can be regarded as outliers? Give reasons for your answer.

....

....

....

....

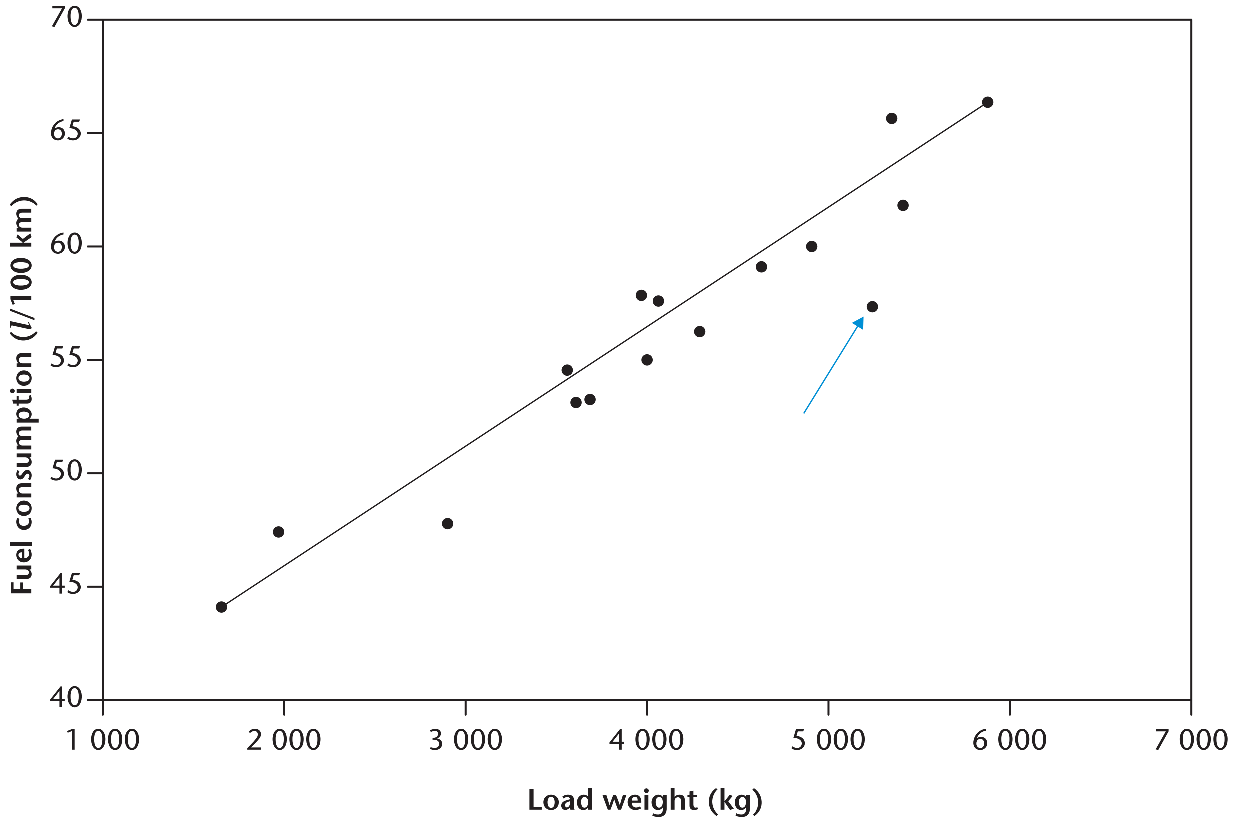
....

....

Maths2_Gr9_LB_Book.indb 172 2014/09/08 09:07:11 AM

Outliers in data sets can be very important. We need to decide whether there is a particular reason for the value being so different to the others. Sometimes it gives us important information. In some cases, the data collected for that point could be wrong.

The scatter plot below is for data collected by a transport company.



The company uses just one type of truck. Before each transport job, the company has to specify the price for the job. In order to specify a price before a job, the company needs to estimate how much their costs will be for doing the job. One of the main costs is the cost of fuel, and the main factor influencing the amount of fuel used is the distance. The load weight also plays a role: the greater the load weight, the higher the fuel consumption (litres/100 km).

The table on the next page gives information that was recorded for previous transport jobs. The jobs are numbered from 1 to 16 and for each job the values of the four variables *distance*, *load weight*, *amount of fuel used* and *fuel consumption rate* are given.

2. (a) Which of the four variables are represented on the scatter plot given above?

....

(b) What are the values of these two variables for the point indicated by the blue arrow on the scatter plot?

....

Job number	Distance (km)	Load weight (kg)	Fuel used (litres)	Fuel consumption (litres/100 km)
1	1 304	5 445	879	67.4
2	1 320	2 954	639	48.4
3	1 151	4 705	698	60.6
4	1 371	4 378	787	57.4
5	325	3 673	176	54.2
6	1 630	5 995	1 113	68.3
7	1 023	5 357	600	58.7
8	620	4 988	382	61.6
9	73	1 992	35	47.9
10	1 071	5 529	680	63.5
11	370	4 140	218	58.9
12	1 423	4 062	843	59.2
13	394	4 068	221	56.1
14	1 536	1 678	682	44.4
15	1 633	3 736	887	54.3
16	435	3 644	241	55.4

3. (a) Consider the scatter plot and the data set. What is the effect of load weight on fuel consumption?

....

(b) Is job 7 an exception in this respect? Explain your answer.

....

....

4. Further investigations revealed that the driver for jobs 2 and 7 was the same person, and that he was not the driver for any other jobs. What may this indicate?

....

....

FIND OUTLIERS

Researchers collected data on the population of some African countries plus the Seychelles, the income per person, and the percentage of the income spent on health.

Country	Total population (in 1 000s)	Total annual national income per person (US$)	Percentage of income spent on health
Angola	18 498	4 830	4,6
Botswana	1 950	13 310	10,3
DRC	66 020	280	2,0
Lesotho	2 067	1 970	8,2
Malawi	15 263	810	6,2
Mauritius	1 288	12 580	5,7
Mozambique	22 894	770	5,7
Namibia	2 171	6 250	5,9
Seychelles	84	19 650	4,0
South Africa	50 110	9 790	8,5
Swaziland	1 185	5 000	6,3
Tanzania	43 739	1 260	5,1
Zambia	12 935	1 230	4,8

1. What are the three variables in this table?

....

....

....

2. Why do you think it is important to look at income per person in this case, rather than the total income?

....

....

....

3. Plot the points for the national income per person and the percentage spent on health care for each country.

4. Write a short report on the data in the table and what the scatter plot shows you about the data. Comment on the general trend and any outliers.

Chapter 11
Probability

In this chapter you will learn about the idea of probability, and what information probabilities provide about what may happen in future. You will also learn about compound events.

1	4	3	4	6	1	4	2	1	4	3	1	1	6	1	3	3	3	6	5	2	1	4	6	6	5
6	5	3	2	4	2	6	6	1	5	4	6	5	3	1	2	2	2	2	4	6	2	6	4	5	3
6	3	6	2	2	5	6	6	2	4	4	1	2	4	2	1	2	1	2	5	4	3	3	6	2	4
6	2	5	2	5	4	4	6	2	3	1	1	6	2	2	5	6	3	2	6	2	3	2	6	1	2
1	4	2	3	4	1	3	1	1	1	3	5	4	2	6	6	1	4	2	2	6	1	3	2	2	5
5	4	1	1	6	4	1	3	3	5	2	2	3	3	1	3	6	4	3	2	1	2	2	4	5	3
5	5	6	3	6	1	5	3	3	2	1	2	3	3	4	1	5	4	2	4	2	5	2	3	6	3
4	3	6	6	6	5	3	3	2	4	3	4	6	6	5	4	5	6	1	6	5	2	1	2	2	5
6	1	4	4	4	6	3	1	5	4	2	4	6	3	2	4	3	6	5	5	4	3	5	2	6	2
2	4	5	6	5	1	4	6	1	5	3	6	4	4	3	2	2	5	5	3	5	1	4	4	4	1
6	1	1	5	2	5	1	5	2	2	6	1	2	3	1	2	4	5	3	1	5	6	4	6	1	4
1	1	5	6	6	1	2	4	1	5	5	6	3	4	1	4	2	3	3	6	2	2	6	3	1	4
4	4	6	5	1	6	5	1	6	5	5	4	5	4	3	2	5	6	5	1	3	3	3	5	2	2
4	3	6	1	1	6	1	6	2	1	4	1	1	4	5	3	3	1	2	1	5	6	6	1	3	1
2	6	3	6	4	1	3	3	2	5	4	2	3	1	3	2	2	2	2	4	3	3	6	3	1	2
3	3	3	6	3	6	3	3	6	4	4	2	5	3	3	1	3	3	3	5	1	2	5	1	2	5
4	1	6	4	2	6	4	5	2	1	3	5	2	1	5	5	4	4	3	4	5	1	6	2	1	5
2	3	6	5	3	2	4	6	1	1	4	6	4	6	4	1	3	2	3	1	2	1	1	1	1	2
1	3	4	1	6	3	6	2	3	4	5	6	5	2	5	4	2	2	5	5	6	5	5	6	5	3
4	3	6	5	4	3	6	6	3	5	6	6	2	3	6	3	5	4	1	1	3	6	1	1	5	5
5	6	6	6	4	2	1	6	2	2	4	4	2	3	1	3	3	5	3	2	1	6	3	5	3	5
6	2	1	3	5	5	3	5	2	6	1	4	4	2	6	2	1	5	1	5	5	5	1	4	4	2
1	2	6	4	2	2	3	6	4	3	2	1	3	5	3	4	4	5	5	5	2	2	2	3	4	3
5	3	1	3	5	2	4	2	5	2	6	6	6	6	2	1	1	3	4	4	2	3	5	4	6	2
5	3	6	2	5	2	4	2	4	5	3	4	6	5	6	5	6	2	1	1	6	3	5	2	6	6
4	6	1	1	6	3	2	5	5	5	3	4	5	1	2	3	4	3	4	5	6	6	2	5	2	4
4	1	6	4	6	3	2	4	2	2	3	1	3	5	2	1	1	6	5	2	2	4	3	1	3	6
3	6	3	4	1	4	2	6	5	6	1	5	2	4	3	2	6	6	1	3	5	5	5	1	2	6
6	2	5	5	6	4	1	2	1	2	3	6	2	1	4	2	2	6	1	3	5	1	4	5	5	2
1	6	5	5	4	6	5	5	5	4	2	3	2	5	5	1	3	2	3	3	6	2	5	2	3	2
5	1	3	5	5	2	6	6	6	1	1	3	5	4	2	3	5	5	6	2	5	5	5	1	5	4
5	5	6	6	5	5	3	3	2	2	2	3	3	2	6	4	1	2	6	3	6	2	6	2	1	6
3	1	1	4	1	3	4	6	6	3	2	6	5	3	2	6	1	3	1	6	4	3	1	4	1	2
2	4	5	2	6	4	4	1	6	6	4	5	1	2	5	2	3	5	1	6	4	6	3	5	1	6
1	6	6	6	6	2	2	2	1	2	2	3	5	5	3	3	5	2	2	5	2	4	1	2	2	4
2	6	4	6	2	6	4	1	5	5	6	4	1	3	3	2	2	5	4	2	2	1	3	4	5	5
6	6	5	2	4	2	2	5	2	6	6	3	2	2	4	1	3	5	4	2	4	3	4	1	2	4
5	4	6	6	1	4	2	5	5	4	6	1	4	5	1	2	1	1	2	6	5	2	1	3	4	4
1	6	6	5	4	4	4	5	1	1	1	1	1	6	1	1	3	1	2	3	3	5	2	4	2	5
6	5	4	6	1	6	3	5	6	2	1	1	6	2	6	1	4	4	3	2	3	1	5	3	5	1
4	6	4	1	1	5	6	2	1	6	3	2	3	5	4	1	4	5	3	2	3	5	1	6	1	1
4	5	1	3	4	3	5	3	5	1	2	2	6	3	4	6	6	6	1	2	6	4	1	6	6	5
3	2	6	5	3	1	5	5	2	6	3	4	5	4	1	6	6	3	5	6	1	1	5	4	2	2
4	6	6	3	2	6	6	5	3	5	5	1	5	3	3	3	5	2	5	3	5	4	5	2	6	5
3	2	5	2	6	2	2	2	1	6	1	6	3	3	4	1	1	5	1	6	4	5	4	2	2	1

11 Probability

11.1 Simple events

REVISION

yellow	green	pink	blue	red	brown	grey	black

1. (a) Suppose the 8 coloured buttons above are in a bag and you draw one button from the bag without looking. Can you tell what colour you will draw? ....

   (b) Suppose you repeatedly draw a button from the bag, note its colour, then put it back. Can you tell in approximately what fraction of all the trials the button will be yellow?

   ....

   ....

Archie has a theory. Because the 8 possible outcomes are equally likely, he believes that if you perform 8 trials in a situation like the above you will draw each colour once.

> Each time you draw a button from the bag without looking you perform a **trial**. If you do this and put the button back, and repeat the same actions 8 times, you have performed 8 trials.
>
> The number of times an event occurs during a set of trials is called the **frequency** of the event.
>
> When the frequency of an event is expressed as a fraction of the total number of trials, it is called the **relative frequency**.

2. If Archie's theory is correct, how many times will each colour be drawn if 40 trials are performed?

   ....

   ....

3. If Archie's theory is correct, in what fraction of the total number of trials will each colour be drawn?

   ....

Maths2_Gr9_LB_Book.indb 179 2014/09/08 09:07:13 AM

4. If Archie's theory is correct, how many times will each of the colours be drawn if a total of 40 trials is performed? Write your answers in the second row of the table below. Write the predicted relative frequencies in row 3 as fortieths, and in row 4 as twohundredths.

colour	yellow	green	pink	blue	red	brown	grey	black
frequencies predicted by Archie								
relative frequencies predicted by Archie expressed in 40ths								
relative frequencies predicted by Archie expressed in 200ths								

The relative frequency for each colour that Archie predicted is called the **probability** of drawing that colour. If all the outcomes are equally likely, then

$$\textbf{probability of an outcome} = \frac{\textbf{1}}{\textbf{the total number of equally-likely outcomes}}$$

You will now investigate whether Archie's theory is correct.

5. (a) Make 8 small cards and write the name of one of the above colours on each card, so that you have cards with the eight colour names. Perform 8 trials to check whether Archie's theory is correct. Record your results (your tally marks 1 and your frequencies 1) in the relevant row of the table below.
   (b) Find out what any four of your classmates found when they did the experiment. Enter their results in your table too (Friend 1, 2, 3, 4 frequencies).

Table for the results of the experiments

colour	yellow	green	pink	blue	red	brown	grey	black
your tally marks (1)								
your frequencies (1)								
Friend 1 frequencies								
Friend 2 frequencies								
Friend 3 frequencies								
Friend 4 frequencies								
Total frequencies for 5 experiments								

6. (a) What was the total number of trials in the five experiments you recorded in the above table? ....

(b) What is the total of the frequencies for the different colours, in the last row of your table? ....

7. Is Archie's theory correct? ....

Bettina has a different theory to Archie's. She believes that if one does many trials with the eight buttons in a bag, each colour will be drawn in **approximately** one-eighth of the cases. In other words Bettina believes that the relative frequency of each outcome will be close to the probability of that outcome, but may not be equal to it.

8. (a) You and your four classmates performed 40 trials in total. Enter the results in the second row of the table below. Also express each frequency as a fraction of 40, in fortieths and in twohundredths.

colour	yellow	green	pink	blue	red	brown	grey	black
actual frequencies obtained in your experiments (40 trials)								
relative frequencies as 40ths								
relative frequencies as 200ths								
probability as 200ths								

(b) Do your experiments show that Bettina's theory is correct or not?

....

Jayden believes that **when more trials are performed, the relative frequencies will get closer to the probabilities**.

You will now do an investigation to investigate whether Jayden's theory is true.

INVESTIGATE WHAT HAPPENS WHEN MORE TRIALS ARE DONE

1. Perform 40 trials by drawing one card at a time from eight small cards with the names of the colours written on them, and enter your results in the second and third rows of the table below.

colour	yellow	green	pink	blue	red	brown	grey	black
tally marks								
frequencies								
relative frequencies as 40ths								
relative frequencies as 200ths								
probabilities as 200ths								

2. Make a copy of the above table, without the row for tally marks, and without the row for the relative frequencies as fortieths and the row for the probabilities, on a loose sheet of paper. Exchange it with a classmate. Enter the results of your classmate on table 1 and 2 on the next page. Also enter your own results for question 1 on the tables.

3. Get hold of the data reports of three other classmates, and enter these on the tables on the next page too.

4. Add the frequencies of the various colours in the five sets of data for 40 trials each, and calculate the relative frequencies expressed as twohundredths.

5. Is the range of relative frequencies for 200 trials smaller than the ranges for the five different sets of 40 trials each? What does this indicate with respect to Jayden's theory?

....

....

When only a small number of trials are done, the actual relative frequencies for different outcomes may differ a lot from the probabilities of the outcomes.

When many trials are done, the actual relative frequencies of the different outcomes are quite close to the probabilities of the outcomes.

Table 1: Frequencies for 5 sets of 40 trials each

colour	yellow	green	pink	blue	red	brown	grey	black
frequencies for your own 40 trials in question 1								
frequencies for 40 trials by classmate 1								
frequencies for 40 trials by classmate 2								
frequencies for 40 trials by classmate 3								
frequencies for 40 trials by classmate 4								
total frequencies for 200 trials								
relative frequencies for 200 trials as 200ths								

Table 2: Relative frequencies for each of the 5 sets of 40 trials each

(expressed as 200ths)

colour	yellow	green	pink	blue	red	brown	grey	black
relative frequencies for your own 40 trials								
relative frequencies for 40 trials by classmate 1								
relative frequencies for 40 trials by classmate 2								
relative frequencies for 40 trials by classmate 3								
relative frequencies for 40 trials by classmate 4								

6. How many different three-digit numbers can be formed with the symbols 3 and 5, if no other symbols are used? You may use one, two or three of the symbols in each number, and you may repeat the same symbol.

....

11.2 Compound events

TOSSING A COIN AND GIVING BIRTH

1. Simon threw a coin and the outcome was heads. He will now throw the coin again.
   (a) What are the possible outcomes? ....
   (b) What is the probability of each of the possible outcomes? ....
   (c) What are the possible outcomes if Simon throws the coin for a third time?
   ....
   (d) What is the probability of each of the possible outcomes for the third throw?
   ....

What happens when a coin is thrown for a second time has nothing to do with what happened when it was thrown the first time.

The first throw and the second throws are called **independent events**: what happened on the first throw cannot influence what will happen on the second throw.

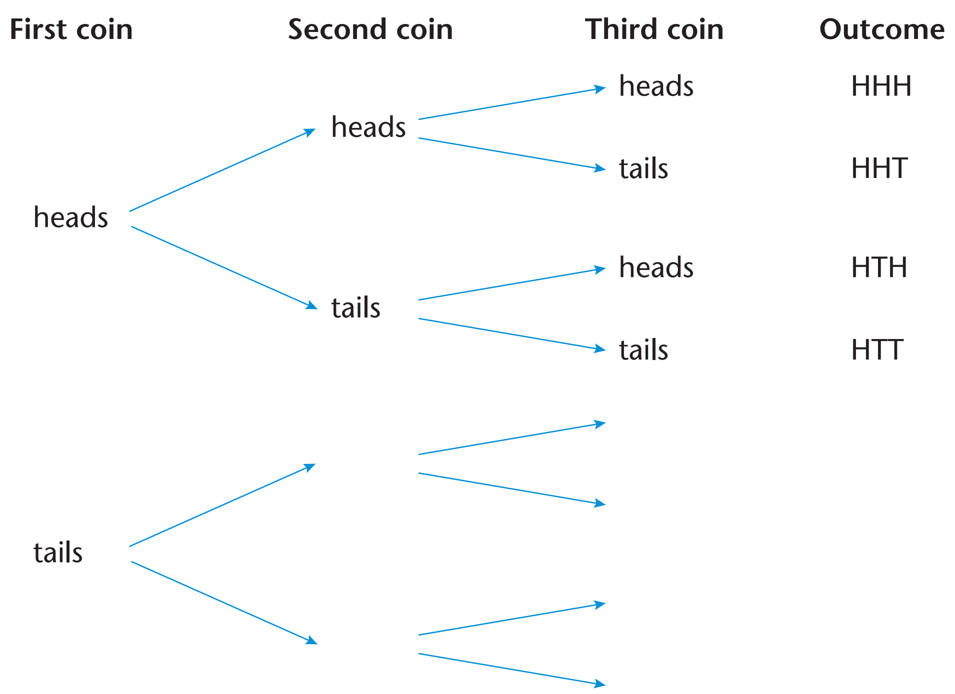
2. (a) If an event has four different equally-likely outcomes, what is the probability of each of the four outcomes? ....
   (b) Does that mean that if the event is repeated 4 times, each of the four outcomes will happen once? ....
   (c) Does your answer in (a) means that if the event is repeated 100 times, each of the four outcomes will happen 25 times? ....
   ....

3. (a) What are the possible outcomes when two coins are thrown? Use the **two-way table** below to answer this question. One possible outcome is already given.

	Heads	Tails
Heads		H T
Tails		

   (b) Do you think these four outcomes are equally likely? ....
   (c) What is the probability of each of the four outcomes? ....
   (d) What is the probability of getting a head and a tail? ....

4. Let us consider the possible outcomes if three coins are thrown.
Below is a tree diagram that can help you figure out what the different possible outcomes are. Complete the diagram by filling in the missing information.



5. (a) Do you think the eight different outcomes in question 4 are equally likely?

....

(b) What is the probability of each of the eight outcomes? ....

(c) What is the probability of throwing two heads and one tail?

....

6. In question 6 on page 183 you were asked to write down the various numbers that can be formed by using symbols 3 and 5. Think of all the four-letter codes that you can form by using only two letters, P and Q. Any letter can be used more than once in one code. First think about how you will go about finding all the possibilities in a systematic way and then try to set up a tree diagram to help you.

(a) Draw a tree diagram in your exercise book to help you to solve this problem. List all the outcomes.

....

....

(b) If the codes are formed by randomly choosing the letters, what is the probability that the code will consist of the using the same letter four times?

....

(c) What is the probability that the code will consist of two P's and two Q's?

....

When a woman is pregnant, the baby can be a boy or a girl. Suppose we make the assumption that the two possibilities are equally likely, so the probability of a boy is $\frac{1}{2}$ and the probability of a girl is $\frac{1}{2}$.

7. (a) Complete this two-way table to show the possible outcomes of the gender of the two children in a family

	Boy	Girl

(b) List the possible outcomes.

....

(c) What is the probability that the two children in the family will be of the same gender?

....

(d) What is the probability that the eldest child will be a boy and then they will have a girl?

....

8. A certain woman already has one child, which is a boy. She now expects a second child. What is the probability of it being a boy again, if we make the assumption that a baby being a boy or a girl are equally likely events?

....

The assumption that a boy or a girl being born are equally likely events may not actually be true. However, probabilities can only be calculated and used to make predictions if it is assumed that outcomes are equally likely.

9. (a) A woman gets married and plans to have a baby in one year and another baby in the next year. What is the probability that both babies will be girls?

.....

(b) A woman gets married and plans to have a baby in each of the first three years of the marriage. What is the probability that she will have a boy in the first year, and girls in the second and third years?

....