MATHEMATICS

Grade 9

Book 1

CAPS

Learner Book

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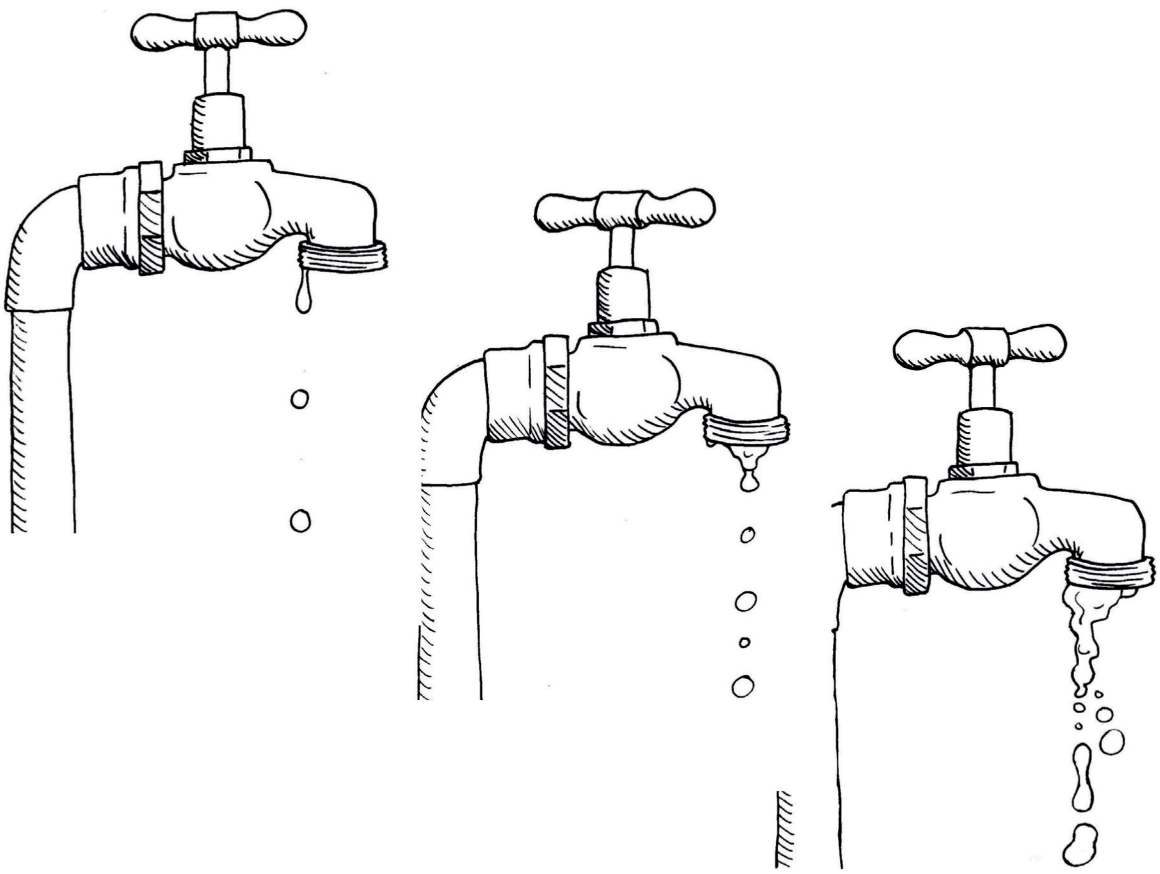
Chapter 1
Whole numbers

In this chapter you will engage with different kinds of numbers that are used for counting, measuring, solving equations and many other purposes.



				99	100	101	102	103
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1 Whole numbers

1.1 Properties of numbers

DIFFERENT TYPES OF NUMBERS

The natural numbers

The numbers that we use to count are called **natural numbers**:

1 2 3 4 5 6 7 8 9 10 11 12 13 14

Natural numbers have the following properties:

When you add two or more natural numbers, you get a natural number again.

When you multiply two or more natural numbers, you get a natural number again.

Mathematicians describe this by saying: The system of natural numbers is **closed under addition and multiplication**.

However, when a natural number is subtracted from another natural number the answer is not always a natural number again. For example, there is no natural number that provides the answer to 5 – 20.

Similarly, when a natural number is divided by another natural number the answer is not always a natural number again. For example, there is no natural number that provides the answer to 10 ÷ 3.

The system of natural numbers is **not closed under subtraction or division**.

When subtraction or division is done with natural numbers, the answers are not always natural numbers.

1. (a) Is there a smallest natural number, that means a natural number that is smaller than all other natural numbers? If so, what is it? ....

   (b) Is there a largest natural number, in other words, a natural number that is larger than all other natural numbers? If so, what is it? ....

2. In each of the following cases, say whether the answer is a natural number or not.

   (a) 100 + 400 ....

   (b) 100 – 400 ....

   (c) 100 × 400 ....

   (d) 100 ÷ 400 ....

The whole numbers

Although we don't use 0 for counting, we need it to write numbers. Without 0, we would need a special symbol for 10, all multiples of 10 and some other numbers. For example, all the numbers that belong in the yellow cells below would need a special symbol.

	41	42	43	44	45	46	47	48	49
	51	52	53	54	55	56	57	58	59
	61	62	63	64	65	66	67	68	69
	71	72	73	74	75	76	77	78	79
	81	82	83	84	85	86	87	88	89
	91	92	93	94	95	96	97	98	99
	111	112	113	114	115	116	117	118	119

The natural numbers combined with 0 is called the system of **whole numbers**.

If you are working with natural numbers and you add two numbers, the answer will always be different from any of the two numbers added. For example: 21 + 25 = 46 and 24 + 1 = 25. If you are working with whole numbers, in other words including 0, this is not the case. When 0 is added to a number the answer is just the number you start with: 24 + 0 = 24.

For this reason, 0 is called the **identity element** for addition. In the set of natural numbers there is no identity element for addition.

3. Is there an identity element for multiplication in the whole numbers? Explain your answer.

   ....

4. (a) What is the smallest natural number? ....

   (b) What is the smallest whole number? ....

The integers

In the set of whole numbers, no answer is available when you subtract a number from a number smaller than itself. For example there is no whole number that is the answer for 5 – 8. But there is an answer to this subtraction in the system of integers.

5 – 8 = –3. The number –3 is read as "negative 3" or "minus 3".

The whole numbers start with 0 and extend in one direction:

0 1 2 3 4 5 6 → → → ......

The integers extend in both directions:

...... ← ← ← –5 –4 –3 –2 –1 0 1 2 3 4 5 6 → → → ......

All whole numbers are also integers. The set of whole numbers forms part of the set of integers. For each whole number, there is a negative number that corresponds with it. The negative number –5 corresponds to the whole number 5 and the negative number –120 corresponds to the whole number 120.

Within the set of integers, the sum of two numbers can be 0.

For example 20 + (–20) = 0 and 135 + (–135) = 0.

20 and –20 are called **additive inverses** of each other.

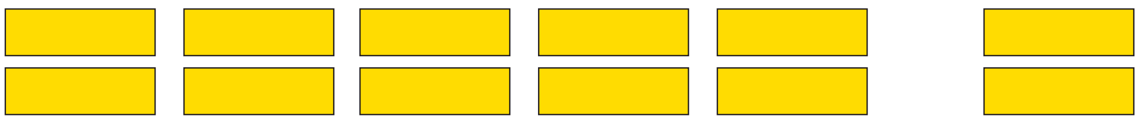
5. Calculate the following without using a calculator.

   (a) 100 – 165 .... (b) 300 – 700 ....

6. You may use a calculator to calculate the following:

   (a) 123 – 765 .... (b) 385 – 723 ....

The rational numbers



7. Five people share 12 slabs of chocolate equally among them.

   (a) Will each person get more or less than two full slabs of chocolate? ....

   (b) Can each person get another half of a slab? ....

   (c) How much more than two full slabs can each person get, if the two remaining slabs are divided as shown here?

   ....

   (d) Will each person get 2,4 or $2\frac{2}{5}$ slab? ....

The system of integers does not provide an answer for all possible division questions. For example, as we see above, the answer for 12 ÷ 5 is not an integer.

To have answers for all possible division questions, we have to extend the number system to include fractions and negative fractions, in other words, numbers of the form $\frac{\text{integer}}{\text{integer}}$. This system of numbers is called the **rational numbers**. We can represent rational numbers as common fractions or as decimal numbers.

8. Express the answers for each of the following division problems in two ways: using the common fraction notation and using the decimal notation for fractions.

   (a) 23 ÷ 10 (b) 23 ÷ 5

   ....

   (c) 230 ÷ 100 (d) 8 ÷ 10

   ....

9. Answer the statement by writing 'yes' or 'no' in the appropriate cell.

Statement	**Natural numbers**	**Whole numbers**	**Integers**	**Rational numbers**
The sum of two numbers is a number of the same kind (closed under addition).				
The sum of two numbers is always bigger than either of the two numbers.				
When one number is subtracted from another, the answer is a number of the same kind (closed under subtraction).				
When one number is subtracted from another, the answer is always smaller than the first number.				
The product of two numbers is a number of the same kind (closed under addition).				
The product of two numbers is always bigger than either of the two numbers.				
The quotient of two numbers is a number of the same kind (closed under division).				
The quotient of two numbers is always smaller than the first of the two numbers.				

Irrational numbers

Rational numbers do not provide for all situations that may occur in mathematics. For example, there is no rational number which will produce the answer 2 when it is multiplied by itself.

(number) × (same number) = 2

2 × 2 = 4 and 1 × 1 = 1, so clearly, this number must be between 1 and 2.

But there is no number which can be expressed as a fraction, in either the common fraction or the decimal notation, which will solve this problem. Numbers like these are called **irrational numbers**.

Here are some more examples of irrational numbers:

$\sqrt{5}$ $\quad$ $\sqrt{10}$ $\quad$ $\sqrt{3}$ $\quad$ $\sqrt{7}$ $\quad$ π

The rational and the irrational numbers together are called the **real numbers**.

1.2 Calculations with whole numbers

Do not use a calculator at all in Section 1.2.

ESTIMATING, ROUNDING OFF AND COMPENSATING

1. A shop owner wants to buy chickens from a farmer. The farmer wants R38 for each chicken. Answer the following questions without doing written calculations.
   (a) If the shop owner has R10 000 to buy chickens, do you think he can buy more than 500 chickens? ....
   (b) Do you think he can buy more than 200 chickens? ....
   (c) Do you think he can buy more than 250 chickens? ....

What you were trying to do in question 1 is called **estimation**. To estimate, when working with numbers, means to try to get close to an answer without actually doing the calculations. However, you can do other, simpler calculations to estimate.

When the goal is not to get an accurate answer, numbers may be rounded off. For example, the cost of 51 chickens at R38 each may be **approximated** by calculating 50 × 40. This is clearly much easier than calculating 51 × R38.

> To approximate something means to try to find out more or less how much it is, without measuring or calculating it precisely.

2. (a) How much is 5 × 4? ....
   (b) How much is 5 × 40? ....
   (c) How much is 50 × 40? ....

The cost of 51 chickens at R38 each is approximately R2 000.

This approximation was obtained by rounding both 51 and 38 off to the nearest multiple of 10, and then calculating with the multiples of 10.

3. In each case, estimate the cost by rounding off to calculate the approximate cost, without using a calculator. In each case make two estimates. First make a rough estimate by rounding the numbers off to the nearest 100 before calculating. Then make a better estimate by rounding the numbers off to the nearest 10 before calculating.
   (a) 83 goats are sold for R243 each.
   ....
   ....
   (b) 121 chairs are sold for R258 each.
   ....
   ....
   (c) R5 673 is added to R3 277.
   ....
   ....
   (d) R874 is subtracted from R1 234.
   ....
   ....

Suppose you have to calculate R823 – R273.
An estimate can be made by rounding the numbers off to the nearest 100:
R800 – R300 = R500.

4. (a) By working with R800 instead of R823, an error was introduced into your answer. How can this error be corrected: by adding R23 to the R500, or by subtracting it from R500? ....

(b) Correct the error to get a better estimate. ....

(c) Now also correct the error that was made by subtracting R300 instead of R273.

....

What you did in question 4 is called **compensating for errors**.

5. Estimate each of the following by rounding off the numbers to the nearest 100.

(a) 812 – 342

....

(b) 2 342 – 1 876

....

(c) 812 + 342

....

(d) 2 342 + 1 876

....

(e) 9 + 278

....

(f) 3 231 – 1 769

....

(g) 8 234 – 2 776

....

(h) 5 213 – 3 768

....

6. Find the exact answer for each of the calculations in question 5, by working out the errors caused by rounding, and compensating for them.

(a)

....
....
....

(b)

....
....
....

(c)

....
....
....

(d)

....
....
....

(e) ....

(f) ....

(g) ....

(h) ....

ADDING IN COLUMNS

1. (a) Write 8 000 + 1 100 + 130 + 14 as a single number: ....

   (b) Write 3 000 + 700 + 50 + 8 as a single number: ....

   (c) Write 5 486 in expanded notation, as shown in 1(b).

   ....

You can calculate 3 758 + 5 486 as shown on the left below.

	3 758
	5 486
Step 1	8 000
Step 2	1 100
Step 3	130
Step 4	14
	9 244

You can do this in short, as shown on the right. This is a bit harder on the brain, but it saves paper!

3 758
5 486
9 244

2. Explain how the numbers in each of steps 1 to 4 are obtained.

....
....
....
....

It is only possible to use the shorter method if you add the units first, then add the tens, then the hundreds and finally, the thousands. You can then do what you did in question 1(a), without writing the separate terms of the expanded form.

3. Calculate each of the following without using a calculator.

   (a) 3 878 + 3 784

   (b) 298 + 8 594

   (c) 10 921 + 2 472

   (d) 1 298 + 18 782

4. A farmer buys a truck for R645 840, a tractor for R783 356, a plough for R83 999 and a bakkie for R435 690.

   (a) Estimate to the nearest R100 000 how much these items will cost altogether.

   (b) Use a calculator to calculate the total cost.

5. An investor makes R543 682 in one day on the stock market and then loses R264 359 on the same day.

   (a) Estimate to the nearest R100 000 how much money she has made in total on that day.

   (b) Use a calculator to determine how much money she has made.

MULTIPLYING IN COLUMNS

1. (a) Write 3 489 in expanded notation: ....

   (b) Write an expression without brackets that is equivalent to

   7 × (3 000 + 400 + 80 + 9): ....

7 × 3 489 may be calculated as shown on the left below.

	3 489
	× 7
Step 1	63
Step 2	560
Step 3	2 800
Step 4	21 000
	24 423

A shorter method is shown on the right.

3 489
× 7
24 423

2. Explain how the numbers in each of steps 1 to 4 on the above left are obtained.

   ....

47 × 3 489 may be calculated as shown on the left below.

	3 489
	× 47
Step 1	63
Step 2	560
Step 3	2 800
Step 4	21 000
Step 5	360
Step 6	3 200
Step 7	16 000
Step 8	120 000
	163 983

A shorter method is shown on the right.

3 489
× 47
24 423
139 560
163 983

3. Explain how the numbers in each of steps 5 to 8 on the above left are obtained.

   ....

4. Explain how the number 139 560 that appears in the shorter form on the above right is obtained.

   ....

SUBTRACTING IN COLUMNS

1. Write each of the following as a single number.
   (a) 8 000 + 400 + 30 + 2 ....
   (b) 7 000 + 1 300 + 120 + 12 ....
   (c) 3 000 + 900 + 50 + 7 ....

2. If you worked correctly you should have obtained the same answers for questions 1(a) and 1(b). If this was not the case, redo your work.

The expression 7 000 + 1 300 + 120 + 12 was formed from 8 000 + 400 + 30 + 2 by

- taking 1 000 away from 8 000 and adding it to the hundreds term to get 1 400,
- taking 100 away from 1 400 and adding it to the tens term to get 130, and
- taking 10 away from 130 and adding it to the units term to get 12.

3. Form an expression like the expression in 1(b) for each of the following:
   (a) 8 000 + 200 + 100 + 4 (b) 3 000 + 400 + 30 + 1

   ....

4. Write expressions like in question 1(b) for the numbers below.
   (a) 7 214 .... (b) 8 103 ....

8 432 – 3 957 can be calculated as shown below.

	8 432
	– 3 957
Step 1	5
Step 2	70
Step 3	400
Step 4	4 000
Step 5	4 475

To do the subtraction in each column, you need to think of 8 432 as 8 000 + 400 + 30 + 2, in fact you have to think of it as 7 000 + 1 300 + 120 + 12.

In step 1, the 7 of 3 957 is subtracted from 12.

5. (a) How is the 70 in step 2 obtained? ....

   ....

   (b) How is the 400 in step 3 obtained? ....

   ....

   (c) How is the 4 000 in step 4 obtained? ....

   ....

   (d) How is the 4 475 in step 5 obtained? ....

   ....

Because of the zeros obtained in steps 2, 3 and 4, the answers need not be written separately as shown above. The work can actually be shown in the short way on the right.

$$\begin{array}{r} 8\,432 \\ -\ \underline{3\,957} \\ 4\,475 \end{array}$$

6. Calculate each of the following without using a calculator.
   (a) 9 123 – 3 784
   (b) 8 284 – 3 547

7. Use a calculator to check your answers. If your answers are wrong, try again.

8. Calculate each of the following without using a calculator.
   (a) 7 243 – 3 182
   (b) 6 221 – 1 888

You may use a calculator to do the questions below.

9. Bettina has R87 456 in her savings account. She withdraws R44 800 to buy a car. How much money is left in her savings account?

10. Liesbet starts a savings account by making a deposit of R40 000. Over a period of time she does the following transactions on the savings account:

    a withdrawal of R4 000
    a withdrawal of R2 780
    a deposit of R1 200
    a deposit of R7 550
    a withdrawal of R5 230
    a deposit of R8 990
    a deposit of R1 234

    How much money does she have in her savings account now? ....

11. (a) R34 537 – R13 267
    (b) R135 349 – R78 239

LONG DIVISION

Study this method for calculating 13 254 ÷ 56:

	13 254	
200 × 56 = 11 200	11 200	(200 is a rough estimate of the answer for 13 254 ÷ 56)
	2 054	(2 054 remains after 11 200 is taken from 13 254)
30 × 56 = 1 680	1 680	(30 is a rough estimate of the answer for 2 054 ÷ 56)
	374	(374 remains after 1 680 is taken from 2 054)
6 × 56 = 336	336	(6 is an estimate of the answer for 374 ÷ 56)
236 × 56 = 13 216	38	(38 remains)

So 13 254 ÷ 56 = 236 remainder 38, or 13 254 ÷ 56 = $236\frac{38}{56} = 236\frac{19}{28}$,
which can also be written as 236,68 (correct to two decimal figures).

The work can also be set out as follows:

$$\begin{array}{r|r}
 & 6 \\
 & 30 \\
 & 200 \\
\hline
56 & 13\ 254 \\
 & \underline{11\ 200} \\
 & 2\ 054 \\
 & \underline{1\ 680} \\
 & 374 \\
 & \underline{336} \\
 & 38
\end{array}
\quad \text{or more briefly as} \quad
\begin{array}{r|r}
 & 236 \\
\hline
56 & 13\ 254 \\
 & \underline{11\ 200} \\
 & 2\ 054 \\
 & \underline{1\ 680} \\
 & 374 \\
 & \underline{336} \\
 & 38
\end{array}$$

1. (a) Mlungisi's work to do a certain calculation is shown on the right. What is the question that Mlungisi tries to answer?

$$\begin{array}{lr|r}
 & & 463 \\
\hline
 & 78 & 36\ 177 \\
\text{Step 1} & & \underline{31\ 200} \\
\text{Step 2} & & 4\ 977 \\
\text{Step 3} & & \underline{4\ 680} \\
\text{Step 4} & & 297 \\
\text{Step 5} & & \underline{234} \\
 & & 63
\end{array}$$

....

(b) Where does the number 31 200 in step 1 come from? How did Mlungisi obtain it, and for what purpose did he calculate it?

....

....

(c) Explain step 2 in the same way as you explained step 1.

....

(d) Explain step 3.

....

2. Calculate each of the following without using a calculator.

(a) 33 030 ÷ 63

(b) 18 450 ÷ 27

3. Use a calculator to check your answers to question 2. If your answers are wrong, try again. It is important that you learn to do long division correctly.

4. Calculate each of the following without using a calculator.

(a) 76 287 ÷ 287

(b) 65 309 ÷ 44

Use your calculator to do questions 5 and 6 below.

5. A municipality has budgeted R85 000 for putting up new street name boards. The street name boards cost R72 each. How many new street name boards can be put up, and how much money will be left in the budget?

6. A furniture dealer quoted R840 000 for supplying 3 450 school desks. A school supply company quoted R760 000 for supplying 2 250 of the same desks. Which provider is cheapest, and what do the two providers actually charge for one school desk?

1.3 Multiples and factors

LOWEST COMMON MULTIPLES AND PRIME FACTORISATION

1. Consecutive multiples of 6, starting at 6 itself, are shown in the table below.

6	12	18	24	30	36	42	48	54	60
66	72	78	84	90	96	102	108	114	120
126	132	138	144	150	156	162	168	174	180
186	192	198	204	210	216	222	228	234	240

(a) The table below also shows multiples of a number. What is the number? ....

15	30	45	60	75	90	105	120	135	150
165	180	195	210	225	240	255	270	285	300
315	330	345	360	375	390	405	420	435	450
465	480	495	510	525	540	555	570	585	600

(b) Draw rough circles around all the numbers that occur in both the above tables.

(c) What is the smallest number that occurs in both tables? ....

90 is a multiple of 6, it is also a multiple of 15.
90 is called a **common multiple** of 6 and 15, it is a multiple of both.
The smallest number that is a multiple of both 6 and 15 is the number 30.
30 is called the **lowest common multiple** or **LCM** of 6 and 15.

2. Calculate, without using a calculator.

(a) 2 × 3 × 5 × 7 × 11

....

(b) 2 × 2 × 5 × 7 × 13

....

(c) 2 × 3 × 3 × 3 × 5 × 13

....

(d) 3 × 5 × 5 ×17

....

Check your answers by using a calculator or by comparing with some classmates.
2 is a factor of each of the numbers 2 310, 1 820 and 3 510.
Another way of saying this is: 2 is a **common factor** of 2 310, 1 820 and 3 510.

3. (a) Is 2 × 3, in other words, 6, a common factor of 2 310 and 3 510? ....

   (b) Is 2 × 3 × 5, in other words, 30, a common factor of 2 310 and 3 510? ....

   (c) Is there any bigger number than 30 that is a common factor of 2 310 and 3 510?

   ....

30 is called the **highest common factor** or **HCF** of 2 310 and 3 510.

In question 2 you can see the list of **prime factors** of the numbers 2 310, 1 820, 3 510 and 1 275.

> The LCM of two numbers can be found by multiplying all the prime factors of both numbers, without repeating (except where a number is repeated as a factor in one of the numbers).
>
> The HCF of two numbers can be found by multiplying the factors that are common to the two numbers, i.e. in the list of prime factors of both numbers.

4. In each case find the HCF and LCM of the numbers.

   (a) 1 820 and 3 510

   ....
   ....
   ....
   ....

   (b) 2 310 and 1 275

   ....
   ....
   ....
   ....

   (c) 1 820 and 3 510 and 1 275

   ....
   ....
   ....
   ....
   ....

   (d) 2 310 and 1 275 and 1 820

   ....
   ....
   ....
   ....
   ....

   (e) 780 and 7 700

   ....
   ....
   ....
   ....

   (f) 360 and 1 360

   ....
   ....
   ....
   ....

1.4 Solving problems about ratio, rate and proportion

RATIO AND RATE PROBLEMS

You may use a calculator in this section.

1. Moeneba collects apples in the orchard. She picks about 5 apples each minute. Approximately how many apples will Moeneba pick in each of the following periods of time?

   (a) 8 minutes .... (b) 11 minutes ....

   (c) 15 minutes .... (d) 20 minutes ....

In the situation described in question 1, Moeneba picks apples **at a rate of** about 5 apples **per minute**.

2. Garth and Kate also collect apples in the orchard, but they both work faster than Moeneba. Garth collects at a rate of about 12 apples per minute, and Kate collects at a rate of about 15 apples per minute. Complete the following table to show approximately how many apples they will each collect in different periods of time.

Period of time in min	1	2	3	8	10	20
Moeneba	5			40		
Garth	12					
Kate	15					
The three together	32					

In this situation, the number of apples picked is **directly proportional** to the time taken.

If you filled the table in correctly, you will notice that during any period of time, Kate collected 3 times as many apples as Moeneba. We can say that during any time interval, the **ratio** between the numbers of apples collected by Moeneba and Kate is **3 to 1**, which can be written as **3:1**. For any period of time, the ratio between the numbers of apples collected by Garth and Moeneba is 12:5.

3. (a) What is the ratio between the numbers of apples collected by Kate and Garth during a period of time? ....

   (b) Would it be correct to also say that the ratio between the numbers of apples collected by Kate and Garth is 5:4? Explain your answer.

   ....

4. To make biscuits of a certain kind, 5 parts of flour has to be mixed with 2 parts of oatmeal, and 1 part of cocoa powder. How much oatmeal and how much cocoa powder must be used if 500 g of flour is used?

....

5. A motorist covers a distance of 360 km in exactly 4 hours.
   (a) Approximately how far did the motorist drive in 1 hour? ....
   (b) Do you think the motorist covered exactly 90 km in each of the 4 hours?

   Explain your answer briefly. ....

   ....
   (c) Approximately how far will the motorist drive in 7 hours? ....
   (d) Approximately how long will the motorist need to travel 900 km? ....

Some people use these formulae to do calculations like those in question 5.

$\textbf{average speed} = \dfrac{\textbf{distance}}{\textbf{time}}$, which here means distance ÷ time

$\textbf{distance} = \textbf{average speed} \times \textbf{time}$

$\textbf{time} = \dfrac{\textbf{distance}}{\textbf{average speed}}$, which here means distance ÷ average speed

6. For each of questions 5(c) and 5(d), state which formula will produce the correct answer.

....

7. A motorist completes a journey in three sections, making two long stops to eat and relax between sections. During section A he covers 440 km in 4 hours. During section B he covers 540 km in 6 hours. During section C he covers 280 km in 4 hours.
   (a) Calculate his average speed over each of the three sections.

   ....

   ....

   ....
   (b) Calculate his average speed for the journey as a whole.

   ....

   ....

   ....
   (c) On the next day, the motorist has to travel 874 km. How much time (stops excluded) will he need to do this? Justify your answer with calculations.

   ....

   ....

8. Different vehicles travel at different average speeds. A large transport truck with a heavy load travels much slower than a passenger car. A small bakkie is also slower than a passenger car. In the following table, some average speeds and the times needed are given for different vehicles that all have to be driven for the same distance of 720 km. Complete the table.

Time in hours	12	9	8	6	5
Average speed in km/h	60				

9. Look at the table you have just completed.
   (a) What happens to the time needed if the average speed increases?

   ....

   (b) What happens to the average speed if the time is reduced? ....

   (c) What can you say about the product average speed × time, for the numbers in the table? ....

In the situation above, the average speed is said to be **indirectly proportional** to the time needed for the journey.

1.5 Solving problems in financial contexts

You may use a calculator in Section 1.5.

DISCOUNT, PROFIT AND LOSS

1. (a) R12 800 is divided equally between 100 people.
   How much money does each person get? ....
   (b) How much money do eight of the people together get?

   ....

Another word for hundredths is **percent**.

Instead of $\frac{5}{100}$ we can write 5%. The symbol % means exactly the same as $\frac{}{100}$.
In question 1(a) you calculated $\frac{1}{100}$ or 1% of R12 800, and in question 1(b) you calculated $\frac{8}{100}$ or 8% of R12 800.

The amount that a dealer pays for an article is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after discount is the **selling price**.

2. The marked prices of some articles are given below. A discount of 15% is offered to customers who pay cash. In each case calculate how much a customer who pays cash will actually pay.
   (a) R850 ....
   (b) R140 ....
   (c) R32 600 ....
   (d) R138 ....

Lina bought a couch at a sale. It was marked R3 500 but she paid only R2 800. She was given a discount of R700.

What percentage discount was given to Lina?

This question means:

How many hundredths of the marked price were taken off?

To answer the question we need to know how much $\frac{1}{100}$ (one hundredth) of the marked price is.

3. (a) How much is $\frac{1}{100}$ of R3 500? ....
   (b) How many hundredths of R3 500 is the same as R700?
   ....
   (c) What percentage discount was given to Lisa: 10% or 20%?
   ....

4. The cost price, marked price and selling price of some articles are listed below.
   Article A: Cost price = R240, marked price = R360, selling price = R324.
   Article B: Cost price = R540, marked price = R700, selling price = R560.
   Article C: Cost price = R1 200, marked price = R2 000, selling price = R1 700.
   The profit is the difference between the cost price and the selling price.
   For each of the above articles, calculate the percentage discount and profit.
   ....
   ....
   ....
   ....
   ....
   ....
   ....

5. Remey decided to work from home and bought herself a sewing machine for R750. She planned to make 40 covers for scatter cushions. The fabric and other items she needed cost her R3 600. Remey planned to sell the covers at R150 each.

   (a) How much profit could Remey make if she sold all 40 covers at this price?

   ....

   ....

   ....

   (b) Remey managed to sell only 25 of the covers and decided to sell the rest at R100 each. Calculate her percentage profit.

   ....

   ....

   ....

6. Zadie bakes and sells pies to earn some extra income. The cost of the ingredients for her chicken pies came to about R68. She sold the pies for R60. Did she make a profit or a loss? Calculate the percentage loss or profit.

   ....

HIRE PURCHASE

Sometimes you need an item but do not have enough money to pay the full amount immediately. One option is to buy the item on **hire purchase (HP)**. You will have to pay a deposit and sign an agreement in which you undertake to pay monthly instalments until you have paid the full amount. Therefore:

HP price = deposit + total of instalments

The difference between the HP price and the cash price is the interest that the dealer charges you for allowing you to pay off the item over a period of time.

1. Sara buys a flat screen television on hire purchase. The cash price is R4 199. She has to pay a deposit of R950 and 12 monthly instalments of R360.

   (a) Calculate the total HP price.

   ....

   (b) How much interest does she pay?

   ....

2. Susie buys a car on hire purchase. The car costs R130 000. She pays a 10% deposit on the cash price and will have to pay monthly instalments of R4 600 for a period of three years. David buys the same car, but chooses another option where he has to pay a 35% deposit on the cash price and monthly instalments of R3 950 for two years.
   (a) Calculate the HP price for both options.

   ....

   ....

   (b) Calculate the difference between the total price paid by Susie and by David.

   ....

   (c) Calculate the interest that Susie and David have to pay as a percentage of the cash price.

   ....

   ....

   ....

   ....

SIMPLE INTEREST

When interest is calculated for a number of years on an amount (i.e. a fixed deposit) without the interest being added to the amount each year for the purpose of later interest calculations, it is referred to as simple interest. If the amount is invested for part of a year, the time must be written as a fraction of a year.

1. Interest rates are normally expressed as percentages. This makes it easier to compare rates. Express each of the following as a percentage:

   (a) A rate of R5 for every R100

   ....

   (b) A rate of R7,50 for every R50

   ....

   (c) A rate of R20 for every R200

   ....

   (d) A rate of x rands for every a rands

   ....

Per annum means "per year".

2. Annie deposits R8 345 into a savings account at Bonus Bank. The interest rate is 9% per annum.
   (a) How much interest will she have earned at the end of the first year?

   ....

   (b) Annie decides to leave the deposit of R8 345 with the bank for an indefinite period, and to withdraw only the interest at the end of every year. How much interest does she receive over a period of five years?

   ....

3. Maxi invested R3 500 at an interest rate of 5% per annum. Her total interest was R875. For what period did she invest the amount?

   ....

4. Money is invested for 1 year at an interest rate of 8% per annum. Complete the table of equivalent rates.

Sum invested (R)	1 000	2 500	8 000	20 000	90 000	x
Interest earned (R)						

5. Interest on overdue accounts is charged at a rate of 20% per annum. Calculate the interest due on an account that is 10 days overdue if the amount owing is R260. (Give your answer to the nearest cent.)

   ....

6. A sum of money invested in the bank at 5% per annum, simple interest, amounted to R6 250 after 5 years. This final amount includes the interest. Thuli figured out that the final amount is $(1 + 0{,}05 \times 5) \times$ amount invested.
   (a) Explain Thuli's thinking. ....

   ....

   ....

   ....

   ....

   (b) Calculate the amount that was invested. ....

   ....

   ....

COMPOUND INTEREST

When the interest earned each year is added to the original amount, and the interest for the following year is calculated on this new amount, the result is known as **compound interest**.

Example:

R2 000 is invested at 10% per annum compound interest:

End of 1st year: Amount = R2 000 + R200 interest = R2 200

End of 2nd year: Amount = R2 200 + R220 interest = R2 420

End of 3rd year: Amount = R2 420 + R242 interest = R2 662

1. An amount of R20 000 is invested at 5% per annum compound interest.

   (a) What is the total value of the investment after 1 year?

   ....

   (b) What is the total value of the investment after 2 years?

   ....

   (c) What is the total value of the investment after 3 years?

   ....

2. Bonus Bank is offering an investment scheme over two years at a compound interest rate of 15% per annum. Mr Pillay wishes to invest R800 in this way.

   (a) How much money will be due to him at the end of the two-year period?

   ....

   ....

   (b) How much interest will have been earned during the two years?

   ....

3. Andrew and Zinzi are arguing about interest on money that they have been given for Christmas. They each received R750. Andrew wants to invest his money in ABC Building Society for 2 years at a compound interest rate of 14% per annum, while Zinzi claims that she will do better at Bonus Bank, earning 15% simple interest per annum over 2 years. Who is correct?

   ....

   ....

   ....

4. Mr Martin invests R12 750 for 3 years compounded quarterly (i.e. four times a year) at 5,3%.
   (a) How many conversion periods will his investment have altogether?

   ....

   (b) How much is his investment worth after 3 years?

   ....

   ....

   (c) Calculate the total interest that he earns on his initial investment.

   ....

5. Calculate the interest generated by an investment (P) of R5 000 at 10% (r) compound interest over a period (n) of 3 years. A is the final amount. Use the formula $A = P(1 + \frac{r}{100})^n$ to calculate the interest.

   ....

   ....

EXCHANGE RATE AND COMMISSION

1. (a) Tim bought £650 at the foreign exchange desk at Gatwick Airport in the UK at a rate of R15,66 per £1. The desk also charged 2,5% commission on the transaction. How much did Tim spend to buy the pounds?

   ....

   ....

   (b) What was the value of R1 in British pounds on that day?

   ....

2. Mandy wants to order a book from the internet. The price of the book is $25,86. What is the price of the book in rands? Take the exchange rate as R9,95 for $1.

   ....

3. Bongani is a car salesperson. He earns a commission of 3% on the sale of a car with the value of R220 000. Calculate how much commission he earned.

   ....

Chapter 2
Integers

In this chapter you will work with numbers smaller than 0. These numbers are called negative numbers. They have special properties that make them useful for specific purposes, for example they enable us to solve an equation such as $x + 20 = 10$.

5	+	−15	=		5	−	−15	=		5	×	−15	=	
4	+	−14	=		4	−	−14	=		4	×	−14	=	
3	+	−13	=		3	−	−13	=		3	×	−13	=	
2	+	−12	=		2	−	−12	=		2	×	−12	=	
1	+	−11	=		1	−	−11	=		1	×	−11	=	
0	+	−10	=		0	−	−10	=		0	×	−10	=	
−1	+	−9	=		−1	−	−9	=		−1	×	−9	=	
−2	+	−8	=		−2	−	−8	=		−2	×	−8	=	
−3	+	−7	=		−3	−	−7	=		−3	×	−7	=	
−4	+	−6	=		−4	−	−6	=		−4	×	−6	=	
−5	+	−5	=		−5	−	−5	=		−5	×	−5	=	
−6	+	−4	=		−6	−	−4	=		−6	×	−4	=	
−7	+	−3	=		−7	−	−3	=		−7	×	−3	=	
−8	+	−2	=		−8	−	−2	=		−8	×	−2	=	
−9	+	−1	=		−9	−	−1	=		−9	×	−1	=	
−10	+	0	=		−10	−	0	=		−10	×	0	=	
−11	+	1	=		−11	−	1	=		−11	×	1	=	
−12	+	2	=		−12	−	2	=		−12	×	2	=	
−13	+	3	=		−13	−	3	=		−13	×	3	=	
−14	+	4	=		−14	−	4	=		−14	×	4	=	
−15	+	5	=		−15	−	5	=		−15	×	5	=	

2 Integers

2.1 Which numbers are smaller than 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

Numbers such as –7 and –500, the additive inverses of whole numbers, are included with all the whole numbers and called **integers**.
Fractions can be negative too, e.g. $-\frac{3}{4}$ and –3,46.

The natural numbers are used for counting, and fractions (rational numbers) are used for measuring. Why do we also have negative numbers?

When a larger number is subtracted from a smaller number, the answer may be a negative number: 5 – 12 = –7, and this number is called **negative 7**.

One of the most important reasons for inventing negative numbers was to provide solutions for equations like these:

Equation	Solution	Required property of negative numbers
$17 + x = 10$	$x = -7$ because $17 + (-7) = 17 - 7$ $= 10$	1. Adding a negative number is just like subtracting the corresponding positive number
$5 - x = 9$	$x = -4$ because $5 - (-4) = 5 + 4 = 9$	2. Subtracting a negative number is just like adding the corresponding positive number
$20 + 3x = 5$	$x = -5$ because $3 \times (-5) = -15$	3. The product of a positive number and a negative number is a negative number

PROPERTIES OF INTEGERS

1. In each case, state what number will make the equation true. Also state which of the properties of integers in the table above, is demonstrated by the equation.

(a) $20 - x = 50$

(b) $50 + x = 20$

....

....

....

(c) $20 - 3x = 50$

(d) $50 + 3x = 20$

....

....

2.2 Adding and subtracting with integers

Addition and subtraction of negative numbers

Examples: (–5) + (–3) and (–20) – (–7)

This is done in the same way as the addition and subtraction of positive numbers.

(–5) + (–3) = –8 and –20 – (–7) = –13

This is just like 5 + 3 = 8 and 20 – 7 =13, or R5 + R3 = R8, and R20 – R7 = R13.

(–5) + (–3) can also be written as –5 + (–3) or as –5 + –3

Subtraction of a larger number from a smaller number

Examples: 5 – 9 and 29 – 51

Let us first consider the following:

5 + (–5) = 0 10 + (–10) = 0 and 20 + (–20) = 0

If we subtract 5 from 5, we get 0, but then we still have to subtract 4:

$$5 - 9 = \underbrace{5 - 5} - 4$$
$$= 0 - 4$$
$$= -4$$

We know that –9 = (–4) + (–5)

Suppose the numbers are larger, e.g. 29 – 51:

29 – 51 = 29 – 29 – 22

–51 = (–29) + (–22)

How much will be left of the 51, after you have subtracted 29 from 29 to get 0?
How can we find out? Is it 51 – 29?

Addition of a positive and a negative number

Examples: 7 + (–5); 37 + (–45) and (–13) + 45

The following statement is true if the unknown number is 5:

20 – (a certain number) = 15

We also need numbers that will make sentences like the following true:

20 + (a certain number) = 15

But to go from 20 to 15 you have to subtract 5.

The number we need to make the sentence *20 + (a certain number) = 15* true must have the following strange property:

If you **add** this number, it should have the **same effect** as **subtracting 5**.

So mathematicians agreed that the number called negative 5 will have the property that if you add it to another number, the effect will be the same as subtracting the natural number 5.

This means that mathematicians agreed that 20 + (–5) is equal to 20 – 5. In other words, instead of adding *negative* 5 to a number, you may subtract 5.

> Adding a negative number has the same effect as subtracting a corresponding natural number.
> For example: 20 + (–15) = 20 – 15 = 5.

Subtraction of a negative number

We have dealt with cases like –20 – (–7) on the previous page.

The following statement:

25 + (a certain number) = 30

is true if the number is 5

We also need a number to make this statement true:

25 – (a certain number) = 30

If you subtract this number, it should have the same effect that adding 5.

It was agreed that 25 – (–5) is equal to 25 + 5

Instead of subtracting the negative number, you add the corresponding positive number (the additive inverse).

$$\begin{aligned} 8 - (-3) &= 8 + 3 \\ &= 11 \\ -5 - (-12) &= -5 + 12 \\ &= 7 \end{aligned}$$

> We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**.

Subtraction of a positive number from a negative number

For example: –7 – 4 actually means (–7) – 4.

Instead of subtracting a positive number, you add the corresponding negative number.

–7 – 4 can be seen as (–7) + (–4) = –11

CALCULATIONS WITH INTEGERS

Calculate.

1. $-7 + 18$

....

....

2. $24 - 30 - 7$

....

....

3. $-15 + (-14) - 9$

....

....

4. $35 - (-20)$

....

....

5. $30 - 47$

....

....

6. $(-12) - (-17)$

....

....

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate.

   (a) $-7 + -7 + -7 + -7 + -7 + -7 + -7 + -7 + -7 + -7$

   ....

   ....

   (b) $-10 + -10 + -10 + -10 + -10 + -10 + -10$

   ....

   ....

   (c) $10 \times (-7)$

   ....

   (d) $7 \times (-10)$

   ....

2. Say whether you agree (✓) or (✗) disagree with each statement.

   (a) $10 \times (-7) = 70$
   (b) $9 \times (-5) = (-9) \times 5$
   (c) $(-7) \times 10 = 7 \times (-10)$
   (d) $9 \times (-5) = -45$
   (e) $(-7) \times 10 = 10 \times (-7)$
   (f) $5 \times (-9) = 45$

Multiplication of integers is commutative:
$(-20) \times 5 = 5 \times (-20)$

THE DISTRIBUTIVE PROPERTY

1. Calculate each of the following. Note that brackets are used for two purposes in these expressions: to indicate that certain operations are to be done first, and to show the integers.

(a) $20 + (-5)$ (b) $4 \times \mathbf{(}20 + (-5)\mathbf{)}$ (c) $4 \times 20 + 4 \times (-5)$

....

....

(d) $(-5) + (-20)$ (e) $4 \times \mathbf{(}(-5) + (-20)\mathbf{)}$ (f) $4 \times (-5) + 4 \times (-20)$

....

....

2. If you worked correctly, your answers for question 1 should be 15; 60; 60; –25; –100 and –100. If your answers are different, check to see where you went wrong and correct your work.

3. Calculate each of the following where you can.

(a) $20 + (-15)$ (b) $4 \times \mathbf{(}20 + (-15)\mathbf{)}$ (c) $4 \times 20 + 4 \times (-15)$

....

(d) $(-15) + (-20)$ (e) $4 \times \mathbf{(}(-15) + (-20)\mathbf{)}$ (f) $4 \times (-15) + 4 \times (-20)$

....

(g) $10 + (-5)$ (h) $(-4) \times \mathbf{(}10 + (-5)\mathbf{)}$ (i) $(-4) \times 10 + ((-4) \times (-5))$

....

4. What property of integers is demonstrated in your answers for questions 3(a) and (g)? Explain your answer.

....

In question 3(i) you had to multiply two negative numbers. What was your guess?

We can calculate $(-4) \times \mathbf{(}10 + (-5)\mathbf{)}$ as in (h). It is $(-4) \times 5 = -20$

If we want the distributive property to be true for integers, then $(-4) \times 10 + (-4) \times (-5)$ must be equal to –20.

$(-4) \times 10 + (-4) \times (-5) = -40 + (-4) \times (-5)$

Then $(-4) \times (-5)$ must be equal to 20.

5. Calculate:

(a) $10 \times 50 + 10 \times (-30)$ ....

(b) $50 + (-30)$ ....

(c) $10 \times \mathbf{(}50 + (-30)\mathbf{)}$ ....

(d) $(-50) + (-30)$ ....

(e) $10 \times (-50) + 10 \times (-30)$ ....

(f) $10 \times \mathbf{(}(-50) + (-30)\mathbf{)}$ ....

- The product of two positive numbers is a positive number, for example $5 \times 6 = 30$.
- The product of a positive number and a negative number is a negative number, for example $5 \times (-6) = -30$.
- The product of a negative number and a positive number is a negative number, for example $(-5) \times 6 = -30$.

6. (a) Underline the numerical expression below which you would expect to have the same answers. Do not do the calculations.

$16 \times (53 + 68)$ $\quad$ $53 \times (16 + 68)$ $\quad$ $16 \times 53 + 16 \times 68$ $\quad$ $16 \times 53 + 68$

(b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?

....

7. Consider your answers for question 5.

(a) Does multiplication distribute over addition in the case of integers? ....

(b) Illustrate your answer with two examples.

....

....

8. Underline the numerical expression below which you would expect to have the same answers. Do not do the calculations now.

$10 \times \mathbf{(}(-50) - (-30)\mathbf{)}$ $\quad$ $10 \times (-50) - (-30)$ $\quad$ $10 \times (-50) - 10 \times (-30)$

9. Do the three sets of calculations given in question 8.

....

....

10. Calculate (–10) × **(**5 + (–3)**)**.

....

....

11. Now consider the question whether multiplication by a negative number distributes over addition and subtraction of integers. For example, would (–10) × 5 + (–10) × (–3) also have the answer –20, like (–10) × **(**5 + (–3)**)**?

....

....

To make sure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that

(a negative number) × (a negative number) is a positive number,

for example (–10) × (–3) = 30.

12. Calculate each of the following.

(a) (–20) × (–6)

....

....

(b) (–20) × 7

....

....

(c) (–30) × (–10) + (–30) × (–8)

....

(d) (–30) × **(**(–10) + (–8)**)**

....

(e) (–30) × (–10) – (–30) × (–8)

....

....

(f) (–30) × **(**(–10) – (–8)**)**

....

....

Here is a summary of the properties of integers that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0.
  For example, (+12) + (–12) = 0.
- Adding an integer has the same effect as subtracting its additive inverse.
  For example, 3 + (–10) can be calculated by doing 3 – 10, and the answer is –7.
- Subtracting an integer has the same effect as adding its additive inverse.
  For example, 3 – (–10) can be calculated by calculating 3 + 10 is 13.
- The product of a positive and a negative integer is negative.
  For example, (–15) × 6 = –90.
- The product of a negative and a negative integer is positive.
  For example (–15) × (–6) = 90.

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DIVISION WITH INTEGERS

1. Calculate

(a) $5 \times (-7)$ ....

(b) $(-3) \times 20$ ....

(c) $(-5) \times (-10)$ ....

(d) $(-3) \times (-20)$ ....

2. Use your answers in question 1 to determine the following:

(a) $(-35) \div 5$ ....

(b) $(-35) \div (-7)$ ....

(c) $(-60) \div 20$ ....

(d) $(-60) \div (-3)$ ....

(e) $50 \div (-5)$ ....

(f) $50 \div (-10)$ ....

(g) $60 \div (-20)$ ....

(h) $60 \div (-3)$ ....

- The quotient of a positive number and a negative number is a negative number.
- The quotient of two negative numbers is a positive number.

MIXED CALCULATIONS WITH INTEGERS

1. Calculate.

(a) $20(-50 + 7)$ ....

(b) $20 \times (-50) + 20 \times 7$ ....

(c) $20(-50 + -7)$ ....

(d) $20 \times (-50) + 20 \times -7$ ....

(e) $-20(-50 + -7)$ ....

(f) $-20 \times -50 + -20 \times -7$ ....

2. Calculate.

(a) $40 \times (-12 + 8) -10 \times (2 + -8) - 3 \times (-3 - 8)$

....

....

(b) $(9 + 10 - 9) \times 40 + (25 - 30 - 5) \times 7$

....

....

(c) $-50(40 - 25 + 20) + 30(-10 + 7 + 13) - 40(-16 + 15 - 2)$

....

....

(d) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$

....

....

(e) $-3 \times (-14 - 6 + 5) \times (-13 - 7 + 10) \times (20 - 10 - 15)$

....

2.4 Powers, roots and word problems

Answer all questions in this section *without* using a calculator.

1. Complete the tables.

(a)

x	1	2	3	4	5	6	7	8	9	10	11	12
x^2												
x^3												

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
x^2												
x^3												

3^2 is 9 and $(-3)^2$ is also 9.
3^3 is 27 and $(-5)^3$ is -125.
Both (–3) and 3 are **square roots** of 9
3 may be called the **positive square root** of 9, and (–3) may be called the **negative square root** of 9.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.
10^2 is 100 and $(-10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.

The symbol $\sqrt{\ }$ means that you must take the **positive square root** of the number.

2. Calculate the following:

(a) $\sqrt{4} - \sqrt{9}$

....

....

(b) $\sqrt[3]{27} + (-\sqrt[3]{64})$

....

....

(c) $-(3^2)$

....

(d) $(-3)^2$

....

(e) $4^2 - 6^2 + 1^2$

....

....

(f) $3^3 - 4^3 - 2^3 - 1^3$

....

....

(g) $\sqrt{81} - \sqrt{4} \times \sqrt[3]{125}$

....

....

....

(h) $-(4^2)(-1)^2$

....

....

....

(i) $\dfrac{(-5)^2}{\sqrt{37-12}}$

....

....

(j) $\dfrac{-\sqrt{36}}{-1^3 - 2^3}$

....

....

3. Determine the answer to each of the following:

(a) The overnight temperature in Polokwane drops from 11 °C to –2 °C. By how many degrees has the temperature dropped?

....

(b) The temperature in Estcourt drops from 2 °C to –1 °C in one hour, and then another two degrees in the next hour. How many degrees in total did the temperature drop over the two hours?

....

(c) A submarine is 75 m below the surface of the sea. It then rises by 21 m. How far below the surface is it now?

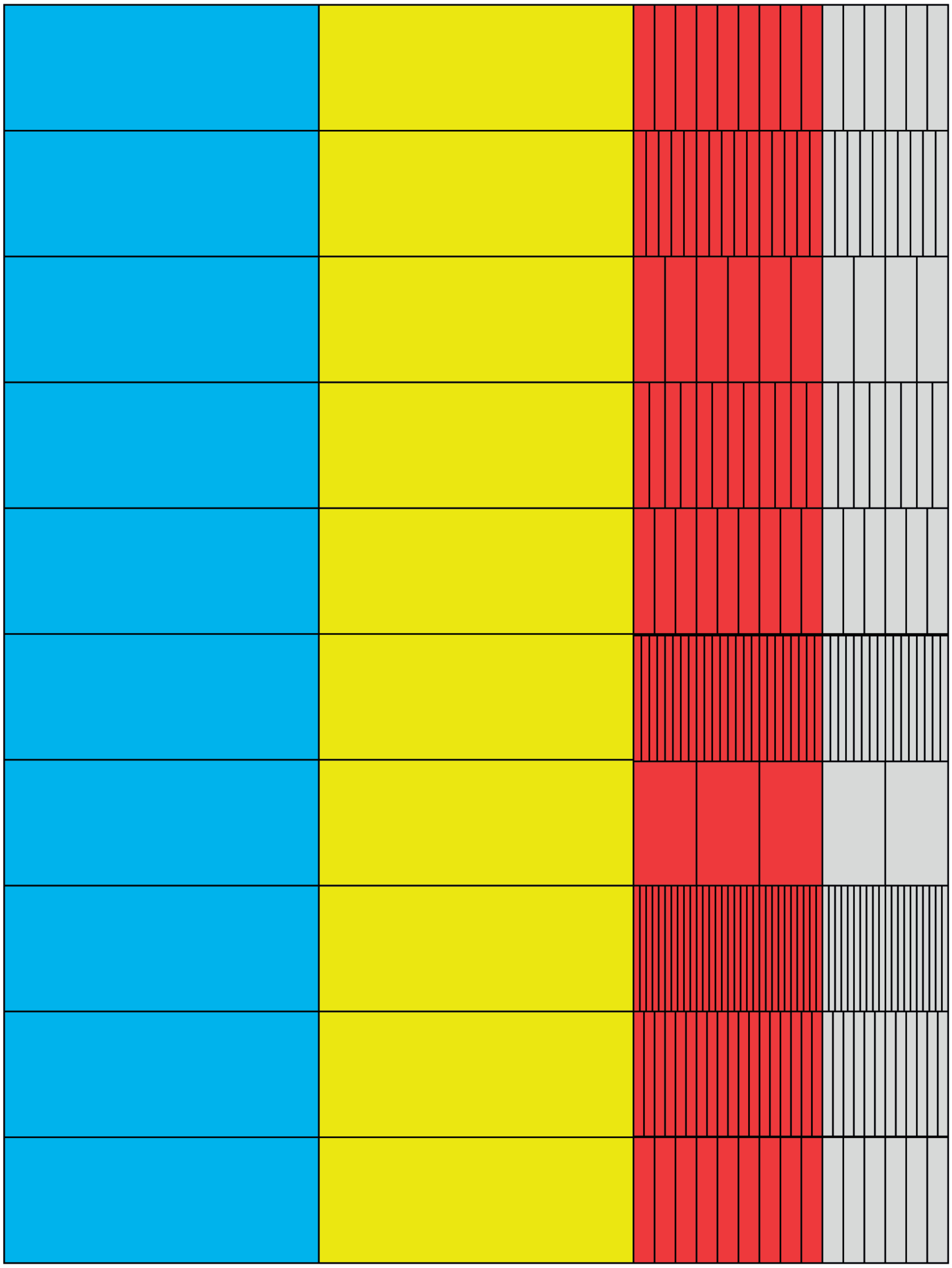
....

(d) A submarine is 37 m below the surface of the sea. It then sinks a further 15 m. How far below the surface is it now?

....

Chapter 3
Fractions

This chapter is mainly revision of the work on fractions that you have done in previous grades. It is being repeated because it is vital that you are confident working with fractions. Ensure that you complete all your solutions to questions **without using a calculator**, and that you show all steps of your working.



What part of the block is coloured?

3 Fractions

3.1 Equivalent fractions

THE SAME NUMBER IN DIFFERENT FORMS

1. How much money is each of the following amounts?
   (a) $\frac{1}{5}$ of R200 (b) $\frac{2}{10}$ of R200 (c) $\frac{4}{20}$ of R200

   ....

Did you notice that all the answers are the same? That is because $\frac{1}{5}$, $\frac{2}{10}$ and $\frac{4}{20}$ are **equivalent fractions**. They are different ways of writing the same number.

Consider this bar. It is divided into five equal parts.

Each piece is **one fifth** of the whole bar.

2. Draw lines on the bar below so that it is approximately divided into ten equal parts.

   (a) What part of the whole bar is each of your ten parts? ....
   (b) How many tenths is the same as one fifth? ....
   (c) How many tenths is the same as two fifths? ....
   (d) How many fifths is the same as eight tenths? ....

3. Draw lines on the bar below so that it is approximately divided into 25 equal parts.

   (a) How many twenty-fifths is the same as two fifths? ....
   (b) How many fifths is the same as 20 twenty-fifths? ....

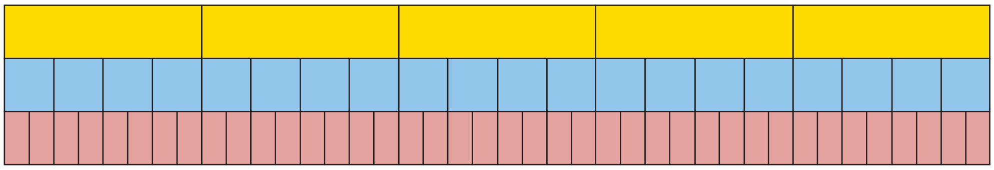
In question 3(b) you found that $\frac{4}{5}$ is equivalent to $\frac{20}{25}$: these are just two different ways to describe the same part of the bar.

This can be expressed by writing $\frac{4}{5} = \frac{20}{25}$ which means that $\frac{4}{5}$ and $\frac{20}{25}$ are equivalent to each other.

4. Write down all the other pairs of equivalent fractions which you found while doing questions 2 and 3.

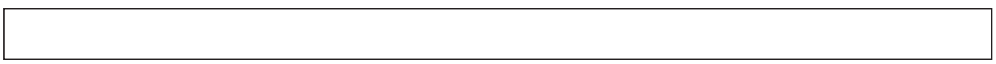
....

The yellow bar is divided into fifths.



5. (a) Into what kind of fraction parts is the blue bar divided? ....
   (b) Into what kind of fraction parts is the red bar divided? ....
   (c) If you want to mark the yellow bar in twentieths like the blue bar, into how many parts do you have to divide each of the fifths? ....
   (d) If you want to mark the yellow bar in fortieths like the red bar, into how many parts do you have to divide each of the fifths? ....
   (e) If you want to mark the yellow bar in *eightieths*, into how many parts do you have to divide each of the fifths? ....
   (f) If you want to mark the blue bar in *eightieths*, into how many parts do you have to divide each of the twentieths? ....

6. Suppose this bar is divided into 4 equal parts, in other words, quarters.



   (a) If the bar is also divided into 20 equal parts, how many of these smaller parts will there be in each quarter? ....
   (b) If each quarter is divided into 6 equal parts, what part of the whole bar will each small part be? ....

7. Complete this table of equivalent fractions, as far as you can using whole numbers. All the fractions in each column must be equivalent.

sixteenths	8	4	2	10	14	12
eighths						
quarters						
twelfths						
twentieths						

Equivalent fractions can be formed by multiplying the numerator and denominator by the same number. For example $\frac{1}{5} = \frac{4 \times 1}{4 \times 5} = \frac{4}{20}$

8. Write down five different fractions that are equivalent to $\frac{3}{4}$.

....

9. Express each of the following numbers as twelfths:

(a) $\frac{2}{3}$ ....

(b) $\frac{3}{4}$ ....

(c) $\frac{5}{6}$ ....

(d) $\frac{1}{6}$ ....

You may divide the numerator and denominator by the same number, instead of multiplying the numerator and denominator by the same number. This gives you a simpler fraction.

The **simplest form** of a fraction has no common factors. For example, you find the simplest form of the fraction $\frac{4}{12}$ is $\frac{1}{3}$ by dividing both the numerator and denominator by the common factor of 4.

10. Convert each of the following fractions to their simplest form:

(a) $\frac{40}{100}$ ....

(b) $\frac{4}{16}$ ....

(c) $\frac{5}{25}$ ....

(d) $\frac{6}{30}$ ....

(e) $\frac{6}{24}$ ....

(f) $\frac{8}{88}$ ....

CONVERTING BETWEEN MIXED NUMBERS AND FRACTIONS

Numbers that have both whole number and fraction parts are called **mixed numbers**.

Examples of mixed numbers: $3\frac{4}{5}$, $2\frac{7}{8}$, and $8\frac{3}{10}$

Mixed numbers can be written in expanded notation, for example:

$3\frac{4}{5}$ means $3 + \frac{4}{5}$ $\quad$ $2\frac{7}{8}$ means $2 + \frac{7}{8}$ $\quad$ $8\frac{3}{10}$ means $8 + \frac{3}{10}$.

To add and subtract mixed numbers, you can work with the whole number parts and the fraction parts separately, for example:

$3\frac{4}{5} + 13\frac{3}{5}$
$= 16\frac{7}{5}$
$= 17\frac{2}{5}$

$13\frac{3}{5} - 3\frac{4}{5}$
$= 12\frac{8}{5} - 3\frac{4}{5}$
$= 9\frac{4}{5}$

(we need to "borrow" a unit from 13, because we cannot subtract $\frac{4}{5}$ from $\frac{3}{5}$)

However, this method can be difficult to do with some examples – and it does not work with multiplication and division.

An alternative and preferred method is to convert the mixed number to an **improper fraction**, as shown in the example below:

$3\frac{4}{5}$
$= 3 + \frac{4}{5}$
$= \frac{15}{5} + \frac{4}{5}$
$= \frac{19}{5}$

NOTE
You can obtain the numerator of 19 in one step by multiplying the denominator (5) by the whole number (3), and then adding the numerator (4).

So you can calculate $3\frac{4}{5} + 13\frac{3}{5}$ using this method:

$3\frac{4}{5} + 13\frac{3}{5}$
$= \frac{19}{5} + \frac{68}{5}$
$= \frac{87}{5}$

The answer must be converted to a mixed number again: $\frac{87}{5} = 17\frac{2}{5}$

1. Convert each of the following mixed numbers to improper fractions:

(a) $5\frac{3}{5}$ (b) $2\frac{3}{8}$

....

(c) $3\frac{4}{7}$ (d) $4\frac{5}{12}$

....

2. Convert each of the following improper fractions to mixed numbers:

(a) $\frac{32}{5}$ (b) $\frac{25}{8}$

....

(c) $\frac{24}{9}$ (d) $\frac{37}{20}$

....

3.2 Adding and subtracting fractions

To add or subtract two fractions, they have to be expressed with the *same* denominators first. To achieve that, one or more of the given fractions may have to be replaced with equivalent fractions.

$\frac{3}{20} + \frac{2}{5}$

$= \frac{3}{20} + \frac{2 \times 4}{5 \times 4}$ to get twentieths.

$= \frac{3}{20} + \frac{8}{20}$

$= \frac{11}{20}$

$\frac{5}{12} + \frac{7}{20}$

$= \frac{5 \times 20}{12 \times 20} + \frac{7 \times 12}{20 \times 12}$

$= \frac{100}{240} + \frac{84}{240}$

$= \frac{184}{240}$

$= \frac{23}{30}$

We will later refer to this method of adding or subtracting fractions as Method A.

In the case of $\frac{5}{12} + \frac{7}{20}$, multiplying by 20 and by 12 was a sure way of making equivalent fractions of the same kind, in this case two-hundred-and-fortieths. However, the numbers became quite big. Just imagine how big the numbers will become if you use the same method to calculate $\frac{17}{75} + \frac{13}{85}$!

Fortunately, there is a method of keeping the numbers smaller (in many cases), when making equivalent fractions so that fractions can be added or subtracted. In this method you first calculate the **lowest common multiple** or LCM of the denominators. In the case of $\frac{5}{12} + \frac{7}{20}$, the smaller multiples of the denominators are:

12:	12	24	36	48	60	72	84
20:	20	40	60	80	100	120	140

The smallest number that is a multiple of both 12 and 20 is 60.

Both $\frac{5}{12}$ and $\frac{7}{20}$ can be expressed in terms of sixtieths:

$\frac{5}{12} = \frac{5 \times 5}{12 \times 5} = \frac{25}{60}$ because to make twelfths into sixtieths you have to divide each twelfth into 5 equal parts, to get $12 \times 5 = 60$ equal parts, i.e. sixtieths.

Similarly, $\frac{7}{20} = \frac{7 \times 3}{20 \times 3} = \frac{21}{60}$.

Hence $\frac{5}{12} + \frac{7}{20} = \frac{25}{60} + \frac{21}{60} = \frac{46}{60} = \frac{23}{30}$

This method may be called the LCM method of adding or subtracting fractions.

ADDING AND SUBTRACTING FRACTIONS

1. Which method of adding and subtracting fractions do you think will be the easiest and quickest for you, Method A or the LCM method? Explain.

....

....

2. Calculate:

(a) $\frac{3}{8} + \frac{2}{5}$

....

....

(b) $\frac{3}{10} + \frac{7}{8}$

....

....

(c) $3\frac{2}{5} + 2\frac{3}{10}$

....

....

(d) $7\frac{3}{8} + 3\frac{11}{12}$

....

....

3. Calculate each of the following:

(a) $\frac{13}{20} - \frac{2}{5}$

(b) $\frac{7}{12} - \frac{1}{4}$

(c) $5\frac{1}{2} - 3\frac{3}{8}$

(d) $4\frac{1}{9} - 5\frac{2}{3}$

4. Paulo and Sergio buy a pizza. Paulo eats $\frac{1}{3}$ of the pizza and Sergio eats two fifths. How much of the pizza is left over?

5. Calculate each of the following. State whether you use Method A or the LCM method.

(a) $\frac{7}{15} + \frac{11}{24}$

(b) $\frac{73}{100} - \frac{7}{75}$

(c) $\frac{3}{25} + \frac{13}{40}$

(d) $\frac{9}{16} - \frac{3}{10}$

(e) $\frac{1}{18} + \frac{7}{20}$

(f) $\frac{11}{35} - \frac{3}{14}$

(g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

3.3 Multiplying and dividing fractions

THINK ABOUT MULTIPLICATION AND DIVISION WITH FRACTIONS

1. Read the questions below, but do not answer them now. Just describe in each case what calculations you think must be done to find the answer to the question. You can think later about how the calculations may be done.

   (a) 10 people come to a party, and each of them must get $\frac{5}{8}$ of a pizza. How many pizzas must be bought to provide for all of them?

   ....

   (b) $\frac{5}{8}$ of the cost of a new clinic must be carried by the 10 doctors who will work there. What part of the cost of the clinic must be carried by each of the doctors, if they have agreed to share the cost equally?

   ....

   (c) If a whole pizza costs R10, how much does $\frac{5}{8}$ of a pizza cost?

   ....

   (d) The owner of a spaza shop has 10 whole pizzas. How many portions of $\frac{5}{8}$ of a pizza each can he make up from the 10 pizzas?

   ....

2. Look at the different sets of calculations shown on the next page.

   (a) Which set of calculations is a correct way to find the answer for question 1(a)?

   ....

   (b) Which set of calculations is a correct way to find the answer for question 1(b)?

   ....

   (c) Which set of calculations is a correct way to find the answer for question 1(c)?

   ....

   (d) Which set of calculations is a correct way to find the answer for question 1(d)?

   ....

Set A: $\frac{10}{10} \times \frac{5}{8} = \frac{50}{80}$

Set B: $\frac{5}{8} = \frac{50}{80}$. 50 eightieths ÷ 10 = $\frac{5}{80}$

Set C: How many eighths in 10 wholes? 80 eighths. How many 5-eighths in 80? 80 ÷ 5 = 16

Set D: $\frac{5}{8}$ is 5 eighths. 10 × 5 eighths = $\frac{50}{8}$

Set E: $\frac{5}{8} \div 10 = \frac{5}{8} \times \frac{10}{1} = \frac{50}{8}$

Multiply a fraction by a whole number

Example:

$8 \times \frac{3}{5}$ = 8 × 3 fifths = 24 fifths = $\frac{24}{5} = 4\frac{4}{5}$

Divide a fraction by a whole number

You can divide a fraction by converting it to an equivalent fraction with a numerator that is a multiple of the divisor.

Example:

$\frac{2}{3} \div 5 = \frac{10}{15} \div 5$ = 10 fifteenths ÷ 5 = 2 fifteenths = $\frac{2}{15}$

A fraction of a whole number, and a fraction of a fraction

Examples:

A $\frac{7}{12}$ of R36.

$\frac{1}{12}$ of R36 is the same as R36 ÷ 12 = R3, so $\frac{7}{12}$ of R36 is 7 × R3 = R21

B $\frac{7}{12}$ of 36 fiftieths.

$\frac{1}{12}$ of 36 fiftieths is the same as 36 fiftieths ÷ 12 = 3 fiftieths,

so $\frac{7}{12}$ of 36 fiftieths is 7 × 3 fiftieths = 21 fiftieths.

$\frac{7}{12} \times \frac{36}{50}$ means $\frac{7}{12}$ of $\frac{36}{50}$, it is the same.

$\frac{1}{12}$ of $\frac{36}{50}$ is the same as $\frac{36}{50} \div 12 = \frac{3}{50}$, so $\frac{7}{12}$ of $\frac{36}{50}$ is $7 \times \frac{3}{50} = \frac{21}{50}$.

3. (a) You calculated $\frac{7}{12} \times \frac{36}{50}$ in the example above. What was the answer?

....

(b) Calculate $\frac{7 \times 36}{12 \times 50}$, and simplify your answer.

....

Example:

$\frac{2}{3} \times \frac{5}{8} = \frac{2}{3}$ of $\frac{15}{24} = \frac{1}{3}$ of $\frac{30}{24} = \frac{10}{24} = \frac{5}{12}$

The same answer is obtained by calculating $\frac{2 \times 5}{3 \times 8}$

To multiply two fractions, you may simply multiply the numerators and the denominators.

$\frac{2}{3} \times \frac{9}{20} = \frac{2 \times 9}{3 \times 20} = \frac{18}{60} = \frac{3}{10}$

Division by a fraction

When we divide by a fraction, we have a very different situation. Think about this:

If you have 40 pizzas, how many learners can have $\frac{3}{5}$ a pizza each?

To find the number of fifths in 40 pizzas: 40 × 5 = 200 fifths of a pizza.

To find the number of 3-fifths: 200 ÷ 3 = 66 portions of $\frac{3}{5}$ pizza and 2 fifths of a pizza left over.

Since the portion for each learner is 3 fifths, the 2 fifths of a pizza that remains is 2 thirds of a portion.

So, to calculate $40 \div \frac{\mathbf{3}}{\mathbf{5}}$, we multiplied by **5** and divided by **3**, and that gave us 66 and 2 thirds of a portion.

In fact, we calculated $40 \times \frac{5}{3}$.

Division is the inverse of multiplication.
So, to divide by a fraction, you multiply by its inverse.

Example:

$\frac{18}{60} \div \frac{2}{3} = \frac{18}{60} \times \frac{3}{2} = \frac{54}{120} = \frac{9}{20}$

MULTIPLYING AND DIVIDING FRACTIONS

1. Calculate each of the following:

(a) $\frac{3}{4}$ of $\frac{12}{25}$

(b) $\frac{3}{4} \times \frac{12}{100}$

....

(c) $\frac{3}{4}$ of $\frac{13}{25}$

(d) $\frac{3}{4} \times 1\frac{1}{2}$

....

(e) $\frac{3}{20} \times \frac{5}{6}$

(f) $\frac{3}{20}$ of $\frac{3}{20}$

....

2. A small factory manufactures copper pans for cooking. Exactly $\frac{3}{50}$ kg of copper is needed to make one pan.

(a) How many pans can they make if $\frac{18}{50}$ kg of copper is available?

....

(b) How many pans can they make if $\frac{20}{50}$ kg of copper is available?

....

....

(c) How many pans can they make if $\frac{2}{5}$ kg of copper is available?

....

....

(d) How many pans can they make if $\frac{3}{4}$ kg of copper is available?

....

(e) How many pans can be made if $\frac{144}{50}$ kg of copper is available?

....

(f) How many pans can be made if 5 kg of copper is available?

....

3. Calculate:

(a) $\frac{18}{50} \div \frac{3}{50}$

(b) $\frac{9}{25} \div \frac{3}{50}$

....

(c) $\frac{144}{50} \div \frac{3}{50}$

(d) $2\frac{44}{50} \div \frac{3}{50}$

....

(e) $2\frac{22}{25} \div \frac{3}{50}$

(f) $\frac{5}{8} \div \frac{3}{50}$

....

(g) $20 \div \frac{3}{50}$

(h) $2 \div \frac{3}{50}$

....

(i) $1 \div \frac{3}{50}$

(j) $\frac{1}{2} \div \frac{3}{50}$

....

4. A rectangle is $3\frac{5}{8}$ cm long and $2\frac{3}{5}$ cm wide.

(a) What is the area of this rectangle?

....

....

(b) What is the perimeter of this rectangle?

....

....

5. A rectangle is $5\frac{5}{6}$ cm long and its area is $8\frac{1}{6}$ cm^2.

How wide is this rectangle?

....

....

6. Calculate.

(a) $2\frac{3}{8}$ of $5\frac{4}{5}$

(b) $3\frac{2}{7} \times 2\frac{7}{12}$

....

....

(c) $8\frac{2}{5} \div 3\frac{3}{10}$

(d) $3\frac{3}{10} \times 3\frac{3}{10}$

....

....

(e) $2\frac{5}{8} \div 5\frac{7}{10}$

(f) $\frac{3}{5} \times 1\frac{2}{3} \times 1\frac{3}{4}$

....

....

7. Calculate:

(a) $\frac{2}{3}(\frac{3}{4} + \frac{7}{10})$

(b) $\frac{2}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{7}{10}$

....

....

(c) $\frac{5}{8}(\frac{4}{5} - \frac{1}{3})$

(d) $\frac{5}{8} \times \frac{4}{5} - \frac{5}{8} \times \frac{1}{3}$

....

....

8. A piece of land with an area of 40 ha is divided into 30 equal plots. The total price of the land is R45 000. Remember that "ha" is the abbreviation for hectares.

(a) Jim buys $\frac{2}{5}$ of the land.

(i) How many plots is this and how much should he pay?

....

....

(ii) What is the area of the land that Jim buys?

....

(b) Charlene buys $\frac{1}{3}$ of the land. How many plots is this and how much should she pay?

....

....

(c) Bongani buys the rest of the land. Determine the fraction of the land that he buys.

....

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

1. Calculate:

(a) $\frac{3}{4} \times \frac{3}{4}$

....

(b) $\frac{7}{10} \times \frac{7}{10}$

....

(c) $2\frac{5}{8} \times 2\frac{5}{8}$

....

(d) $1\frac{5}{12} \times 1\frac{5}{12}$

....

(e) $3\frac{5}{7} \times 3\frac{5}{7}$

....

(f) $10\frac{3}{4} \times 10\frac{3}{4}$

....

$\frac{9}{16}$ is the square of $\frac{3}{4}$, because $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$.

$\frac{3}{4}$ is the square root of $\frac{9}{16}$.

2. Find the square root of each of the following numbers.

(a) $\sqrt{\frac{25}{49}}$

....

(b) $\sqrt{\frac{36}{121}}$

....

(c) $\sqrt{\frac{64}{25}}$

....

(d) $\sqrt{2\frac{46}{49}}$

....

3. Calculate.

(a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

....

(b) $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

....

(c) $\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

....

(d) $\frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}$

....

4. Find the cube root of each of the following numbers.

(a) $\sqrt[3]{\frac{27}{1\,000}}$

(b) $\sqrt[3]{\frac{125}{216}}$

....

(c) $\sqrt[3]{\frac{1\,000}{216}}$

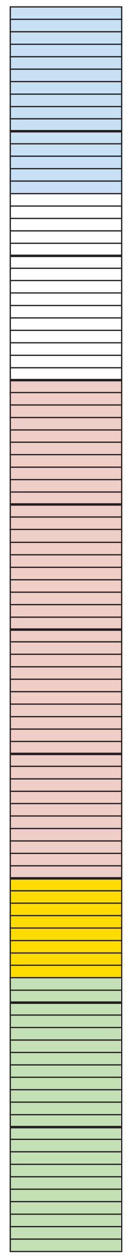
(d) $\sqrt[3]{15\frac{5}{8}}$

....

....

3.4 Equivalent forms

FRACTIONS, DECIMALS AND PERCENTAGE FORMS



1. The rectangle on the right is divided into small parts.

   (a) How many of these small parts are there in the rectangle? ....

   (b) How many of these small parts are there in one tenth of the rectangle?

   ....

   (c) What fraction of the rectangle is blue?

   ....

   (d) What fraction of the rectangle is pink?

   ....

Instead of "6 hundredths" we may say "6 per cent" or, in short, "6%". It means the same thing.
15 per cent of the rectangle on the right is blue.

2. (a) What percentage of the rectangle is green?

   ....

   (b) What percentage of the rectangle is pink?

   ....

0,37 and 37% and $\frac{37}{100}$ are different ways of writing the same value (**37 hundredths**).

3. Express each of the following in three ways: as a decimal; a percentage and a fraction (in simplest form):
   (a) 3 tenths
   (b) 7 hundredths

   ....

   (c) 37 hundredths
   (d) 7 tenths

   ....

   (e) 2 fifths
   (f) 7 twentieths

   ....

4. Fill in the missing values in the table:

Decimal	Percentage	Common fraction (simplest form)
0,2		
	40%	
		$\frac{3}{8}$
0,05		

5. (a) Jannie eats a quarter of a watermelon. What percentage of the watermelon is this?

   ....

   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk in the bottle has he drunk?

   ....

   (c) Jem used 0,18 of the paint in a tin. If he uses half of the remaining amount the next time he paints, what fraction (in simplest form) is left over?

   ....

Chapter 4
The decimal notation for fractions

In this chapter you will do more work with fractions written in the decimal notation. When fractions are written in the decimal notation, calculations can be done in the same way than for whole numbers. It is important to always keep in mind that the common fraction form, the decimal form and the percentage form are just different ways to represent exactly the same numbers. These numbers are called the rational numbers.

$$7 \times 1\ 000\ 000$$

$$+$$

$$4 \times 100\ 000$$

$$+$$

$$7 \times 10\ 000$$

$$+$$

$$6 \times 1\ 000$$

$$+$$

$$3 \times 100$$

$$+$$

$$6 \times 10$$

$$+$$

$$9 \times 1$$

$$+$$

$$3 \text{ tenths} + 7 \text{ thousandths}$$

$$+$$

$$\frac{3}{10\ 000} + \frac{8}{100\ 000} + \frac{7}{1\ 000\ 000}$$

4 The decimal notation for fractions

4.1 Equivalent forms

Decimal fractions and common fractions are simply different ways of expressing the same number. They are different **notations** showing the same value.

To write a decimal fraction as a common fraction:

Write the decimal with a denominator that is a power of ten (10, 100, 1 000, etc.) and then simplify it if possible.

e.g. $0{,}35 = \frac{35}{100} = \frac{7}{20} \times \frac{5}{5} = \frac{7}{20}$

To write a common fraction as a decimal fraction:

Change the common fraction to an equivalent fraction with a power of ten as a denominator.

e.g. $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0{,}75$

If you are permitted to use your calculator, simply type in 3 ÷ 4 and the answer will be given as 0,75. On some calculators you will need to press an additional button to convert the exact fraction to a decimal.

Notation means a set of symbols to show a special thing.

COMMON FRACTIONS, DECIMAL FRACTIONS AND PERCENTAGES

You are not permitted to use a calculator in this exercise.

1. Write the following decimal fractions as common fractions in their simplest form:

(a) 0,56

(b) 3,87

....

(c) 1,9

(d) 5,205

....

2. Write the following common fractions as decimal fractions:

(a) $\frac{9}{20}$

....

(b) $\frac{7}{5}$

....

(c) $\frac{24}{25}$

....

(d) $2\frac{3}{8}$

....

3. Write the following percentages as common fractions in their simplest form:

(a) 70% ....

(b) 5% ....

(c) 12,5% ....

4. Write the following decimal fractions as percentages:

(a) 0,6 ....

(b) 0,43 ....

(c) 0,08 ....

(d) 0,265 ....

(e) 0,005 ....

5. Write the following common fractions as percentages:

(a) $\frac{7}{10}$ ....

(b) $\frac{3}{4}$ ....

(c) $\frac{33}{50}$ ....

(d) $\frac{60}{60}$ ....

(e) $\frac{2}{25}$ ....

(f) $\frac{29}{50}$ ....

6. Jane and Devi are in different schools. At Jane's school the year mark for Mathematics was out of 80, and Jane got 60 out of 80. At Devi's school the year mark was out of 50 and Devi got 40 out of 50.

(a) What fraction of the total marks, in simplest form, did Devi obtain at her school?

....

(b) What percentage did Devi and Jane get for Mathematics?

....

(c) Who performed better, Jane or Devi?

....

7. During a basketball game, Lebo tried to score twelve times. Only four of her attempts were successful.
   (a) What fraction of her attempts was successful?

   ....

   ....

   (b) What percentage of her attempts was not successful?

   ....

4.2 Calculations with decimals

When you **add** and **subtract** decimal fractions:
Add tenths to tenths.
Subtract tenths from tenths.
Add hundredths to hundredths.
Subtract hundreds from hundredths.
And so on!

When you **multiply** decimal fractions, you change the decimals to whole numbers, do the calculation and last, change them back to decimal fractions.

For example: To calculate 13,1 × 1,01, you first calculate 131 × 101 (which equals 13 231). Then, since you have multiplied the 13,1 by 10, and the 1,01 by 100 in order to turn them into whole numbers, you need to divide this answer by 10 × 100 (i.e. 1 000). Thus, the final answer is 13,231

When you **divide** decimal fractions, you can use equivalent fractions to help you.

For example: $21{,}7 \div 0{,}7 = \frac{21{,}7}{0{,}7} = \frac{21{,}7}{0{,}7} \times \frac{10}{10} = \frac{217}{7} = 31$

Notice how you multiply both the numerator and denominator of the fraction by the same number (in this case, 10). Always multiply by the *smallest* power of ten that will convert both values to whole numbers.

CALCULATIONS WITH DECIMALS

You are not permitted to use a calculator in this exercise. Ensure that you show all steps of your working.

1. Calculate the value of the following:

(a) $3{,}3 + 4{,}83$

(b) $0{,}6 + 18{,}3 + 4{,}4$

(c) $9{,}3 + 7{,}6 - 1{,}23$

(d) $(16{,}0 - 7{,}6) - 0{,}6$

(e) $9{,}43 - (3{,}61 + 1{,}14)$

(f) $1{,}21 + 2{,}5 - (2{,}3 - 0{,}23)$

2. Calculate the value of the following:

(a) $4 \times 0{,}5$

(b) $15 \times 0{,}02$

(c) $0{,}8 \times 0{,}04$

(d) $0{,}02 \times 0{,}15$

(e) $1{,}07 \times 0{,}2$

(f) $0{,}016 \times 0{,}02$

3. Calculate the value of the following:

(a) 7,2 ÷ 3 (b) 12 ÷ 0,3 (c) 0,15 ÷ 0,5

....

....

....

(d) 10 ÷ 0,002 (e) 0,3 ÷ 0,006 (f) 0,024 ÷ 0,08

....

....

4. Circle the value that is equal to or closest to the answer to each calculation:

(a) 3 × 0,5

A: 6

B: 1,5

C: 0,15

(b) 4,4 ÷ 0,2

A: 8,8

B: 2,2

C: 22

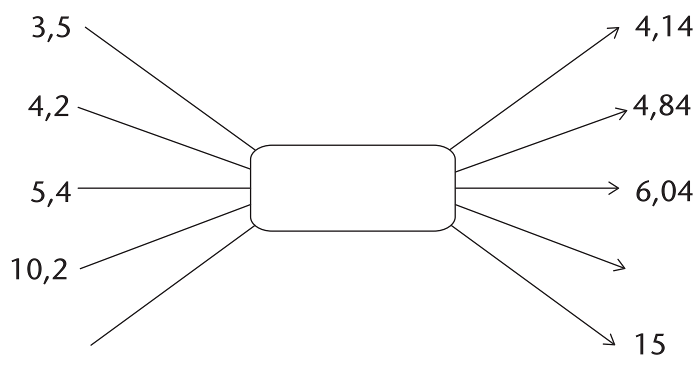
(c) 56 × 1,675

A: more than 56

B: more than 84

C: more than 112

5. Determine the operator and the unknown numbers in the following diagram, and fill them in:



6. Calculate the following:

(a) $(0,1)^2$ (b) $(0,03)^2$ (c) $(2,5)^2$

....

(d) $\sqrt{0{,}04}$ (e) $\sqrt{0{,}16}$ (f) $\sqrt{0{,}49}$

....

(g) $(0{,}2)^3$ (h) $(0{,}4)^3$ (i) $(0{,}03)^3$

....

(j) $\sqrt[3]{0{,}064}$ (k) $\sqrt[3]{0{,}125}$ (l) $\sqrt[3]{0{,}216}$

....

7. Calculate the following:
   (a) $2{,}5 \times 2 \div 10$ (b) $4{,}2 - 5 \times 1{,}2$

   ....

   ....

   (c) $\dfrac{5{,}4 + 7{,}35}{0{,}05}$ (d) $4{,}2 \div 0{,}21 + 0{,}45 \times 0{,}3$

   ....

   ....

   ....

4.3 Solving problems

ALL KINDS OF PROBLEMS

You are not permitted to use a calculator in this exercise. Ensure that you show all steps of working.

1. Is $6{,}54 \times 0{,}81 = 0{,}654 \times 8{,}1$? Explain your answer.

   ....

   ....

   ....

2. You are given that $45 \times 24 = 1\,080$. Use this result to determine:
   (a) $4{,}5 \times 2{,}4$ (b) $4{,}5 \times 24$ (c) $4{,}5 \times 0{,}24$

   ....

(d) $0,045 \times 24$ (e) $0,045 \times 0,024$ (f) $0,045 \times 24$

3. Without actually dividing, choose which answer in brackets is the correct answer, or the closest to the correct answer

(a) $14 \div 0,5$ (7; 28; 70)

(b) $0,58 \div 0,7$ (8; 80; 0,8)

(c) $2,1 \div 0,023$ (10; 100; 5)

4. (a) John is asked to calculate $6,5 \div 0,02$. He does the following:

Step 1: $6,5 \div 2 = 3,25$

Step 2: $3,25 \times 100 = 325$

Is he correct? Why?

(b) Use John's method in part (a) to calculate:

(i) $4,8 \div 0,3$

(ii) $21 \div 0,003$

5. Given: $0,174 \div 0,3 = 0,58$. Using this fact, write down the answers for the following without doing any further calculations:

(a) $0,3 \times 0,58$

(b) $1,74 \div 3$

(c) $17,4 \div 30$

(d) $174 \div 300$

(e) $0,0174 \div 0,03$

(f) $0,3 \times 5,8$

4.4 More problems

MORE PROBLEMS AND CALCULATIONS

You may use a calculator for this exercise.

1. Calculate the following, rounding off all answers correct to 2 decimal places:

   (a) 8,567 + 3,0456 (b) 2,781 – 6,0049

   ....

   (c) 1,234 × 4,056 (d) $\frac{5,678 + 3,245}{1,294 - 0,994}$

   ....

2. What is the difference between 0,890 and 0,581?

   ....

3. If a rectangle is 12,34 cm wide and 31,67 cm long.

   (a) What is the perimeter of the rectangle?

   ....

   ....

   (b) What is the area of the rectangle? Round off your answer to two decimal places.

   ....

   ....

4. Alison buys a cooldrink for R5,95, a chocolate for R3,25 and a packet of chips for R4,60. She pays with a R20 note.

   (a) How much did she spend?

   ....

   ....

   ....

   (b) How much change did she get?

   ....

   ....

5. A tractor uses 11,25 ℓ of fuel in 0,75 hours. How many litres does it use in one hour?

....

....

6. Mrs Ruka received her municipal bill.
   (a) Her water consumption charge for one month is R32,65. The first 5,326 kℓ are free, then she pays R5,83 per kilolitre for every kilolitre thereafter.
   How much water did the Ruka household use?

....

....

   (b) The electricity charge for Mrs Ruka for the same month was R417,59. The first 10 kWh are free. For the next 100 kWh the charge is R1,13 per kWh, and thereafter for each kWh the charge is R1,42.
   How much electricity did the Ruka household use?

....

....

....

7. A roll of material is 25 m long. To make one dress, you need 1,35 m of material. How many dresses can be made out of a roll of material and how much material is left over?

....

....

....

8. If 1 litre of petrol weighs 0,679 kg, what will 28,6 ℓ of petrol weigh?

....

9. The reading on a water meter at the beginning of the month is 321,573 kℓ. At the end of the month the reading is 332,523 kℓ. How much water was used during this month, in ℓ?

....

....

4.5 Decimals in algebraic expressions and equations

DECIMALS IN ALGEBRA

1. Simplify the following:

(a) $\sqrt{0,09x^{36}}$

(b) $7,2x^3 - 10,4x^3$

(c) $(2,4x^2y^3)(10y^3x)$

(d) $11,75x^2 - 1,2x \times 5x$

(e) $\dfrac{3,4x - 1,2x}{1,1x \times 4}$

(f) $\sqrt[3]{0,008x^{12}} + \sqrt{0,16x^8}$

(g) $3x^2 + 0,1x^2 - 45,6 + 3,9$

(h) $\dfrac{0,4y + 1,2y}{0,6x - 3x}$

2. Simplify the following:

(a) $\dfrac{0,5x^9}{0,02x^3}$

(b) $\dfrac{0,325}{x^2} - \dfrac{1,675}{x^2}$

(c) $\frac{3,6x}{1,5y^3} \times \frac{5y}{0,6x}$

. . . .

. . . .

. . . .

(d) $\frac{9,5x^2}{1,2y^2} \div \frac{0,05x}{0,04y^8}$

. . . .

. . . .

. . . .

3. Solve the following equations:

(a) $0,24 + x = 0,31$

. . . .

. . . .

. . . .

(b) $x + 5,61 = 7,23$

. . . .

. . . .

. . . .

(c) $x - 3,14 = 9,87$

. . . .

. . . .

. . . .

(d) $4,21 - x = 2,74$

. . . .

. . . .

. . . .

(e) $0,96x = 0,48$

. . . .

. . . .

. . . .

(f) $x \div 0,03 = 1,5$

. . . .

. . . .

. . . .

WORKSHEET

You are not permitted to use a calculator in this exercise, except for question 5. Ensure that you show all steps of working, where relevant.

1. Complete the following table:

Percentage	Common fraction	Decimal fraction
2,5%		
	$\frac{15}{250}$	
		0,009

2. Calculate the following:

(a) $6{,}78 - 4{,}92$ (b) $1{,}7 \times 0{,}05$ (c) $7{,}2 \div 0{,}36$

....

(d) $4{,}2 - 0{,}4 \times 1{,}2 + 7{,}37$ (e) $(0{,}12)^2$ (f) $\frac{3\sqrt{0{,}04}}{\sqrt[3]{0{,}027}}$

....

....

3. $36 \times 19 = 684$. Use this result to determine:

(a) $3{,}6 \times 1{,}9$ (b) $0{,}036 \times 0{,}19$ (c) $68{,}4 \div 0{,}19$

....

4. Simplify:

(a) $(4{,}95x - 1{,}2) - (3{,}65x + 3{,}1)$ (b) $\frac{2{,}75x^{50}}{0{,}005x^{25}}$

....

....

5. Mulalo went to the shop and purchased 2 tubes of toothpaste for R6,98 each; 3 cans of cooldrink for R6,48 each, and 5 tins of baked beans for R7,95 each. If he pays with a R100 note, how much change should he get?

....

Chapter 5
Exponents

In this chapter, you will revise work on exponents that you have done in previous grades. You will extend the laws of exponents to include exponents that are negative numbers.

You will also solve simple equations in exponential form.

In Grade 8 you learnt about scientific notation. In this chapter we will extend the scientific notation to include very small numbers such as 0,0000123.

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5 Exponents

5.1 Revision

Remember that exponents are a shorthand way of writing repeated multiplication of the same number by itself. For example: $5 \times 5 \times 5 = 5^3$. The **exponent**, which is 3 in this example, stands for however many times the value is being multiplied. The number that is being multiplied, which is 5 in this example, is called the **base**.

If there are mixed operations, then the powers should be calculated before multiplication and division. For example: $5^2 \times 3^2 = 25 \times 9$.

You learnt these laws for working with exponents in previous grades:

Law	Example
$a^m \times a^n = a^{m+n}$	$3^2 \times 3^3 = 3^{2+3} = 3^5$
$a^m \div a^n = a^{m-n}$	$5^4 \div 5^2 = 5^{4-2} = 5^2$
$(a^m)^n = a^{m \times n}$	$(2^3)^2 = 2^{2 \times 3} = 2^6$
$(a \times t)^n = a^n \times t^n$	$(3 \times 4)^2 = 3^2 \times 4^2$
$a^0 = 1$	$32^0 = 1$

THE EXPONENTIAL FORM OF A NUMBER

1. Write the following in exponential notation:

(a) $2 \times 2 \times 2 \times 2 \times 2$ (b) $s \times s \times s \times s$ (c) $(-6) \times (-6) \times (-6)$

....

(d) $2 \times 2 \times 2 \times 2 \times s \times s \times s \times s$ (e) $3 \times 3 \times 3 \times 7 \times 7$ (f) $500 \times (1{,}02) \times (1{,}02)$

....

2. Write each of the numbers in exponential notation in some different ways if possible:

(a) 81 (b) 125 (c) 1 000

....

(d) 64 (e) 216 (f) 1 024

....

ORDER OF OPERATIONS

1. Calculate the value of $7^2 - 4$.

   Bathabile did the calculation like this: $7^2 - 4 = 14 - 4 = 10$

   Nathaniel did the calculation differently: $7^2 - 4 = 49 - 4 = 45$

   Which learner did the calculation correctly? Give reasons for your answer.

   ....

   ....

2. Calculate: $5 + 3 \times 2^2 - 10$, with explanations.

   ....

   ....

   ....

3. Explain how to calculate $2^6 - 6^2$.

   ....

   ....

   ....

4. Explain how to calculate $(4 + 1)^2 + 8 \times \sqrt[3]{64}$

   ....

   ....

   ....

   ....

   ....

LAWS OF EXPONENTS

1. Use the laws of exponents to calculate the following:

   (a) $2^2 \times 2^4$ (b) $3^4 \div 3^2$ (c) $3^0 + 3^4$

   ....

   ....

   ....

(d) $(2^3)^2$

(e) $(2 \times 5)^2$

(f) $(2^2 \times 7)^3$

....

....

....

2. Complete the table. Substitute the given number for y. The first column has been done as an example.

	y	2	3	4	5
(a)	$y \times y^4$	2×2^4 $= 2^{1+4}$ $= 2^5$ $= 32$			
(b)	$y^2 \times y^3$	$2^2 \times 2^3$ $= 2^{2+3}$ $= 4 \times 8$ $= 32$			
(c)	y^5	$2^5 = 32$			

3. Are the expressions $y \times y^4$; $y^2 \times y^3$ and y^5 equivalent? Explain.

....

....

....

4. Complete the table. Substitute the given number for y.

	y	2	3	4	5
(a)	$y^4 \div y^2$	$2^4 \div 2^2$ $= 16 \div 4$ $= 4$			
(b)	$y^3 \div y^1$	$2^3 \div 2^1$ $= 8 \div 2$ $= 4$			
(c)	y^2	$2^2 = 4$			

5. (a) From the table, is $y^4 \div y^2 = y^3 \div y^1 = y^2$? Explain why.

....

....

....

(b) Evaluate $y^4 \div y^2$ for $y = 15$.

....

6. Complete the table:

	x	2	3	4	5
(a)	2×5^x	2×5^2 $= 2 \times 25$ $= 50$			
(b)	$(2 \times 5)^x$	$(2 \times 5)^2$ $= 10^2$ $= 100$			
(c)	$2^x \times 5^x$	$2^2 \times 5^2$ $= 4 \times 25$ $= 100$			

7. (a) From the table above, is $2 \times 5^x = (2 \times 5)^x$? Explain.

....

(b) Which expressions in question 6 are equivalent? Explain.

....

....

8. Below is a calculation that Wilson did as homework. Mark each problem correct or incorrect and explain the mistakes.

(a) $b^3 \times b^8 = b^{24}$

....

....

(b) $(5x)^2 = 5x^2$

....

....

(c) $(-6a) \times (-6a) \times (-6a) = (-6a)^3$

....

5.2 Integer exponents

5^4 means $5 \times 5 \times 5 \times 5$. The exponent 4 indicates the number of appearances of the repeated factor.

What may a negative exponent mean, for example what may 5^{-4} mean?

Mathematicians have decided to use negative exponents to indicate repetition of the multiplicative inverse of the base, for example 5^{-4} is used to indicate $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ or $(\frac{1}{5})^4$, and x^{-3} is used to indicate $(\frac{1}{x})^3$ which is $\frac{1}{x} \times \frac{1}{x} \times \frac{1}{x}$.

This decision was not taken blindly – mathematicians were well aware that it makes good sense to use negative exponents in this meaning. One major advantage is that the negative exponents, when used in this meaning, have the same properties as positive exponents, for example:

$2^{-3} \times 2^{-4} = 2^{(-3)+(-4)} = 2^{-7}$ because $2^{-3} \times 2^{-4}$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$ which is $(\frac{1}{2})^7$.

$2^{-3} \times 2^4 = 2^{(-3)+4} = 2^1$ because $2^{-3} \times 2^4$ means $(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) \times (2 \times 2 \times 2 \times 2)$ which is 2.

NEGATIVE EXPONENTS

1. Express each of the following in the exponential notation in two ways: with positive exponents and with negative exponents.

   (a) $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$

   (b) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

   ....

2. In each case, check whether the statement is true or false. If it is false, write a correct statement. If it is true, give reasons why you say so.

   (a) $10^{-3} = 0{,}001$

   (b) $3^{-5} \times 9^2 = 3$

   ....

   (c) $25^2 \times 10^{-6} \times 2^6 = 5$

   (d) $(\frac{1}{5})^{-4} = 5^4$

   ....

3. Calculate each of the following without using a calculator:

(a) $10^{-3} \times 20^4$

(b) $(\frac{1}{5})^{-4}$

....

4. (a) Use a scientific calculator to determine the decimal values of the given powers.

Example: To find 3^{-1} on your calculator, use the key sequence: **3** y^x **1** ± =

Power	2^{-1}	5^{-1}	$(-2)^{-1}$	$(0{,}3)^{-1}$	0^{-1}	10^{-1}	10^{-2}
Decimal value							

(b) Explain the meaning of 10^{-3}.

....

5. Determine the value of each of the following in two ways:

A. By using the definition of powers (For example, $5^2 \times 5^0 = 25 \times 1 = 25$.)

B. By using the properties of exponents (For example, $5^2 \times 5^0 = 5^{2+0} = 5^2 = 25$.)

(a) $(3^3)^{-2}$

(b) $4^2 \times 4^{-2}$

(c) $5^{-2} \times 5^{-1}$

....

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....

6. Calculate the value of each of the following. Express your answers as common fractions.

(a) 2^{-3}

(b) $3^2 \times 3^{-2}$

(c) $(2 + 3)^{-2}$

....

....

(d) $3^{-2} \times 2^{-3}$ (e) $2^{-3} + 3^{-3}$ (f) 10^{-3}

....

....

....

(g) $2^3 + 2^{-3}$ (h) $(3^{-1})^{-1}$ (i) $(2^{-3})^2$

....

....

....

7. Which of the following are true? Correct any false statement.

(a) $6^{-1} = -6$ (b) $3x^{-2} = \frac{1}{3x^2}$ (c) $3^{-1}x^{-2} = \frac{1}{3x^2}$

....

(d) $(ab)^{-2} = \frac{1}{a^2b^2}$ (e) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$ (f) $\left(\frac{1}{3}\right)^{-1} = 3$

....

....

....

....

....

5.3 Solving simple exponential equations

An exponential equation is an equation in which the variable is in the exponent. So when you solve exponential equations, you are solving questions of the form **"To what power must the base be raised for the statement to be true?"**

To solve this kind of equation, remember that:

If $a^m = a^n$, then $m = n$.

In other words, if the base is the same on either side of the equation, then the exponents are the same.

Example:

$3^x = 243$

$3^x = 3^5$ (rewrite using the same base)

$x = 5$ (since the bases are the same, we equate the exponents)

Some exponential equations are slightly more complex:

Example: $3^{x+3} = 243$

$3^{x+3} = 3^5$ (rewrite using the same base)

$x + 3 = 5$ (equate the exponents)

$x = 2$

Check: LHS $3^{2+3} = 3^5 = 243$

Remember that the exponent can also be negative. However, you follow the same method to solve these kinds of equations.

Example: $2^x = \frac{1}{32}$

$2^x = 2^{-5}$ (rewrite using the same base)

$x = -5$ (equate the exponents)

SOLVING EXPONENTIAL EQUATIONS

1. Use the table to answer questions that follow:

x	2	3	4	5
2^x	4	8	16	32
3^x	9	27	81	243
5^x	25	125	625	3 125

For which value of x is:

(a) $2^x = 32$

(b) $3^x = 81$

(c) $5^x = 3\ 125$

(d) $2^x = 8$

(e) $5^x = 625$

(f) $3^x = 9$

(g) $5^{x+1} = 25$

(h) $3^{x+2} = 27$

(i) $2^{x-1} = 8$

2. Solve these exponential equations. You may use your calculator if necessary.

(a) $4^x = \frac{1}{64}$

(b) $6^{2x} = 1\ 296$

(c) $2^{x-1} = \frac{1}{8}$

(d) $3^{x+2} = \frac{1}{729}$

(e) $5^{x+1} = 15\ 625$

(f) $2^{x+3} = \frac{1}{4}$

(g) $4^{x+3} = \frac{1}{256}$

(h) $3^{2-x} = 81$

(i) $5^{3x} = \frac{1}{125}$

5.4 Scientific notation

Scientific notation is a way of writing numbers that are too big or too small to be written clearly in decimal form. The diameter of a hydrogen atom, for example, is a very small number. It is 0,000000053 mm. The distance from the sun to the earth is, on average, 150 000 000 km.

In scientific notation the diameter of the hydrogen molecule is written as $5{,}3 \times 10^{-8}$ and the distance from the sun to the earth as $1{,}5 \times 10^{8}$. It is easier to compare and to calculate numbers like these, as it is very cumbersome to count the zeros when you work with these numbers.

Look at more examples below:

Decimal notation	Scientific notation
6 130 000	$6{,}13 \times 10^{6}$
0,00001234	$1{,}234 \times 10^{-5}$

A number written in scientific notation is written as the product of two numbers, in the form $\pm a \times 10^{n}$ where a is a decimal number between 1 and 10, and n is an integer.

Any number can be written in scientific notation, for example:

$40 = 4{,}0 \times 10$

$2 = 2 \times 10^{0}$

The decimal number 324 000 000 is written as $3{,}24 \times 10^{8}$ in scientific notation, because the decimal comma is moved 8 places to the left to form 3,24.

The decimal number 0,00000065 written in scientific notation is $6{,}5 \times 10^{-7}$, because the decimal point is moved 7 places to the right to form the number 6,5.

WRITING VERY SMALL AND VERY LARGE NUMBERS

1. Express the following numbers in scientific notation:

   (a) 134,56

   ....

   (b) 0,0000005678

   ....

   (c) 876 500 000

   ....

   (d) 0,0000000000321

   ....

   (e) 0,006789

   ....

   (f) 89 100 000 000 000

   ....

   (g) 0,001

   ....

   (h) 100

   ....

2. Express the following numbers in ordinary decimal notation:

   (a) $1,234 \times 10^{6}$

   ....

   (b) 5×10^{-1}

   ....

   (c) $4,5 \times 10^{5}$

   ....

   (d) $6,543 \times 10^{-11}$

   ....

3. Why do we say that 34×10^{3} is not written in scientific notation? Rewrite it in scientific notation.

   ....

4. Is each of these numbers written in scientific notation? If not, rewrite it so that it is in scientific notation.

   (a) $90,3 \times 10^{-5}$

   ....

   ....

   (b) 100×10^{2}

   ....

   ....

   (c) $1,36 \times 10^{5}$

   ....

   ....

   (d) $2,01 \times 10^{-2}$

   ....

   ....

   (e) $0,01 \times 10^{3}$

   ....

   ....

   (f) $0,6 \times 10^{8}$

   ....

   ....

CALCULATIONS USING SCIENTIFIC NOTATION

Example: 123 000 × 4 560 000

$= 1{,}23 \times 10^5 \times 4{,}56 \times 10^6$	(write in scientific notation)
$= 1{,}23 \times 4{,}56 \times 10^5 \times 10^6$	(multiplication is commutative)
$= 5{,}6088 \times 10^{11}$	(Use a calculator to multiply the decimals, but add the powers mentally.)

1. Use scientific notation to calculate each of the following. Give the answer in scientific notation.

(a) 135 000 × 246 000 000

(b) 987 654 × 123 456

(c) 0,000065 × 0,000216

(d) 0,000000639 × 0,0000587

Example: $5 \times 10^3 + 4 \times 10^4$

$= 0{,}5 \times 10^4 + 4 \times 10^4$	(Form like terms)
$= 4{,}5 \times 10^4$	(Combine like terms)

2. Calculate the following. Leave the answer in scientific notation.

(a) $7{,}16 \times 10^5 + 2{,}3 \times 10^3$

(b) $2{,}3 \times 10^{-4} + 6{,}5 \times 10^{-3}$

(c) $4{,}31 \times 10^7 + 1{,}57 \times 10^6$

(d) $6{,}13 \times 10^{-10} + 3{,}89 \times 10^{-8}$

Chapter 6
Patterns

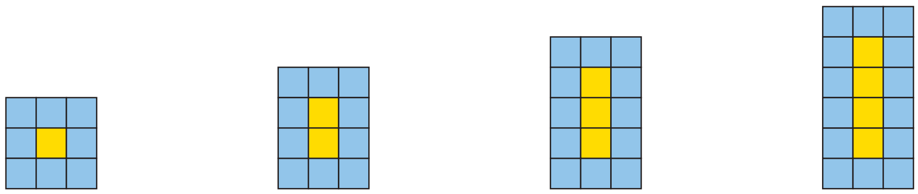
In this chapter you will learn about different kinds of number patterns. Some number patterns are found within geometric patterns. You will learn to identify how patterns are formed, and to make your own patterns. You will learn to make formulae that can be used to describe number patterns.

1	2	3	5	8	13	21	34	55
2	3	5	8	13	21	34	55	89
3	5	8	13	21	34	55	89	144
5	8	13	21	34	55	89	144	233
8	13	21	34	55	89	144	233	377
13	21	34	55	89	144	233	377	610
21	34	55	89	144	233	377	610	987
34	55	89	144	233	377	610	987	1597
55	89	144	233	377	610	987	1597	2584
89	144	233	377	610	987	1597	2584	4181
144	233	377	610	987	1597	2584	4181	6765
233	377	610	987	1597	2584	4181	6765	10946
377	610	987	1597	2584	4181	6765	10946	17711
610	987	1597	2584	4181	6765	10946	17711	28657
987	1597	2584	4181	6765	10946	17711	28657	46368

6 Patterns

6.1 Geometric patterns

INVESTIGATING AND EXTENDING



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

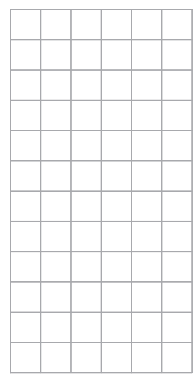
1. Blue and yellow square tiles are combined to form the above arrangements.
   (a) How many yellow tiles are there in each arrangement?

   ....

   (b) How many blue tiles are there in each arrangement?

   ....

   (c) If more arrangements are made in the same way, how many blue tiles and how many yellow tiles will there be in arrangement 5? Check your answer by drawing the arrangement on the grid on the right.

   ....

   (d) Complete this table.

Number of yellow tiles	1	2	3	4	5	8
Number of blue tiles						

   (e) How many blue tiles will there be in a similar arrangement with 26 yellow tiles?

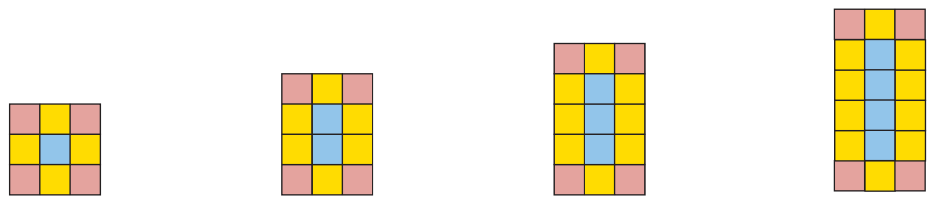
   ....

   (f) How many blue tiles will there be in a similar arrangement with 100 yellow tiles?

   ....

(g) Describe how you thought to produce your answer for (f)?

2. (a) In these arrangements there are red tiles too. Complete this table.



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

Number of blue tiles	1	2	3	4	5	6	7
Number of yellow tiles							
Number of red tiles							

(b) How many red tiles are there in each arrangement?

(c) How many yellow tiles are there in each arrangement?

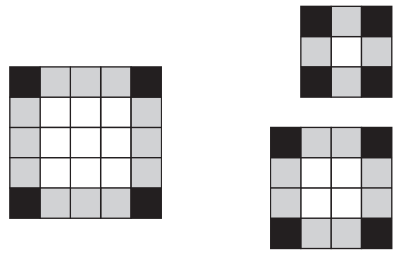
The number of red tiles in arrangements like those in question 2 is **constant.** It is always 4, no matter how many blue and yellow tiles there are.

The number of blue tiles is different for different arrangements. We can say the number of blue tiles **varies.** We can also say the number of blue tiles is a **variable**.

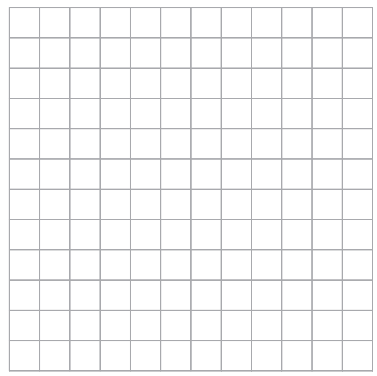
3. Is the number of yellow tiles in the above arrangements a constant or is it a variable?

....

4. Look at these three arrangements. They consist of black squares, grey squares and white squares.



(a) Draw another arrangement of the same kind, but with a different length, on the grid provided on the right.



(b) Describe what is constant in these arrangements.

....

(c) What are the variables in these arrangements?

....

....

The smallest arrangement above may be called arrangement 1, the next bigger one may be called arrangement 2, and so on.

5. (a) Complete the table for arrangements like those in question 4.

Arrangement number	1	2	3	4	5	6	7	10	20
Number of black squares									
Number of grey squares									
Number of white squares									

(b) How many grey squares do you think there will be in arrangement 15? Explain your answer.

....

....

(c) How many black squares do you think there will be in arrangement 15? Explain your answer.

....

(d) How many white squares do you think there will be in arrangement 15? Explain your answer.

....

....

The numbers of grey squares in the different arrangements in question 4 form a pattern: 4; 8; 12; 16; 20; 24; . . . , and so on.

The numbers of white squares in the different arrangements also form a pattern: 1; 4; 9; 16; 25; 36; 49; . . . , and so on.

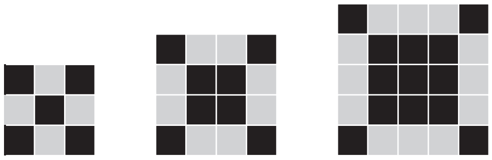
6. What are the next five numbers in each of the above patterns?

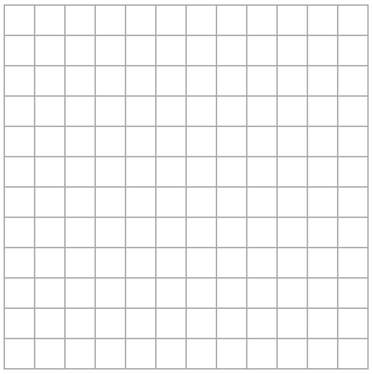
....

....

....

7. (a) Draw the next arrangement that follows the same pattern.





(b) How many black tiles are there in the arrangement you have drawn? ....

(c) How many black tiles will there be in each of the next four arrangements?

....

DO SOMETHING MORE

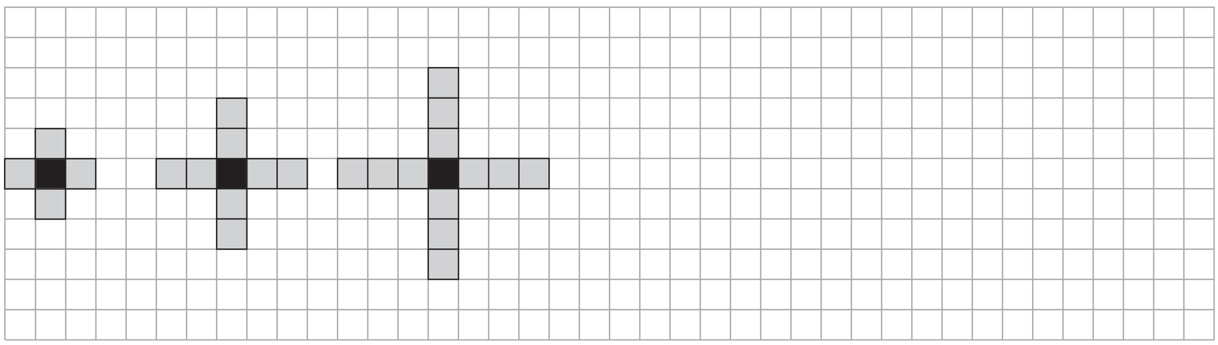
Consider the arrangements in question 4 again. If there are 20 grey tiles in such an arrangement, how many white tiles are there? Enter your answer in the table below. Also complete the table.

Number of grey squares	20	36	52			
Number of white squares				256	225	625

6.2 More patterns

DRAWING AND INVESTIGATING

1. (a) Make two more arrangements of black and grey squares so that a pattern is formed.



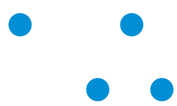
(b) Is there a constant in your pattern? If yes, what is its value?

....

(c) Is there a variable in your pattern? If yes, give the values of the variable.

....

2. (a) Make three more arrangements with dots to form the sequence 1; 3; 6; 10; 15 . . .



(b) How many dots will there be in the sixth and seventh arrangements? Explain how you got your answer.

....

....

....

(c) How many dots are there in arrangements 1 and 2 together? ....

(d) How many dots are there in arrangements 2 and 3 together? ....

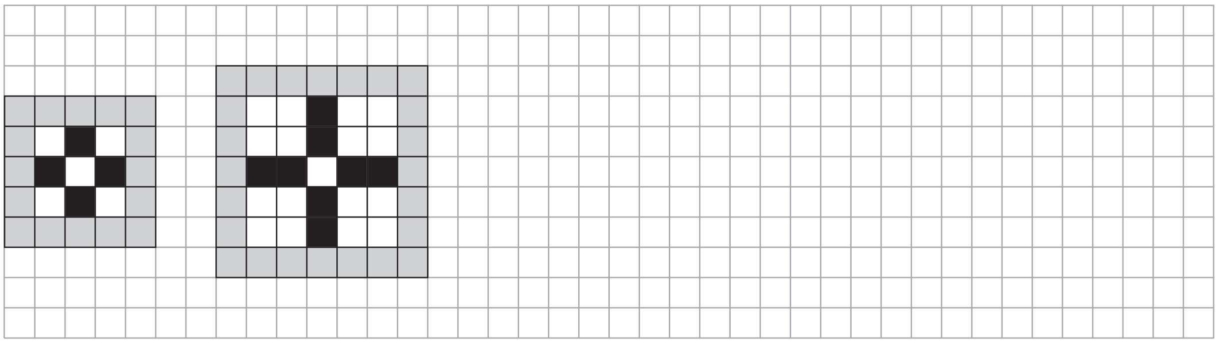
(e) How many dots are there in arrangements 3 and 4 together? ....

(f) How many dots are there in arrangements 4 and 5 together? ....

(g) Describe the pattern in your answers for (c), (d), (e) and (f).

....

3. (a) Draw two more arrangements to make a pattern.



(b) What are the variables in your pattern?

....

(c) The number of black squares is a variable in these arrangements. The value of this variable is 4 in the first arrangement and 8 in the second arrangement. What is the value of this variable in the third arrangement? ....

(d) What are the values of each of the variables in the fifth arrangement in your pattern? Explain your answers.

....

....

....

4. (a) Now make a pattern of your own.

(b) Use this table to describe the variables in your pattern, and their values.

Arrangement number	1	2	3	4	5	6

6.3 Different kinds of patterns in sequences

DO THE SAME THING REPEATEDLY

1. (a) Write the next three numbers in each of the sequences below.

   Sequence A: 5 9 13 17 21 ....

   Sequence B: 5 10 20 40 80 ....

   Sequence C: 5 10 17 26 37 ....

   (b) Describe the differences in the ways in which the three sequences are formed.

   ....

   ....

   ....

2. You will now make a sequence with the first term 5. Write 5 on the left on the line below. Then add 8 to the first term (5) to form the second term of your sequence. Write the second term next to the first term (5) in the line below. Now add 8 to the second term to form the third term. Continue like this to form ten more terms.

   ....

The numbers in a sequence are also called the **terms** of the sequence.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between consecutive terms in a sequence is **constant**.

A sequence can be formed by repeatedly multiplying or dividing. In this case the **ratio** between consecutive terms is **constant**.

A sequence can also be formed in such a way that neither the difference nor the ratio between consecutive terms is constant.

To write more terms of sequence A in question 1(a), you **added** 4 **repeatedly**.

To write more terms of sequence B in question 1(a), you **multiplied** by 2 **repeatedly**.

To write more terms of sequence C in question 1(a) you did not add the same number each time, nor did you multiply by the same number.

3. Write the next three terms of each sequence. In each case also describe what the pattern is, for example "there is a constant difference of –5 between consecutive terms".
   (a) 100; 92; 84; 76; ....

   ....
   (b) 1; 4; 9; 16; ....

   ....

   ....
   (c) 2; 8; 18; 32; ....

   ....

   ....
   (d) 3; 6; 11; 18; ....

   ....

   ....
   (e) 640; 320; 160; ....

   ....

   ....
   (f) 1; 2; 4; 7; 11; ....

   ....

   ....

4. In each case, follow the instruction to make a sequence with eight terms.
   (a) Start with 1 and multiply by 2 repeatedly.

   ....
   (b) Start with 256 and subtract 32 repeatedly.

   ....
   (c) Start with 256 and divide by 2 repeatedly.

   ....

The sequence that you made in question 2 can be represented with a table, as shown below.

Term number	1	2	3	4	5	6	7	8	9	10
Term value	5	13	21	29	37	45	53	61	69	77

5. In each case make a sequence by following the instructions. Write the term numbers and the term values in the given table.

   (a) Term 1 = 10. Add 15 repeatedly.

Term number								
Term value								

   (b) Term 1 = 10. Term value = 15 × term number – 5.

Term number								
Term value								

   (c) Term 1 = 10. Multiply by 2 repeatedly.

Term number						
Term value						

   (d) Term 1 = 20. Term value = $10 \times 2^{\text{term number}}$

Term number						
Term value						

   (e) Term 1 = 10. Term value = $10 \times 2^{\text{term number} - 1}$

Term number						
Term value						

   (f) **Term 4** = 30. Add 5 repeatedly.

Term number								
Term value								

6. Instructions for forming a sequence are given in two different ways in question 5. How would you describe the two different ways for giving instructions to form a sequence?

   ....

   ....

6.4 Formulae for sequences

The formula for a number sequence can be written in two different ways:

- A description of the **relationship between consecutive terms**. In other words the calculations that you do to a term to produce the next term, as in questions 5(a), (c) and (f) on the previous page. The first (or another) term must be given. This kind of formula has two parts, the first term, and the relationship between terms.
- A description of the **relationship between the value of the term and its position in the sequence**. This relationship describes the calculations that can be done **on the term number** to produce the **term value**, as in question 5(b), (d) and (e) on the previous page.

MAKE TWO FORMULAE FOR THE SAME SEQUENCE

1. Choose any whole number smaller than 10 as the first term of a sequence.
   (a) Use your chosen first term to form a sequence by adding 5 repeatedly.

   ....

   (b) Multiply each term number below by 5 to form a sequence:

Term number	1	2	3	4	5	6	7	8
Term value								

   (c) What is similar about the two sequences you have formed?

   ....

   (d) Now fill in your own sequence in the same table:

Term number	1	2	3	4	5	6	7	8
Term value in (b)								
Term value of your own sequence in (a)								

(e) What must you add to or subtract from each term value in (b) to get the same sequence as the one you made in (a)?

....

(f) Fill in the following to write a formula for each sequence:

For the sequence in (b): Term value = ... (term number) ....

For the sequence in (a): Term value = ... (term number) ....

2. Now you are going to repeat what you did in question 1, with a different set of sequences.

In this sequence, the term number is multiplied by 3 to get the term value.

Term number	1	2	3	4	5	6	7	8
Term value	3	6	9	12	15	18	21	24

Now make a formula describing the relationship of the **term value** to the **term number** for each of these sequences:

(a) The sequence that starts with 8 and is formed by adding 3 repeatedly.

....

....

(b) The sequence that starts with 12 and is formed by adding 3 repeatedly.

....

....

(c) The sequence that starts with 2 and is formed by adding 3 repeatedly.

....

....

3. Write the first eight terms of each of the following sequences, and in each case describe how each term can be calculated from the previous term.

(a) Term value = 10 × term number + 5

Term number	1	2	3	4	5	6	7	8
Term value								

....

(b) Term value = 5 × term number – 3

Term number	1	2	3	4	5	6	7	8
Term value								

....

4. For each sequence, write a formula to obtain each term from the previous term, and also try to write formula which relates each term to its position in the sequence. Check both your formulae by applying them, and write the results in the table.

(a) 7 11 15 19 23 27 31 35 39 43

A. Relationship between consecutive terms: ....

....

B. Relationship between term value and its position in sequence: ....

....

Term number	1	2	3	4	5
Term value using A					
Term value using B					

(b) 60 57 54 51 48 45 42 39 36

A. Relationship between consecutive terms: ....

....

B. Relationship between term value and its position in sequence: ....

....

Term number	1	2	3	4	5
Term value using A					
Term value using B					

(c) 1 2 4 8 16 32 64 128

A. Relationship between consecutive terms: ....

B. Relationship between term value and its position in sequence: ....

....

Term number	1	2	3	4	5
Term value using A					
Term value using B					

Chapter 7
Functions and relationships

In this chapter you will work with relationships between sets of numbers called input numbers and output numbers. You will find the output numbers that correspond to given input numbers, and the other way round. You will use rules to calculate the output numbers, and you will solve equations to find the input numbers. The rules to calculate the output numbers can be given in words (verbally), as flow diagrams or as formulae.

-10	100	-19	-36	-51	-64	-75	-84	-91
-9	81	-17	-32	-45	-56	-65	-72	-77
-8	64	-15	-28	-39	-48	-55	-60	-63
-7	49	-13	-24	-33	-40	-45	-48	-49
-6	36	-11	-20	-27	-32	-35	-36	-35
-5	25	-9	-16	-21	-24	-25	-24	-21
-4	16	-7	-12	-15	-16	-15	-12	-7
-3	9	-5	-8	-9	-8	-5	0	7
-2	4	-3	-4	-3	0	5	12	21
-1	1	-1	0	3	8	15	24	35
0	0	1	4	9	16	25	36	49
1	1	3	8	15	24	35	48	63
2	4	5	12	21	32	45	60	77
3	9	7	16	27	40	55	72	91
4	16	9	20	33	48	65	84	105
5	25	11	24	39	56	75	96	119
6	36	13	28	45	64	85	108	133
7	49	15	32	51	72	95	120	147
8	64	17	36	57	80	105	132	161
9	81	19	40	63	88	115	144	175
10	100	21	44	69	96	125	156	189

7 Functions and relationships

7.1 Find output numbers for given input numbers

TWO DIFFERENT SETS OF INPUT NUMBERS

In this activity you will do some calculations with:

Set A: the natural numbers smaller than 10: the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Set B: multiples of 10 that are bigger than 10 but smaller than 100: the numbers 20, 30, 40, 50, 60, 70, 80 and 90.

1. You are going to choose a number, multiply it by 5, and subtract the answer from 50.

   (a) Choose any number from set A and do the above calculations.

   ....

   (b) Choose any number from set B and do the above calculations.

   ....

   (c) If you choose any other number from set B, do you think the answer will also be a negative number?

   ....

   ....

2. (a) Write down all the different output numbers that will be obtained when the calculations $50 - 5x$ are performed on the different numbers in set A.

Output numbers are numbers that you obtain when you apply the rule to the input numbers.

   ....

   ....

   ....

   (b) Write down the output numbers that will be obtained when the formula $50 - 5x$ is applied to set B.

   ....

3. (a) Complete the following table for set A:

Input numbers	1	2	3	4	5	6	7	8	9
Values of $50 - 5x$									

(b) Complete the following table for set B:

Input numbers	20	30	40	50	60	70	80	90
Values of $50 - 5x$								

4. In this question your set of input numbers will be the even numbers 2; 4; 6; 8; 10; ...

(a) What will all the output numbers be if the rule $2n + 1$ is applied to the set of even numbers? Write a list.

....

....

(b) What will the output numbers be if the rule $2n - 1$ is applied?

....

(c) What will the output numbers be if the rule $2n + 5$ is applied?

....

(d) What will the output numbers be if the rule $3n + 1$ is applied?

....

5. (a) What kind of output numbers will be obtained by applying the rule $x - 1\ 000$ to natural numbers smaller than 1 000?

....

(b) What kind of output numbers will be obtained by applying the rule $\frac{x}{10} + 10$ to natural numbers smaller than 10?

....

....

(c) If you use the rule $30x + 2$, and use input numbers that are positive fractions with denominators 2, 3 and 5, what kind of output numbers will you obtain?

....

....

7.2 Different ways to represent the same relationship

Consider the work that you did in Section 6.4 of Chapter 6. In each question, there were two variable quantities.

A quantity that changes is called a **variable quantity** or just a **variable**.

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. You can sometimes work out which number is linked to a specific value of the other variable.

An algebraic expression such as $10x + 5$ describes what calculations must be done to find the output number that corresponds to a given input number.

The output number can also be called the output value, or the value of the expression, which is $10x + 5$ in this case.

For any input number, application of the rule $10x + 5$ produces only one output number, and it is very clear what that number is. For instance if the formula is applied to $x = 3$, the output number is 35.

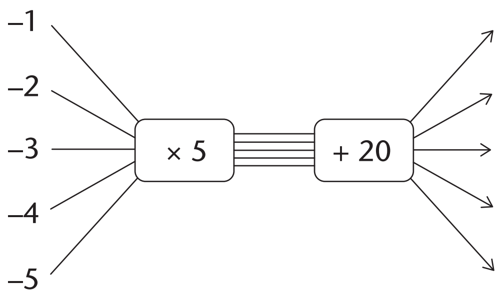
A relationship between two variables in which there is only one output number for each input number, is called a **function**.

Functions can be represented in different ways:

- With a table that shows some values of the two variables. A table shows clearly which value of the output variable corresponds to each particular value of the input variable.
- A flow diagram, which shows what calculations are to be done to calculate the output number that corresponds to a given input variable.
- A formula, which also describes what calculations are to be done to calculate the output number that corresponds to a given input variable.
- A graph.

Examples of these four ways of describing a function are given on the next two pages.

1. Complete the flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you have completed shows the following information:

- Each input number is multiplied by 5, then 20 is added, to produce the output numbers.
- Which output numbers correspond to which input numbers.

The calculations that need to be done can also be described with an expression. The expression $5x + 20$ describes what calculations you did in question 1. One may also write this as a formula:

A verbal formula:
output number = 5 × input number + 20

An algebraic formula:
output number = $5x + 20$

The output numbers of a function are also called **function values**. Hence the formula can also be written as *function value* = $5x + 20$

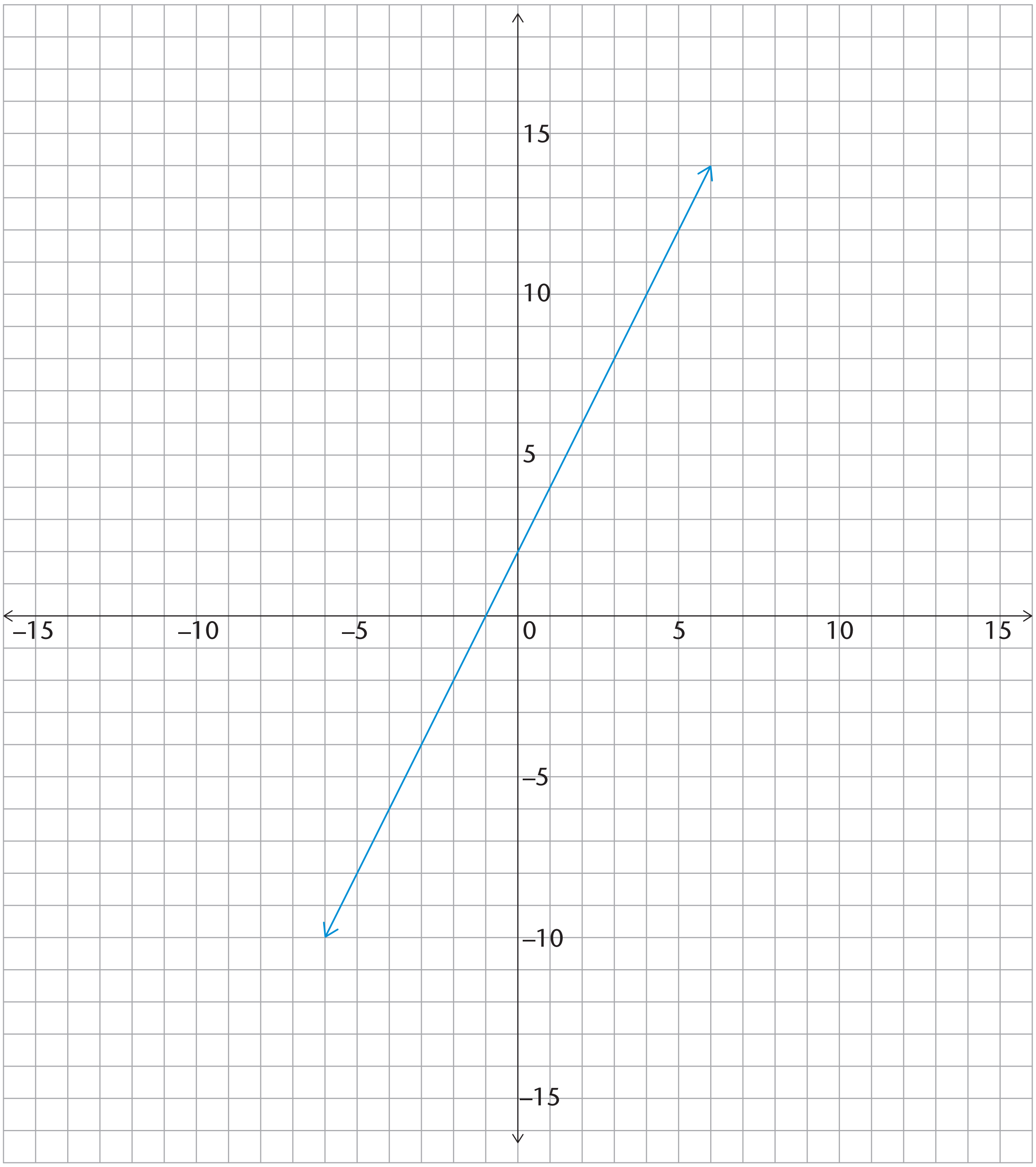
2. Complete this table for the function described by $5x + 20$:

Input numbers	–1	–2	–3	–4	–5
Function values					

3. Draw a graph of this function on the next page.

4. A graph of a certain function is given below. Complete the table for this function.

Input numbers					
Function values					



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7.3 Different representations of the same relationship

Do this work on the following pages. There is a page for each question.

Represent each of the following functions with

(a) a flow diagram

(b) a table of values for the set of integers from –5 to 5

(c) a graph

1. The relationship described by the expression $3x + 4$

2. The relationship described by the expression $2x - 5$

3. The relationship described by the expression $\frac{1}{2}x + 2$

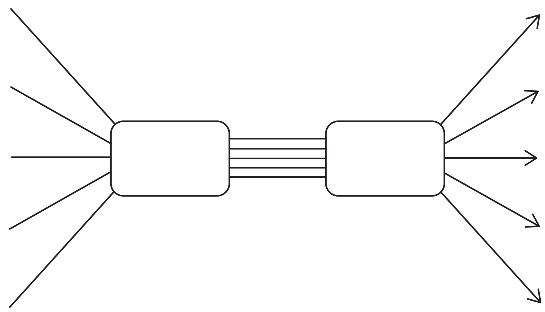
4. The relationship described by the expression $-3x + 4$

5. The relationship described by the expression $2{,}5x + 1{,}5$

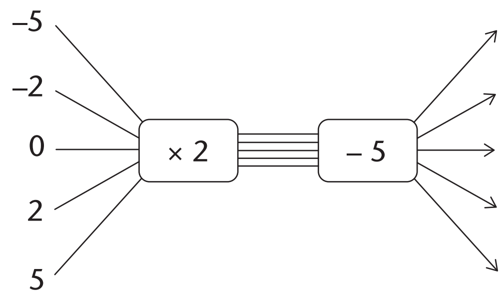
6. The relationship described by the expression $0{,}2x + 1{,}4$

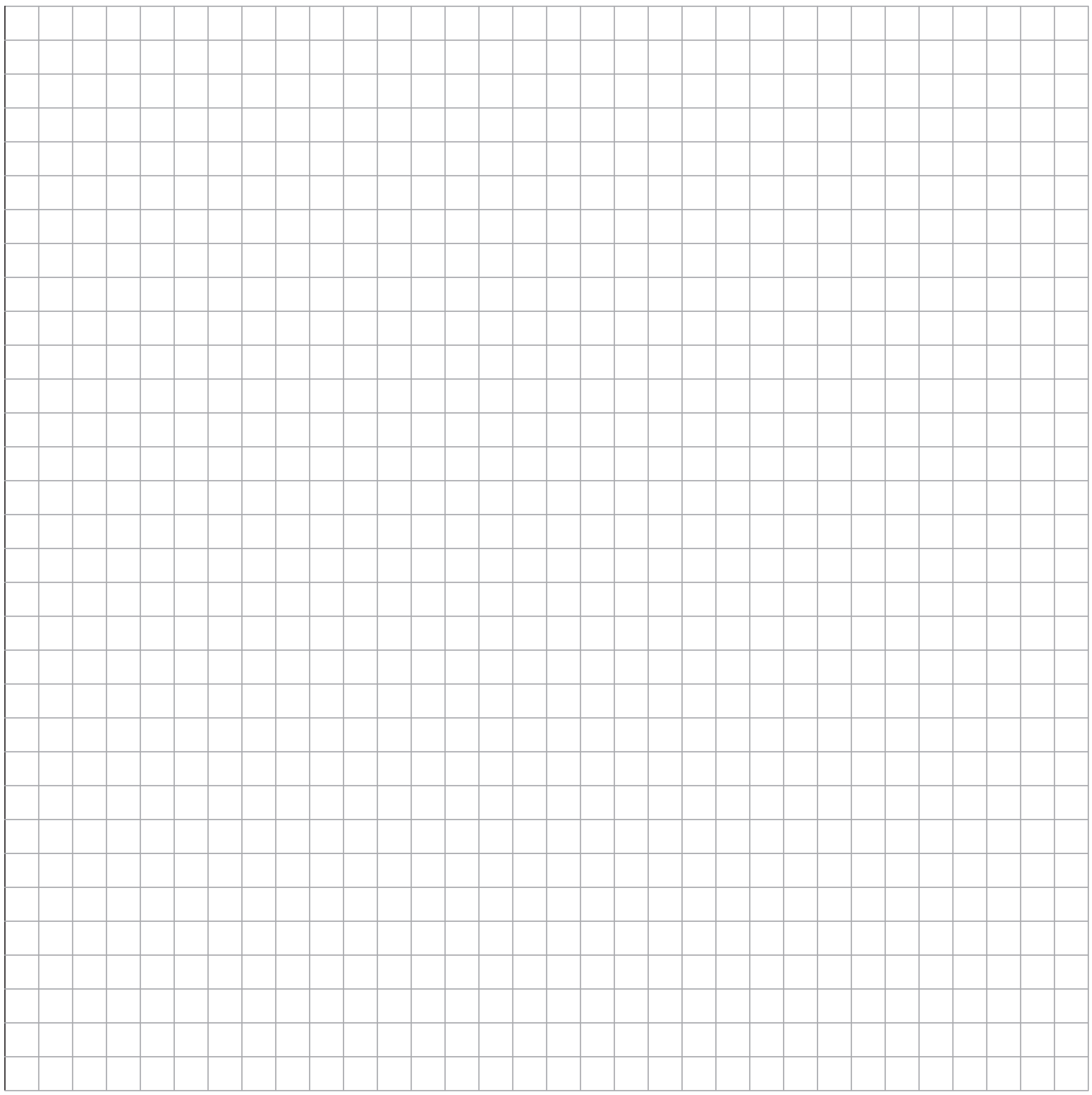
7. The relationship described by the expression $-2x - 4$

1.



2.





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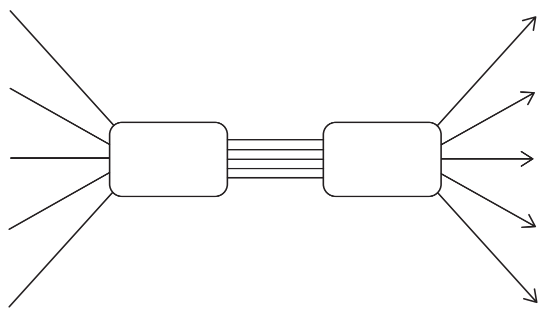

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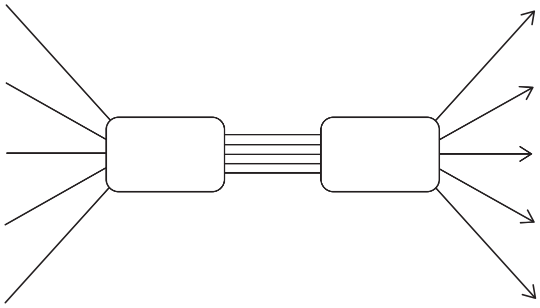

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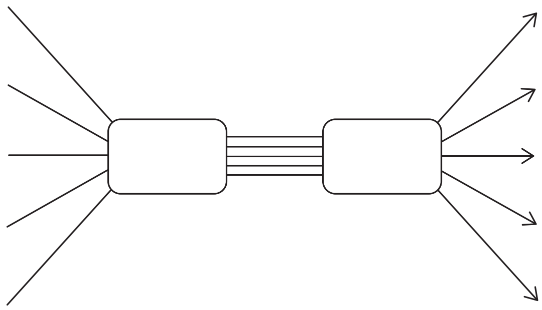

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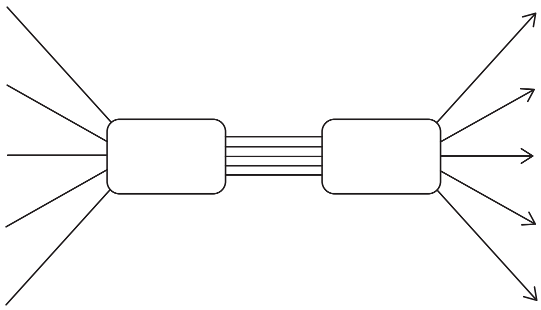

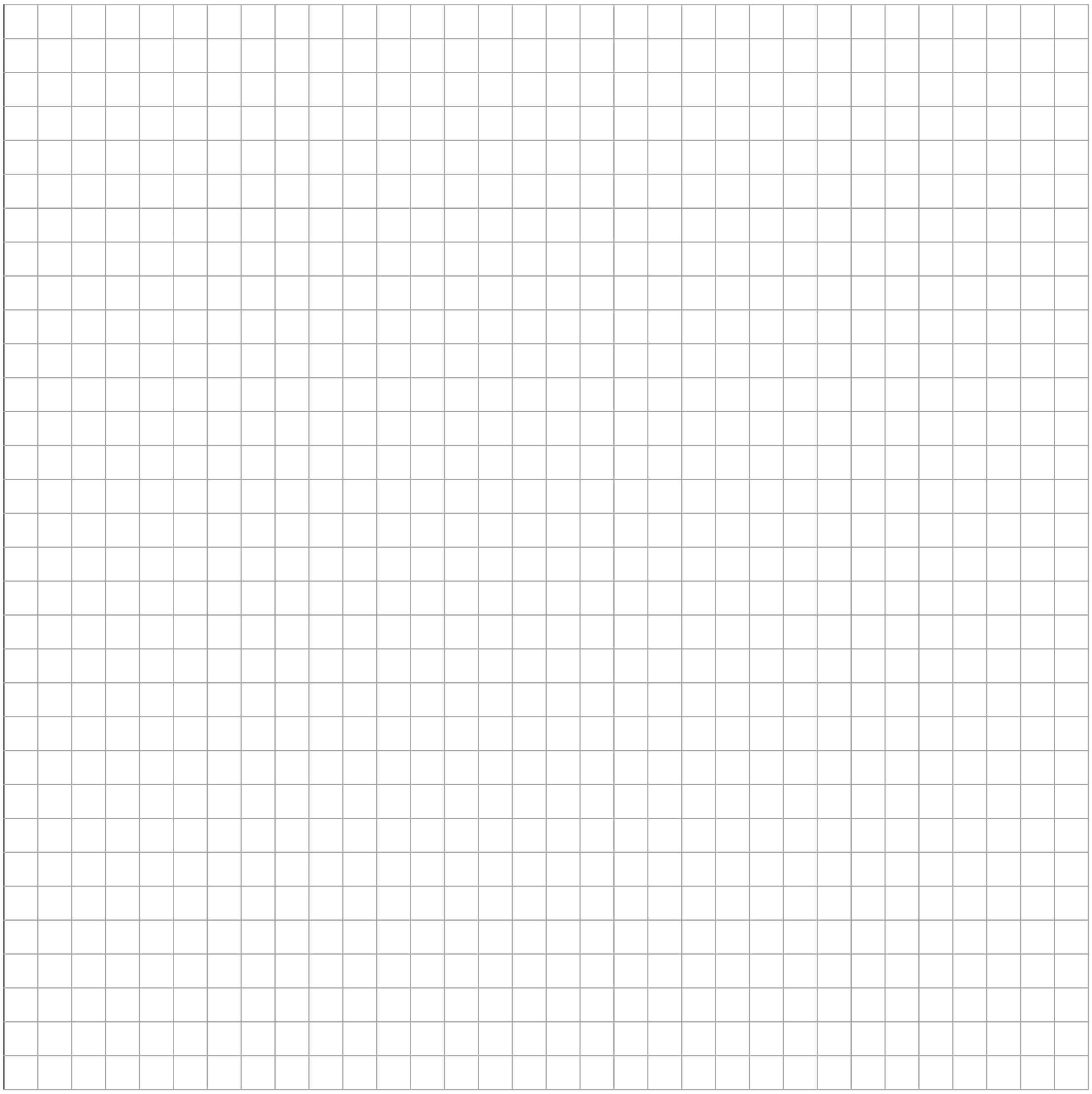

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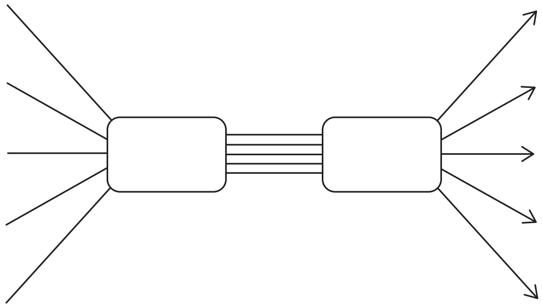

Chapter 8
Algebraic expressions

An algebraic expression is a description of a set of operations that are to be done in a certain order. In this chapter, you will learn to specify a different set of operations that will produce the same results as a given set of operations. Two different expressions that produce the same results are called equivalent expressions.

8 Algebraic expressions

8.1 Algebraic language

WORDS, DIAGRAMS AND EXPRESSIONS

1. Complete this table.

	Words	Flow diagram	Expression
	Multiply a number by 5 and then subtract 3 from the answer.	→ [× 5] → [– 3] →	$5x - 3$
(a)	Add 5 to a number and then multiply the answer by 3.	→ [ ] → [ ] →	
(b)		→ [– 3] → [× 5] →	
(c)			$3(2x + 3)$

An **algebraic expression** indicates **a sequence of operations** that can also be described in words. In some cases they can be described with flow diagrams.

Expressions in brackets should always be calculated first. If there are no brackets in an algebraic expression, it means that multiplication and division must be done first, and addition and subtraction afterwards.

For example, if $x = 5$ the expression $12 + 3x$ means "multiply 5 by 3, then add 12". It does **not** mean "add 12 and 3, then multiply by 5".

If you wish to say "add 12 and 3, then multiply by 5", the numerical expression should be $5 \times (12 + 3)$ or $(12 + 3) \times 5$.

2. Describe each of these sequences of calculations with an algebraic expression:
   (a) Multiply a number by 10, subtract 5 from the answer, and multiply the answer by 3.

   ....

   (b) Subtract 5 from a number, multiply the answer by 10, and multiply this answer by 3.

   ....

3. Evaluate each of these expressions for $x = 10$:

(a) $200 - 5x$ ....

(b) $(200 - 5)x$ ....

(c) $5x + 40$ ....

(d) $5(x + 40)$ ....

(e) $40 + 5x$ ....

(f) $5x + 5 \times 40$ ....

SOME WORDS WE USE IN ALGEBRA

An expression with one term only, like $3x^2$, is a **monomial**.
An expression which is a sum of two terms, like $5x + 4$, is called a **binomial**.
An expression which is a sum of three terms, like $3x^3 + 2x + 9$, is called a **trinomial**.

The symbol x is often used to represent the **variable** in an algebraic expression, but other letter symbols may also be used.

In the monomial $3x^2$, the 3 is called the **coefficient** of x^2.

In the binomial $5x + 4$, and the trinomial $3x^2 + 2x + 9$ the numbers 4 and 9 are called **constants**.

1. Complete the table, using the completed first row as an example.

Expression	Type of expression	Symbol used to represent the variable	Constant	Coefficient of
$x^2 + 6x + 10$	Trinomial	x	10	the second term is 6
$6s^3 + s^2 + 5$				s^2 is ......
$\frac{k}{3} + 12$				the first term is
$4p^{10}$				p^{10} is ......

2. Consider the pattern-polynomial starting with $7x^5 + 5x^4 + 3x^3 + x^2 + \ldots$

(a) What is the coefficient of the fourth term? ....

(b) What is the exponent value of the fifth term? ....

(c) Do you think the sixth term will be a constant? Why? ....

....

EQUIVALENT ALGEBRAIC EXPRESSIONS

1. Calculate the numerical value of the expressions for the various values of x. Do the calculations in your exercise book. Then fill in your answers.

	x	-2	-1	0	1	2
(a)	$3x + 2$					
(b)	$2x - 3$					
(c)	$3x + 2 + 2x - 3$					
(d)	$2x - 3 + 3x + 2$					
(e)	$5x - 1$					
(f)	$(3x + 2)(2x - 3)$					
(g)	$3x(2x - 3) + 2(2x - 3)$					
(h)	$6x^2 - 5x - 6$					
(i)	$\frac{(3x+2)(2x-3)}{3x+2}$					
(j)	$\frac{6x^2-5x-6}{3x+2}$					

2. Make a list of all the algebraic expressions above which have the same numerical value for the same value of x, although they may look different:

....

....

....

Equivalent expressions are algebraic expressions that have different sequences of operations, but have the same numerical value for any given value of x.

It is often convenient not to work with a given expression, but to **replace** it with an equivalent expression.

3. Complete this table.

x	2	3	5	10	–5	–10
$12x - 7 + 3x + 10 - 5x$						

4. Complete this table.

x	2	3	5	10	–5	–10
$10x + 3$						

5. (a) Is $10x + 3$ equivalent to $12x - 7 + 3x + 10 - 5x$? Explain your answer.

....

....

....

(b) Suppose you need to know how much $12x - 7 + 3x + 10 - 5x$ is for $x = 37$ and $x = -43$. What do you think is the easiest way to find out? ....

....

CONVENTIONS FOR WRITING ALGEBRAIC EXPRESSIONS

Here are some things that mathematicians have agreed upon, and it makes mathematical work much easier if all people stick to these agreements.

A **convention** is something that people have agreed to do in the same way.

The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$ and $4(x - 5)$ instead of $4 \times (x - 5)$.

It is a convention to write a known number first in a product, i.e. we write $3 \times x$ rather than $x \times 3$, and we write $3x$ **but not** $x3$.

1. Rewrite each of the following in the way in which it is normally written in algebraic expressions.

(a) $x \times 4 + x \times y - y \times 3$

....

(b) $7 \times (10 - x) + (5 \times x + 3)10$

....

People all over the world have agreed that, in expressions that do not contain brackets, addition and subtraction should be performed as they appear from left to right in the expression.

According to this convention, $x - y + z$ means that you first have to subtract y from x, then add z. For example if $x = 10$, $y = 5$ and $z = 3$, $x - y + z$ is $10 - 5 + 3$ and it means $10 - 5 = 5$, then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10 - 8 = 2$.

2. Calculate $50 - 20 + 30$ and $50 + 30 - 20$ and $50 - 30 + 20$

....

3. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$.

(a) $x + y - z$

....

(b) $x - z + y$

....

(c) $10y - 3x + 5z - 4y$

....

(d) $10y - 3x - 5z + 4y + 3x$

....

People have also agreed that, in expressions that do not contain brackets, we should do **multiplication** (and division) **before addition and subtraction**.

Hence $5 + 3 \times 4$ should be understood as "multiply 4 by 3, then add the answer to 5" and not as "add 5 and 3 then multiply the answer by 4".

Also, $3 \times 4 + 5$ should be understood to mean "multiply 4 by 3, then add 5 to the answer", and not as "add 4 and 5 then multiply the answer by 3".

4. Do each of the following calculations.

(a) multiply 4 by 3, then add 5 to the answer

....

(b) add 4 and 5 then multiply the answer by 3

....

(c) multiply 4 by 3, then add the answer to 5

....

(d) add 5 and 3 then multiply the answer by 4

....

5. Rewrite the instructions in 4(a) and 4(c) without using words.

....

....

6. Calculate each of the following.

(a) $10 \times 5 + 30$

....

(b) $30 + 10 \times 5$

....

(c) $10 \times 5 - 30$

....

(d) $30 - 10 \times 5$

....

7. (a) Add 4 and 5, then subtract the answer from 20.

....

(b) Subtract 4 from 20 and then add 5.

....

(c) Add 4 and 5, then multiply the answer by 3.

....

(d) Multiply 3 by 5 and then add the answer to 4.

....

If we want to specify the calculations in 7(a) and 7(c) without using words we face challenges.

We cannot write $20 - 4 + 5$ for "*add 4 and 5 then subtract the answer from 20*", because that would mean "*subtract 4 from 20 then add 5*". We need a way to indicate, without using words, that we want the addition to be performed before the subtraction in this case.

Similarly we cannot write $4 + 5 \times 3$ for "*add 4 and 5 then multiply the answer by 3*", because that would mean "*multiply 3 by 5 and then add the answer to 4*". We need a way to indicate, without using words, that we want the addition to be performed before the multiplication in this case.

Mathematicians have agreed to use brackets to address the above challenges. The following convention is used all over the world:

> Whenever there are brackets in an expression, the calculations within the brackets should be performed first.

Hence 20 – (4 + 5) means *add 4 and 5 then subtract the answer from 20*, but 20 – 4 + 5 means *subtract 4 from 20 then add 5*.

(4 + 5) × 3 or 3 × (4 + 5) means *add 4 and 5 then multiply the answer by 3*, but 4 + 5 × 3 means *multiply 3 by 5 then add the answer to 4*.

10 + 2(5 + 9) means *add 5 and 9, multiply the answer by 2, then add this answer to 10*:

5 + 9 = 14   14 × 2 = 28   28 + 10 = 38

8. Calculate each of the following.

(a) 100 + 50 – 30

....

(b) 100 + (50 – 30)

....

(c) 100 – 50 + 30

....

(d) 100 – (50 + 30)

....

(e) 3(10 – 4) + 2

....

(f) 10(5 + 7) + 3(18 – 8)

....

(g) 250 – 10 × (18 + 2) + 35

....

(h) (20 + 20) × (20 – 10)

....

(i) (250 – 10) × (18 + 2) + 35

....

(j) 20 + 20 × (20 – 10)

....

(k) 200 + (100 × 2(15 + 5))

....

(l) (200 + 100) × 2 × 15 + 5

....

In algebra, we normally write $3(x + 2y)$ instead of $(x + 2y) \times 3$, and we write $3(x - 2y)$ instead of $(x - 2y) \times 3$. Don't let this conventional way of writing in algebra confuse you. The expression $3(x + 2y)$ does not mean that multiplication by 3 is the first thing you should do when you evaluate the expression for certain values of x and y. The first thing you should do is to add the values of x and y. That is what the brackets tell you!

However, performing the instructions $3(x + 2y)$ is not the only way in which you can find out how much $3(x + 2y)$ is for any given values of x and y. Instead of working out $3(x + 2y)$, you may work out $3x + 6y$. In this case you will multiply each term before you add them together.

9. Evaluate each of the following expressions for $x = 10$, $y = 5$ and $z = 2$.

(a) $xy + z$

....

(b) $x(y + z)$

....

(c) $x + yz$ (d) $xy + xz$

....

(e) $xy - z$ (f) $x(y - z)$

....

(g) $x - yz$ (h) $xy - yz$

....

(i) $x + (y - z)$ (j) $x - (y - z)$

....

(k) $x - (y + z)$ (l) $x - y - z$

....

(m) $x + y - z$ (n) $x - y + z$

....

8.2 Properties of operations

1. Calculate the following:
   (a) $5(3 + 4)$ (b) $5 \times 3 + 5 \times 4$

   ....

   (c) $6 \times 3 + (4 + 6)$ (d) $(6 + 4) + 3 \times 6$

   ....

   (e) $3 \times (4 \times 5)$ (f) $(3 \times 4) \times 5$

   ....

You should have noticed that for each row the results are the same. This is because operations with numbers have certain properties, namely the **distributive**, **commutative** and **associative** properties.

The **distributive** property is used each time you multiply a number in parts. For example:

The number thirty-four is actually 30 + 4. You may calculate 5 × 34 by calculating 5 × 30 and 5 × 4, and then adding the two answers:

$$5 \times 34 = 5 \times 30 + 5 \times 4$$

The word "distribute" means to spread out. The distributive property may be described as follows:

$a(b + c) = ab + ac$

where a, b and c can be any numbers.

We may say: "multiplication distributes over addition"

2. Calculate each of the following:

(a) $5(x - y)$ for $x = 10$ and $y = 8$

....

(b) $5x - 5y$ for $x = 10$ and $y = 8$

....

(c) $5(x - y)$ for $x = 100$ and $y = 30$

....

(d) $5x - 5y$ for $x = 100$ and $y = 30$

....

(e) $5(x - y + z)$ for $x = 10$, $y = 3$, $z = 2$

....

(f) $5x - 5y + 5z$ for $x = 10$, $y = 3$, $z = 2$

....

3. We say "multiplication distributes over addition".
Does multiplication also distribute over subtraction?
Give examples to support your answer.

....

....

For any values of x and y,

- $x + y$ and $y + x$ give the same answers, and
- xy and yx give the same answers.

This is called the **commutative property** of addition, and multiplication.

4. We say "addition is commutative" and "multiplication is commutative".
Is subtraction also commutative? Demonstrate your answer with an example.

....

The **associative property** allows you to arrange three or more numbers in any sequence when adding or multiplying. For any values of x, y and z, the following expressions all have the same answer:

$x + y + z$ $\quad\quad$ $y + x + z$ $\quad\quad$ $z + y + x$

5. Calculate 16 + 33 + 14 + 17 in the easiest possible way.

....

The associative property of multiplication allows you to simplify something like the following.

$abc + bca + cba$

Because the order of multiplication does not change the result we can rewrite this expression as: $abc + abc + abc$.

This then can be simplified by adding like terms to be $3abc$. You will be able to use these properties throughout this chapter and when you do algebraic manipulations.

When you form an expression that is equivalent to a given expression you say that you *manipulate* the expression.

6. Replace each of the following expressions with a simpler expression that will give the same answer. **Do not do any calculations now.** In each case state why your replacement will be easier to do.
   (a) 17 × 43 + 17 × 57

   ....

   ....

   (b) 7 × 5 × 8 × 4 + 12 × 8 × 4 × 7 – 9 × 4 × 5 × 8

   ....

   ....

   (c) 43 × 17 + 57 × 17

   ....

   ....

   (d) $43x + 57x$ (for $x = 213$ or any other value)

   ....

   ....

7. Which properties of operations did you use in each part of question 6?

....

....

8.3 Combining like terms in algebraic expressions

REARRANGE TERMS, THEN COMBINE LIKE TERMS

To check whether two expressions are possibly equivalent, you can evaluate both expressions for several different values of the variable.

1. In each case below, first predict whether the two expressions are equivalent and then check by evaluating both for $x = 1$, $x = 10$, $x = 2$ and $x = -2$ in the tables.

   (a) $x(x + 3)$ and $x^2 + 3$

   ....

   ....

   (b) $x(x + 3)$ and $x^2 + 3x$

   ....

   ....

Some expressions can be simplified by rearranging the terms and combining "like terms".

In the expression $5x^2 + 13x + 7 + 2x^2 - 8x - 12$, the terms $5x^2$ and $2x^2$ are like terms.

Two or more like terms can be combined to form a single term.

$5x^2 + 2x^2$ can be replaced by $7x^2$ because for any value of x, for example $x = 2$ or $x = 10$, calculating $5x^2 + 2x^2$ and $7x^2$ will produce the same output value (try it!).

2. Complete the table.

x	10	2	5	1
$5x^2 + 2x^2$				
$7x^2$				
$13x - 8x$				
$5x$				

It is difficult to see the like terms in a long expression like $3x^2 + 13x + 7 + 2x^2 - 8x - 12$. Fortunately, you can rearrange the terms in an expression so that the like terms are next to each other.

3. (a) Complete the second and third rows of the table below. You will complete the next two rows when you do question (g).

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$				
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$				

(b) What do you observe? ....

(c) How does the one expression in the above table differ from the other one?

....

(d) Combine like terms in $3x^2 + 2x^2 + 13x - 8x + 7 - 12$ to make a shorter equivalent expression.

....

(e) Evaluate your shorter expression for $x = 10$, $x = 2$ and $x = 5$.

....

....

(f) Is your shorter expression equivalent to $3x^2 + 13x + 7 + 2x^2 - 8x - 12$? Explain how you know whether it is or is not.

....

....

....

(g) Evaluate $5x^2 + 5x - 5$ and $5(x^2 + x - 1)$ for $x = 10$, $x = 2$, $x = 5$ and $x = 1$, and write your answers in the last two rows of the above table.

4. Simplify each expression:

(a) $(3x^2 + 5x + 8) + (5x^2 + x + 4)$

....

(b) $(7x^2 + 3x + 5) + (2x^2 - x - 2)$

....

(c) $(6x^2 - 7x - 4) + (4x^2 + 5x + 5)$

....

(d) $(2x^2 - 5x - 9) - (5x^2 - 2x - 1)$

....

(e) $(-2x^2 + 5x - 3) + (-3x^2 - 9x + 5)$

....

(f) $(y^2 + y + 1) + (y^2 - y - 1)$

....

5. Complete the table. (Hint: Save yourself some work by simplifying first!)

....

....

....

x	2,5	3,7	6,4	12,9	35	–4,7	–0,04
$(3x + 6,5) + (7x + 3,5)$							
$(13x - 6) + (26 - 12x)$							

6. Simplify:

(a) $(2r^2 + 3r - 5) + (7r^2 - 8r - 12)$

....

(b) $(2r^2 + 3r - 5) - (7r^2 - 8r - 12)$

....

(c) $(2x + 5xy + 3y) - (12x - 2xy - 5y)$

....

(d) $(2x + 5xy + 3y) + (12x - 2xy - 5y)$

....

7. Evaluate the following expressions for $x = 3$, $x = -2$, $x = 5$ and $x = -3$.
   (a) $2x(x^2 - x - 1) + 5x(2x^2 + 3x - 5) - 3x(x^2 + 2x + 1)$

   ....

   ....

   ....

   (b) $(3x^2 - 5x + 7) - (7x^2 + 3x - 5) + (5x^2 - 2x + 8)$

   ....

   ....

   ....

8. Write equivalent expressions without brackets.
   (a) $3x^4 - (x^2 + 2x)$ ....
   (b) $3x^4 - (x^2 - 2x)$ ....
   (c) $3x^4 + (x^2 - 2x)$ ....
   (d) $x - (y + z - t)$ ....

9. Write equivalent expressions without brackets, rearrange so that like terms are grouped together, and then combine the like terms.
   (a) $2y^2 + (y^2 - 3y)$ ....
   (b) $3x^2 + (5x + x^2)$ ....
   (c) $6x^2 - (x^4 + 3x^2)$ ....
   (d) $2t^2 - (3t^2 - 5t^3)$ ....
   (e) $6x^2 + 3x - (4x^2 + 5x)$ ....
   (f) $2r^2 - 5r + 7 + (3r^2 - 7r - 8)$ ....
   (g) $5(x^2 + x) + 2(x^2 + 3x)$ ....
   (h) $2x(x - 3) + 5x(x + 2)$ ....

10. Write equivalent expressions without brackets and simplify these expressions as far as possible.

   Example $5r^2 - 2r(r + 5) = 5r^2 - 2r^2 - 10r$
   $= 3r^2 - 10r$

   (a) $3x^2 + x(x + 3)$ ....
   (b) $5x + x(7 - 2x)$ ....
   (c) $6r^2 - 2r(r - 5)$ ....
   (d) $2a(a + 3) + 5a(a - 2)$ ....

(e) $6y(y + 1) - 3y(y + 2)$

....

(f) $4x(2x - 3) - 3x(x + 2)$

....

(g) $2x^2(x - 5) - x(3x^2 - 2)$

....

(h) $x(x - 1) + x(2x + 3) - 2x(3x + 1)$

....

8.4 Multiplication of algebraic expressions

MULTIPLY POLYNOMIALS BY MONOMIALS

1. (a) Calculate 3 × 38 and 3 × 62 and add the two answers.

....

....

(b) Add 38 and 62, then multiply the answer by 3.

....

(c) If you do not get the same answer for (a) and (b), you have made a mistake. Rework until you get it right.

The fact that if you work correctly, you get the same answer in questions 1(a) and 1(b), is a demonstration of the **distributive property**.

> The distributive property may be described as follows:
> $a(b + c) = ab + ac$ and
> $a(b - c) = ab - ac$,
> where a, b and c can be any numbers.

What you saw in question 1 was that
3 × 100 = 3 × 38 + 3 × 62.

This can also be expressed by writing 3(38 + 62) = 3 × 38 + 3 × 62.

2. (a) Calculate 10 × 56 ....

(b) Calculate 10 × 16 + 10 × 40 ....

3. (a) Write down any two numbers smaller than 100. Let us call them x and y. Add your two numbers, and multiply the answer by 3.

....

(b) Calculate 3 × x and 3 × y and add the two answers.

....

(c) If you do not get the same answers for (a) and (b) you have made a mistake somewhere. Correct your work.

4. Complete the table.

x	12	50	5
y	4	30	10
$5x - 5y$			
$5(x - y)$			
$5x + 5y$			
$5(x + y)$			

Performing the instructions $5(x + y)$ is not the only way in which you can find out how much $5(x + y)$ is for any given values of x and y. Instead of doing $5(x + y)$ you may do $5x + 5y$. In this case you will multiply first, and again, before you add.

5. (a) For $x = 10$ and $y = 20$, evaluate $8(x + y)$ by first adding 10 and 20, and then multiplying by 8.

   ....

   (b) Now evaluate $8(x + y)$ by doing $8x + 8y$, in other words first calculate 8×10 and 8×20.

   ....

6. In question 5 you evaluated $8(x + y)$ in two different ways for the given values of x and y. Now also evaluate $20(x - y)$ in two different ways, for $x = 5$ and $y = 3$.

   ....

   ....

7. Use the distributive property in each of the following cases to make a different expression that is equivalent to the given expression.

   (a) $a(b + c)$ (b) $a(b + c + d)$

   ....

(c) $x(x + 1)$

....

(d) $x(x^2 + x + 1)$

....

(e) $x(x^3 + x^2 + x + 1)$

....

(f) $x^2(x^2 - x + 3)$

....

What you do in this question is sometimes called "multiplication of a polynomial by a monomial". One may also say that in each case you **expand** the expression, or you write an equivalent expression in **expanded form**.

(g) $2x^2(3x^2 + 2)$

....

(h) $3x^3(2x^2 + 4x - 5)$

....

(i) $-2x^4(x^3 - 2x^2 - 4x + 5)$

....

(j) $a^2b(a^3 - a^2 + a + 1)$

....

(k) $x^2y^3(3x^2y + xy^2 - y)$

....

(l) $-2x(x^3 - y^3)$

....

(m) $2a^2b(3a^2 + 2a^2b^2 + 4b^2)$

....

(n) $2ab^2(3a^3 - 1)$

....

8. Expand the parts of each expression and simplify.
   Then evaluate the expression for $x = 5$.

(a) $5(x - 2) + 3(x + 4)$

....

....

(b) $x(x + 4) - 4\ (x + 4)$

....

....

(c) $x(x - 4) + 4(x - 4)$

....

....

(d) $x(x^2 + 3x + 9) - 3(x^2 + 3x + 9)$

....

....

(e) $x(x^2 - 3x + 9) + 3(x^2 - 3x + 9)$

....

....

(f) $x^2(x^2 - 3x + 4) - x(x^3 + 4x^2 + 2x + 3)$

....

....

9. Write in expanded form.

(a) $x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2)$

....

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(b) $x^2y(x^2 - 2xy + y^2) - xy^2(2x^2 - 3xy - y^2)$

....

(c) $ab^2c(b^2c^2 - ac) + b^2c^4(a^2 + abc^2)$

....

(d) $p^2q(pq^2 + p + q) + pq(p - q^2)$

....

SQUARES AND CUBES AND ROOTS OF MONOMIALS

1. Evaluate each of the following expressions for $x = 2$, $x = 5$ and $x = 10$.

(a) $(3x)^2$

....

(b) $9x^2$

....

(c) $(2x)^2$

....

(d) $4x^2$

....

(e) $(2x)^3$

....

(f) $8x^3$

....

(g) $(2x + 3x)^2$

....

(h) $(10x - 7x)^2$

....

2. In each case, write an equivalent monomial without brackets.

(a) $(5x)^2$

....

(b) $(5x)^3$

....

(c) $(20x)^2$

....

(d) $(10x)^3$

....

(e) $(2x + 7x)^2$

....

(f) $(20x - 13x)^3$

....

The square root of $16x^2$ is $4x$, because $(4x)^2 = 16x^2$.

3. Write down the square root of each of the following expressions.

(a) $\sqrt{(7x)^2}$

....

(b) $\sqrt{9x^2}$

....

(c) $\sqrt{(20x)^2}$

....

(d) $\sqrt{100x^2}$

....

(e) $\sqrt{(20x - 15x)^2}$ (f) $\sqrt{25x^2}$

....

(g) $\sqrt{(21x - 16x)^2}$ (h) $\sqrt{(5x)^2}$

....

The cube root of $64x^3$ is $4x$, because $(4x)^3 = 64x^3$

4. Write down the cube root of each of the following expressions.

(a) $\sqrt[3]{(7x)^3}$ (b) $\sqrt[3]{27x^3}$

....

(c) $\sqrt[3]{(20x)^3}$ (d) $\sqrt[3]{1\,000x^3}$

....

(e) $\sqrt[3]{(20x - 15x)^3}$ (f) $\sqrt[3]{125x^3}$

....

8.5 Dividing polynomials by integers and monomials

1. Complete the table.

x	20	10	5	–5	–10	–20
$(100x - 5x^2) \div 5x$						
$20 - x$						

Can you explain your observations?

....

....

2. (a) R240 prize money must be shared equally between 20 netball players. How much should each one get?

....

(b) Mpho decided to do the calculations below. Do not do Mpho's calculations, but think about this: Will Mpho get the same answer that you got for question (a)?

$(140 \div 20) + (100 \div 20)$ ....

(c) Gert decided to do the calculations below. Without doing the calculations, say whether Gert will get the same answer that you got for question (a).

$(240 \div 12) + (240 \div 8)$ ....

3. Do the necessary calculations to find out whether the following statement are true or false:
   (a) $(140 + 100) \div 20 = (140 \div 20) + (100 \div 20)$

   ....

   (b) $240 \div (12 + 8) = (240 \div 12) + (240 \div 8)$

   ....

   (c) $(300 - 60) \div 20 = (300 \div 20) - (60 \div 20)$

   ....

Division is **right-distributive** over addition and subtraction, for example,
$(2 + 3) \div 5 = (2 \div 5) + (3 \div 5)$.
The division symbol is to the right of the brackets.
But it is not left-distributive, for example,
$10 \div (2 + 4) \neq (10 \div 2) + (10 \div 4)$.

For example $(200 + 40) \div 20 = (200 \div 20) + (40 \div 20) = 10 + 2 = 12$, and
$(500 + 200 - 300) \div 50 = (500 \div 50) + (200 \div 50) - (300 \div 50)$

4. Evaluate each expression for $x = 2$ and $x = 10$
   (a) $(10x^2 + 5x) \div 5$ ....

   ....

   (b) $(10x^2 \div 5) + (5x \div 5)$ ....

   ....

   (c) $2x^2 + x$

   ....

   (d) $(10x^2 + 5x) \div 5x$ ....

   ....

   (e) $(10x^2 \div 5x) + (5x \div 5x)$ ....

   ....

   (f) $2x + 1$

   ....

The distributive property of division can be expressed like this:

$(x + y) \div z = (x \div z) + (y \div z)$
$(x - y) \div z = (x \div z) - (y \div z)$

5. (a) Do not do any calculations. Which of the following expressions do you *think* will have the same value as $(10x^2 + 20x - 15) \div 5$, for $x = 10$ as well as $x = 2$?

   $2x^2 + 20x - 15$ $\quad$ $10x^2 + 20x - 3$ $\quad$ $2x^2 + 4x - 3$ ....

   (b) Do the necessary calculations to check your answer.

   ....

6. Simplify:

(a) $(2x + 2y) \div 2$

(b) $(4x + 8y) \div 4$

(c) $(20xy + 16x) \div 4x$

(d) $(42x - 6) \div 6$

(e) $(28x^4 - 7x^3 + x^2) \div x^2$

(f) $(24x^2 + 16x) \div 8x$

(g) $(30x^2 - 24x) \div 3x$

7. Simplify:

(a) $(9x^2 + xy) \div xy$

(b) $(48a - 30ab + 16ab^2) \div 2a$

(c) $(3a^3 + a^2) \div a^2$

(d) $(13a - 17ab) \div a$

(e) $(3a^2 + 5a^3) \div a$

(f) $(39a^2b + 13ab + ab^2) \div ab$

The instruction $72 \div 6$ may also be written as $\frac{72}{6}$.

This notation, which looks just like the common fraction notation, is often used to indicate division.

Hence, instead of $(10x^2 + 20x - 15) \div 5$ we may write $\frac{10x^2 + 20x - 15}{5}$.

Since $(10x^2 + 20x - 15) \div 5$ is equivalent to $(10x^2 \div 5) + (20x \div 5) - (15 \div 5)$, $\frac{10x^2 + 20x - 15}{5}$ is equivalent to $\frac{10x^2}{5} + \frac{20x}{5} - \frac{15}{5}$.

8. Find a simpler equivalent expression for each of the following expressions (clearly, these expressions do not make sense if $x = 0$).

(a) $\frac{16x^2 - 12x}{4x}$

(b) $\frac{16x^3 - 12x}{4x}$

(c) $\frac{16x^3 - 12x^2}{4x}$

(d) $\frac{16x^3 - 12x^2}{4x^2}$

(e) $\frac{16x^3 - 12x^2}{2x}$

(f) $\frac{16x^3 - 12x}{8x}$

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9. In each case check whether the statement is true for $x = 10$; $x = 100$; $x = 5$; $x = 1$ and $x = -2$.

(a) $\frac{x^2}{x} = x$ ....
(b) $\frac{x^3}{x} = x^2$ ....
(c) $\frac{x^3}{x^2} = x$ ....

(d) $\frac{5x^3}{x} = 5x^2$ ....
(e) $\frac{5x^3}{x} = 5^3$ ....

(f) $\frac{5x}{x^2} = \frac{5}{x}$ ....

10. Explain why the equations below are true:

(a) $\frac{100x - 5x^2}{5x} = 20 - x$ for all values of x except $x = 0$

....

(b) $\frac{15x^2 - 10x}{5x}$ is equivalent to $3x - 2$, excluding $x = 0$.

....

11. Complete the table:

x	1,5	2,8	–3,1	0,72
$\frac{3x+12}{3}$				
$\frac{18x^2+6}{6}$				
$\frac{5x^2+7x}{x}$				

(Hint: Simplify the expressions first to save yourself some work!)

12. Simplify each expression to the equivalent form requiring the fewest operations.

(a) $\frac{3a + a^2}{a}$ ....
(b) $\frac{x^3 + 2x^2 - x}{x}$ ....
(c) $\frac{2a + 12ab}{2a}$ ....

(d) $\frac{12x^2 + 10x}{2x}$ ....
(e) $\frac{21ab - 14a^2}{7a}$ ....
(f) $\frac{15a^2b + 30ab^2}{5ab}$ ....

(g) $\frac{7x^3 + 21x^2}{7x^2}$ ....
(h) $\frac{3x^2 + 9x}{3x}$ ....

13. Solve the equations.

(a) $\frac{3x^2 + 15x}{3x} = 20$

....

....

(b) $\frac{30x - 18x^2}{6x} = 2$

....

....

14. Complete the table.

	x	1,1	1,2	1,3	1,4	1,5
(a)	$\frac{x^3 + 2x^2 - x}{x}$					
(b)	$\frac{7x^3 + 21x^2}{7x^2}$					
(c)	$\frac{50x^2 + 5x}{5x}$					

15. Simplify the following expressions.

(a) $\frac{3x(5x + 4) + 6x(5x + 3)}{5x}$

(b) $\frac{14x^2 - 28x}{7x} + \frac{24x - 18x^2}{3x}$

....

....

....

8.6 Products and squares of binomials

How can we obtain the expanded form of $(x + 2)(x + 3)$?

In order to expand $(x + 2)(x + 3)$, you can first keep $(x + 2)$ it is, and apply the distributive property:

$(\mathbf{x + 2})(x + 3)$
$= (\mathbf{x + 2})x + (\mathbf{x + 2})3$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

1. Describe how can you check whether $(x + 2)(x + 3)$ is actually equivalent to $x^2 + 5x + 6$.

....

....

To expand $(x - y)(x + 3y)$ it can be written as $(x - y)x + (x - y)3y$ and the two parts can then be expanded.

$(x - y)(x + 3y)$
$= (x - y)x + (x - y)3y$
$= x^2 - xy + 3xy - 3y^2$
$= x^2 + 2xy - 3y^2$

2. Do some calculations to check whether $(x - y)(x + 3y)$ and $x^2 + 2xy - 3y^2$ are equivalent. Write the results of your calculations in the table below.

x					
y					

3. Expand each of these expressions.

(a) $(x + 3)(x + 4)$

(b) $(x + 3)(4 - x)$

(c) $(x + 3)(x - 5)$

(d) $(2x^2 + 1)(3x - 4)$

(e) $(x + y)(x + 2y)$

(f) $(a - b)(2a + 3b)$

(g) $(k^2 + m)(k^2 + 2m)$

(h) $(2x + 3)(2x - 3)$

(i) $(5x + 2)(5x - 2)$

(j) $(ax - by)(ax + by)$

4. Expand each of these expressions.

(a) $(a + b)(a + b)$

(b) $(a - b)(a - b)$

(c) $(x + y)(x + y)$

(d) $(x - y)(x - y)$

(e) $(2a + 3b)(2a + 3b)$

(f) $(2a - 3b)(2a - 3b)$

(g) $(5x + 2y)(5x + 2y)$

(h) $(5x - 2y)(5x - 2y)$

(i) $(ax + b)(ax + b)$

(j) $(ax - b)(ax - b)$

5. Can you guess the answer to each of the following questions without working it out as you did in question 3? Try them out and then check your answers.
Expand these expressions:

(a) $(m + n)(m + n)$

(b) $(m - n)(m - n)$

(c) $(3x + 2y)(3x + 2y)$

(d) $(3x - 2y)(3x - 2y)$

All the expressions in questions 4 and 5 are **squares of binomials**, for example $(ax + b)^2$ and $(ax - b)^2$

6. Expand:

(a) $(ax + b)^2$

(b) $(ax - b)^2$

(c) $(2s + 5)^2$

(d) $(2s - 5)^2$

(e) $(ax + by)^2$

(f) $(ax - by)^2$

(g) $(2s + 5r)^2$

(h) $(2s - 5r)^2$

7. Expand and simplify.

(a) $(4x + 3)(6x + 4) + (3x + 2)(8x + 5)$

(b) $(4x + 3)(6x + 4) - (3x + 2)(8x + 5)$

8.7 Substitution into algebraic expressions

1. In question 2 you have to find the values of different expressions, for some given values of x. Look carefully at the different expressions in the table. Do you think some of them may be equivalent?

   Simplify the longer expression to check whether you end up with the shorter expression.

2. Complete the table.

	x	13	–13	2,5	10
(a)	$(2x + 3)(3x - 5)$				
(b)	$10x^2 + 5x - 7 + 3x^2 - 4x - 3$				
(c)	$3(10x^2 - 5x + 2) - 5x(6x - 4)$				
(d)	$13x^2 + x - 10$				
(e)	$6x^2 - x - 15$				
(f)	$5x + 6$				

3. Complete this table.

	x	1	2	3	4
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

4. Describe any patterns that you observe in your answers for question 3.

   ....

   ....

   ....

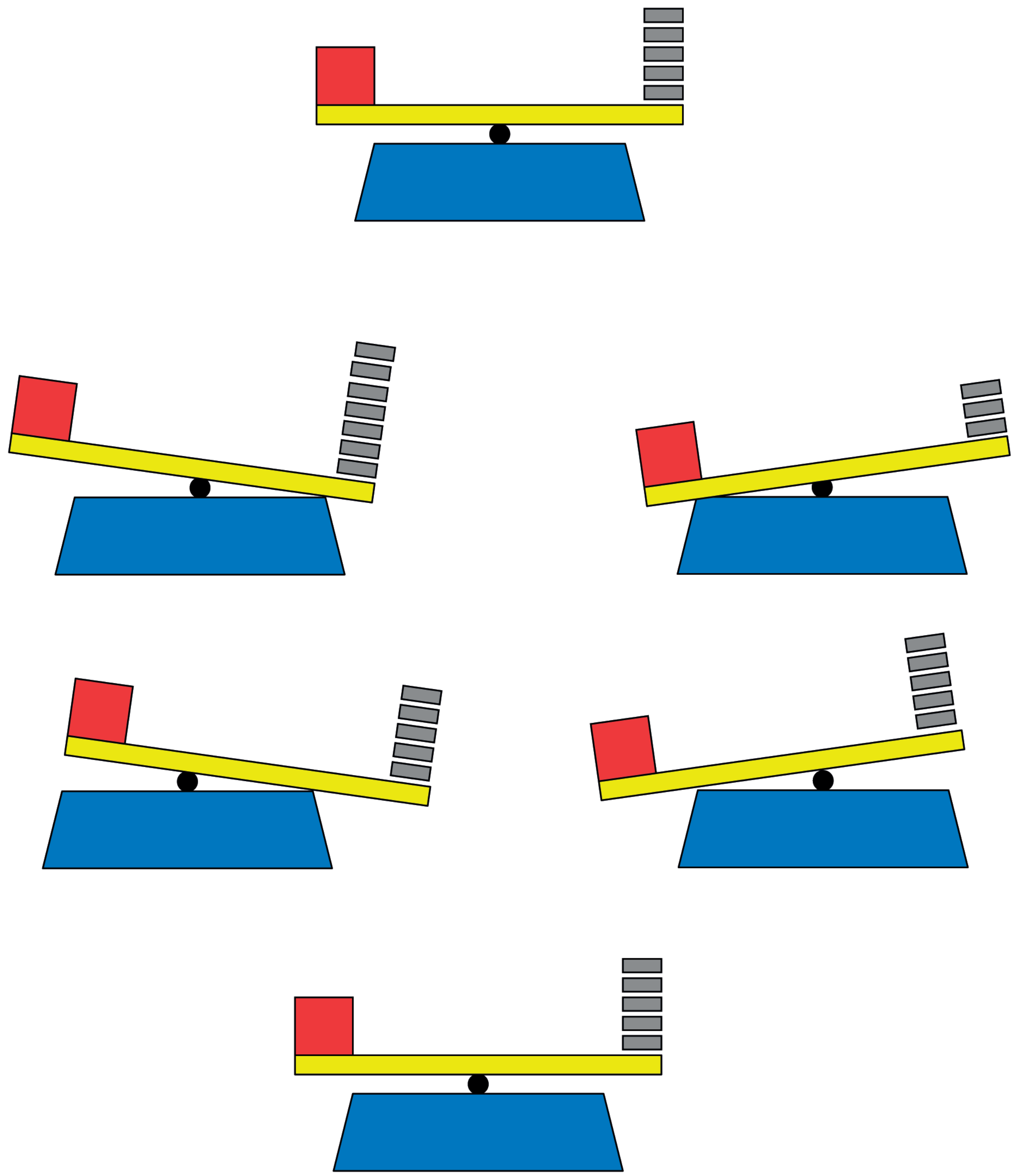
5. Complete this table.

	x	1,5	2,5	3,5	4,5
(a)	$(2x + 3)(5x - 3) + (10x + 9)(1 - x)$				
(b)	$\frac{9x^2 + 30x}{3x}$				
(c)	$3x(10x - 5) - 5x(6x - 4)$				
(d)	$5x(4x + 3) - 2x(7 + 13x) + 2x(3x + 2)$				

CHAPTER 9
Equations

In this chapter, you will find numbers that make statements true. This is called solution of equations. You will solve equations in two different ways, by inspection and by 'reversing' them.

You will find that two equations can have the same solution. Such equations are called equivalent equations. You will also discover that not all statements are algebraic equations. Some statements are algebraic identities and others are in fact algebraic impossibilities. You will learn what the difference is between these three types of statements.



9 Equations

9.1 Solving equations by inspection

1. Six equations are listed below the table. Use the table to find out for which of the given values of x it will be true that the left-hand side of the equation is equal to the right-hand side.

"Searching" for the solution of an equation by using tables is called **solution by inspection**.

x	–3	–2	–1	0	1	2	3	4
$2x + 3$	–3	–1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	–11	–8	–5	–2	1	4	7	10
$10x - 7$	–37	–27	–17	–7	3	13	23	33
$5x + 3$	–12	–7	–2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	–2

(a) $2x + 3 = 5x + 3$

....

(b) $5x + 3 = 9 - x$

....

(c) $2x + 3 = x + 4$

....

(d) $10x - 7 = 5x + 3$

....

(e) $3x - 2 = x + 4$

....

(f) $9 - x = 2x + 3$

....

Two equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

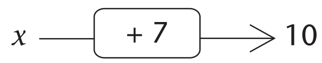
2. Which of the equations in question 1 have the same solutions? Explain.

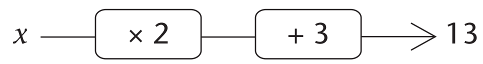
....

....

9.2 Solving equations using additive and multiplicative inverses

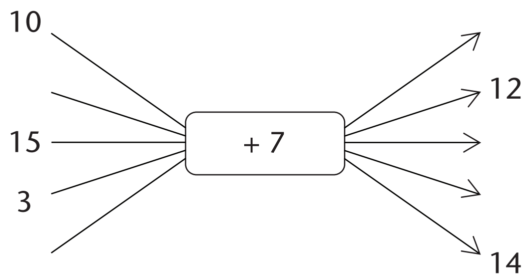
1. In each case find the value of x:

(a) 

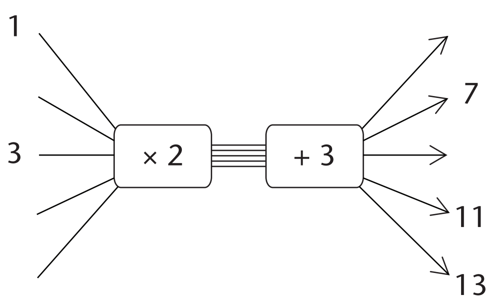
(b) 

....

2. Complete the flow diagrams. You have to fill in all the missing numbers.

(a) 

To find the second input number you may say to yourself, "After I added 7, I had 12. *What did I have before I added 7?*"

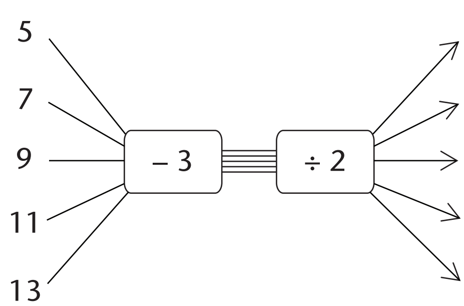
(b) 

To find the input number that corresponds to 13, you may ask yourself, "*What did I have before I added 3?*" and then, "*What did I have before I multiplied by 2?*"

3. Use your answers for question 2 to check your answers for question 1.
4. Describe the instructions in flow diagram 2(b) in words, and also with a symbolic expression.

....

5. Complete the flow diagram.



This flow diagram is called the **inverse** of the flow diagram in question 2(b).

6. Compare the input numbers and the output numbers of the flow diagrams in question 2(b) and question 5. What do you notice?

....

....

7. (a) Add 5 to any number and then subtract 5 from your answer. What do you get?

....

(b) Multiply any number by 10 and then divide the answer by 10. What do you get?

....

If you add a number and then subtract the same number, you are back where you started. This is why addition and subtraction are called **inverse operations**.

If you multiply by a number and then divide by the same number, you are back where you started. This is why multiplication and division are called **inverse operations**.

The expression $5x - 3$ says *"multiply by 5 then subtract 3"*. This instruction can also be given with a flow diagram: → [× 5] → [− 3] →

The equation $5x - 3 = 47$ can also be written as a flow diagram:

→ [× 5] → [− 3] → 47

8. Solve the equations below. You may do this by using the inverse operations. You may write a flow diagram to help you to see the operations.

(a) $2x + 5 = 23$

....

....

(b) $3x - 5 = 16$

....

....

(c) $5x - 60 = -5$

....

....

(d) $\frac{1}{3}x + 11 = 19$

....

....

(e) $10(x + 3) = 88$

....

....

....

(f) $2(x - 13) = 14$

....

....

....

9.3 Setting up equations

CONSTRUCTING EQUATIONS

You can easily make an equation that has 5 as the solution. Here is an example:

Start by writing the solution	$x = 5$
Add 3 to both sides	$x + 3 = 8$
Multiply both sides by 5	$5x + 15 = 40$

1. What is the solution of the equation $5x + 15 = 40$? ....
2. Make your own equation with the solution $x = 3$.

   ....

   ....

   ....

3. Bongile worked like this to make the equation $2(x + 8) = 30$, but he rubbed out part of his work:

Start by writing the solution	$x =$
Add 8 to both sides	$= 15$
Multiply both sides by 2	$2(x + 8) = 30$

   Complete Bongile's writing to solve the equation $2(x + 8) = 30$.

4. This is how Bongile made a more difficult equation:

Start by writing the solution	$x =$
Multiply by 3 on both sides	$3x =$
Subtract 9 from both sides	$3x - 9 = 6$
Add $2x$ to both sides	$5x - 9 = 2x + 6$

   (a) What was on the right-hand side before Bongile subtracted 9? ....

   (b) What is the solution of $5x - 9 = 2x + 6$? ....

5. Bongile started with a solution and he ended up with an equation. Fill in the steps that Bongile took to make the equation, and solve the equation:

   $$x =$$
   $$8x =$$
   $$8x + 3 =$$
   $$3x + 3 = 35 - 5x$$

SOLVING EQUATIONS

To make an equation, you can apply the same operation on both sides (↓)

To solve an equation, you can apply the inverse operation on both sides (↑)

To make (↓)	Equation	To solve (↑)
	$x = 4$	
Multiply by 8	$8x = 32$	Divide by 8
Add 3	$8x + 3 = 35$	Subtract 3
Subtract $5x$	$3x + 3 = 35 - 5x$	Add $5x$

Use any appropriate method to solve the equations below.

1. (a) $5x + 3 = 24 - 2x$ (b) $2x + 4 = -9$

   (c) $3 - x = x - 3$ (d) $6(2x + 1) = 0$

2. (a) $4(1 - 2x) = 12 - 7x$ (b) $8(1 - 3x) = 5(4x + 6)$

   (c) $7x - 10 = 3x + 7$ (d) $1{,}6x + 7 = 3{,}5x + 3{,}2$

NUMBER PATTERNS AND EQUATIONS

1. (a) Which of the following rules will produce the number pattern given in the second row of the table below?

   A. Term value = $8n$ where n is the term number
   B. Term value = $6n - 1$ where n is the term number
   C. Term value = $6n + 2$ where n is the term number
   D. Term value = $10n - 2$ where n is the term number
   E. Term value = $5n + 3$ where n is the term number

   ....

Term number	1	2	3	4	5	6	7	8	9
Term value	8	13	18	23	28	33	38	43	48

   (b) The sixth term of the sequence has the value 33. Which term will have the value 143? You may set up and solve an equation to find out.

   ....

   (c) Apply rule E to your answer, to check whether your answer is correct.

   ....

2. (a) Write the rule that will produce the number pattern in the second row of this table. You may have to experiment to find out what the rule is.

Term number	1	2	3	4	5	6	7	8	9
Term value	5	8	11	14	17	20	23	26	29

   ....

   (b) Which term will have the value 221?

   ....

3. The rule for number pattern A is $4n + 11$, and the rule for pattern B is $7n - 34$.

   (a) Complete the table below for the two patterns.

Term number	1	2	3	4	5	6	7	8	9
Pattern A									
Pattern B									

   (b) For which value of n are the terms of the two patterns equal?

   ....

9.4 Equation and situations

1. Consider this situation:

   To rent a room in a certain building, you have to pay a deposit of R400 and then R80 per day.

   (a) How much money do you need to rent the room for 10 days?

   ....

   (b) How much money do you need to rent the room for 15 days?

   ....

2. Which of the following best describes the method that you used to do question 1(a) and (b)?

   A. Total cost = R400 + R80
   B. Total cost = 400(number of days + 80)
   C. Total cost = 80 × number of days + 400
   D. Total cost = (80 + 400) × number of days

3. For how many days can you rent the room described in question 1, if you have R2 800 to pay?

   ....

   ....

   ....

If you want to know for how many days you can rent the room if you have R720, you can set up an equation and solve it:

You know the total cost is R720 and you know that you can work out the total cost like this:

Total cost = $80x + 400$, where x is the number of days. So, $80x + 400 = 720$ and $x = 4$ days.

In each of the following cases, find the unknown number by setting up an equation and solving it.

4. To rent a certain room, you have to pay a deposit of R300 and then R120 per day.

   (a) For how many days can you rent the room if you can pay a total of R1 740? (If you experience trouble in setting up the equation, it may help you to decide first how you will work out what it will cost to rent the room for 6 days.)

   ....

   ....

(b) What will it cost to rent the room for 10 days, 11 days and 12 days?

....

....

....

(c) For how many days can you rent the room if you have R3 300 available?

....

....

....

(d) For how many days can you rent the room if you have R3 000 available?

....

....

....

5. Ben and Thabo decide to do some calculations with a certain number. Ben multiplies the number by 5 and adds 12. Thabo gets the same answer as Ben when he multiplies the number by 9 and subtracts 16. What is the number they worked with?

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....

6. The cost of renting a certain car for a period of x days can be calculated with the following formula:

   Rental cost in rand = $260x + 310$

   What information about renting this car will you get, if you solve the equation

   $260x + 310 = 2\,910$?

....

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7. Sarah paid a deposit of R320 for a stall at a market, and she also pays R70 per day rental for the stall. She sells fruit and vegetables at the stall, and finds that she makes about R150 profit each day. After how many days will she have earned as much as she has paid for the stall, in total?

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9.5 Solving equations by using the laws of exponents

You may need to look back at Chapter 5 to remember the laws of exponents.

One kind of exponential equation that you deal with in Grade 9 has one or more terms with a base that is raised to a power containing a variable.

Example: $2^x = 16$

When we need to find the unknown value, we are asking the question: **"To what power must the base be raised for the statement to be true?"**

Example:	$2^x = 16$	Make sure that the terms with x are on their own on one side.
	$2^x = 2^4$	Write the known term in the same base as the term with the exponent.
	$x = 4$	Equate the exponents.

In the example above, we can equate the exponents because the two numbers are equal only when they are raised to the same power.

1. Solve for x:

(a) $5^{x-1} = 125$

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(b) $2^{x+3} = 8$

....

....

....

....

(c) $10^x = 10\,000$

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....

....

(d) $4^{x+2} = 64$

....

....

....

....

(e) $7^{x+1} = 1$

....

....

....

(f) $x^0 = 1$

....

....

....

Example: Solve for x: $3^x = \frac{1}{27}$

$3^x = 3^{-3}$ (Rewrite $\frac{1}{27}$ as a number to base 3)

$x = -3$ (Equate the exponents.)

2. Solve for x.

(a) $7^x = \frac{1}{49}$

(b) $10^x = 0{,}001$

(c) $6^x = \frac{1}{216}$

(d) $10^{x-1} = 0{,}001$

(e) $4^{-x} = \frac{1}{16}$

(f) $7^x = 7^{-3}$

In another kind of equation involving exponents, the variable is in the base.

> When we need to find the unknown value, we are asking the question: **"Which number must be raised to the given power for the statement to be true?"**

For these equations, you should remember what you know about the powers of numbers such as 2, 3, 4, 5 and 10.

SOLVING EQUATIONS WITH A VARIABLE IN THE BASE

1. Complete the table below and answer the questions that follow:

	x	2	3	4	5
(a)	x^3	$2^3 = 8$			
(b)	x^5	$2^5 = 32$			
(c)	x^4	$2^4 = 16$			

For what value of x is:

(a) $x^3 = 64$ ....

(b) $x^5 = 32$ ....

(c) $x^4 = 256$ ....

(d) $x^3 = 8$ ....

(e) $x^4 = 16$ ....

(f) $x^5 = 3\ 125$ ....

2. Solve for x and give a reason:

(a) $x^3 = 216$ ....

(b) $x^2 = 324$ ....

(c) $x^4 = 10\ 000$ ....

(d) $8^x = 512$ ....

(e) $18^x = 324$ ....

(f) $6^x = 216$ ....

WORKSHEET

1. Ahmed multiplied a number by 5, added 3 to the answer, and then subtracted the number he started with. The answer was 11. What number did he start with?

....

....

....

2. Use any appropriate method to solve the equations.

(a) $3(x - 2) = 4(x + 1)$

....

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....

....

(b) $5(x + 2) = -3(2 - x)$

....

....

....

....

(c) $1{,}5x = 0{,}7x - 24$

....

....

....

(d) $5(x + 3) = 5x + 12$

....

....

....

(e) $2{,}5x = 0{,}5(x + 10)$

....

....

....

(f) $7(x - 2) = 7(2 - x)$

....

....

....

(g) $\frac{1}{2}(2x - 3) = 5$

....

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....

(h) $2x - 3(3 + x) = 5x + 9$

....

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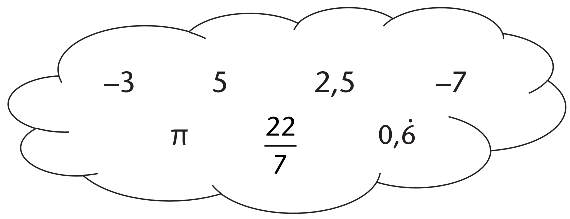
TERM 1
Revision and assessment

Revision

Remember to show all the steps in your working.

WHOLE NUMBERS

1. Write all the numbers from the cloud in the table below, and place a tick in all the column(s) of the type of numbers they are. The first number has been completed for you:



Number value	Number system				
	Real numbers	Natural numbers	Integers	Rational numbers	Irrational numbers
–3	✓		✓	✓	

2. The Ndlovu family is travelling to the Kruger National Park on holiday. Here is a summary of their journey:

Time	Odometer reading (km)	Description
06:12	123 564	Leave home
08:32	123 785	Stop for breakfast and petrol
09:18	123 785	Leave petrol station
11:34	124 011	Stop for toilet break
11:51	124 011	Leave petrol station
13:32	124 175	Reach Kruger gate

(a) Calculate the length of time the journey took, in hours. Give your answer as mixed number.

....

....

(b) Calculate the average speed of the journey, correct to one decimal place.

....

....

3. A car travelling at an average speed of 110 km/h takes $2\frac{1}{4}$ hours to complete a journey. If the return journey needs to be completed in 2 hours, calculate the average speed that must be maintained.

....

....

4. If four tins of bully beef cost R75,80, how much money would seven similar tins cost?

....

....

5. A farmer has enough chicken feed to feed 300 hens for 20 days. If he buys 100 more hens, how long would the same amount of chicken feed last before it runs out?

....

....

6. How long will it take R5 000 invested at 7,2% simple interest p.a. to grow to R5 900?

....

....

7. Chardonnay wishes to buy a new TV set costing R7 499. She does not have enough money and so needs to buy it on hire purchase. The store requires a 10% deposit and then equal monthly payments of Rx for 2 years. If the simple interest charged on the account is 15%, calculate the value of x.

....

....

....

....

8. How much interest will Tebogo get on R12 500 deposited for 21 months in a bank account that provides 5,3% compound interest per annum?

....

....

....

INTEGERS

All the questions in this section should be answered without using a calculator.

1. Write a number in each box to make the calculations correct:

   (a) $\square + \square = -34$

   (b) $\square - \square = -34$

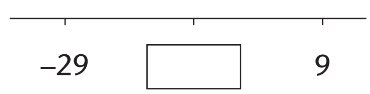
2. These questions show sequences of numbers. Fill the correct values in the boxes:

   (a) 18; 10; 2; $\square$

   (b) 2; –10; 50; $\square$

   (c) –6 386; –6 392; –6 398; $\square$

3. This question shows a number line in which the missing number is halfway between the other two numbers. Fill the correct value into the box:



4. Calculate the following:

   (a) $28 - (-15)$

   (b) $(-5)(12)(-7)$

   ....

   ....

   (c) $5 + 5 \times -6$

   (d) $\dfrac{(\sqrt{81})(-2)^3}{-(-3)^2}$

   ....

   ....

   ....

   ....

   (e) $\dfrac{(-3)^2\sqrt[3]{216}}{(-9)(-3)}$

   ....

   ....

5. Augustus ruled the Roman Empire from 27 BC to AD 14. For how many years did he rule?

   ....

   ....

   ....

FRACTIONS

All the questions in this section should be answered without using a calculator.

1. Simplify the following:

(a) $\sqrt{\frac{36}{81}x^8}$

(b) $\frac{5}{2}x^2 - \frac{5}{4}x^2$

....

....

(c) $(\frac{3}{4}xy^3)(\frac{4}{9}y)$

....

....

2. Simplify the following:

(a) $\frac{4x^{10}}{8x^5}$

(b) $\frac{5}{x} - \frac{1}{x}$

....

....

(c) $\frac{5x}{6y^2} \times \frac{3y}{15x}$

(d) $\frac{x+2}{4z^2} \div \frac{4(x+2)}{2z^3}$

....

....

THE DECIMAL NOTATION FOR FRACTIONS

All the questions in this section should be answered without using a calculator.

1. Calculate the following:

(a) 27,49 – 6,99

(b) 0,03 × 1,4

(c) 1,44 ÷ 0,012

....

....

2. Simplify the following:

(a) $\sqrt{0{,}04x^{16}}$

(b) $3{,}5x^2 - 4{,}6x^2$

(c) $(1{,}2x^2y^3)(5yx^2)$

....

....

3. Simplify the following:

(a) $\dfrac{0,2x^{15}}{0,01x^{5}}$

(b) $\dfrac{0,45}{x} - \dfrac{1,35}{x}$

....

....

(c) $\dfrac{0,5x^{3}}{4,5y^{2}} \times \dfrac{3y}{2,5x}$

(d) $\dfrac{2,5x^{3}}{2y^{2}} \div \dfrac{0,5x}{0,03y^{6}}$

....

....

EXPONENTS

All the questions in this section should be answered without using a calculator, unless otherwise specified in the question.

1. Write the following numbers in scientific notation:

(a) 2 500 001

(b) 0,000 304 5

....

....

2. Write the following number in "normal" notation: $9,45 \times 10^{-5}$.

....

3. Which of the following numbers is bigger: $4,7 \times 10^{-9}$ or $5,12 \times 10^{-10}$?

....

4. Calculate the following, giving your answer in scientific notation:

(a) $(5,9 \times 10^{6}) - (4,7 \times 10^{6})$

(b) $(5,9 \times 10^{6}) + (4,7 \times 10^{5})$

....

....

(c) $(7,2 \times 10^{-4}) \times (2 \times 10^{2})$

....

....

5. Calculate the following, giving your answer as an ordinary decimal number. A calculator may be used:
   (a) $(6,3 \times 10^{-4}) - (1,9 \times 10^{-3})$
   (b) $(5,8 \times 10^{-7}) \div (8 \times 10^{-11})$

6. Simplify the following, leaving all answers with positive exponents:
   (a) 3^{-2}
   (b) $2^7 \times 6^{-3} \times 3^2$
   (c) $\frac{2y^{-3}}{y^3}$
   (d) $(2x^6)^{-3}$
   (e) $(2x^7)(2,5x^{-8})$
   (f) $(-3a^2bc)^2(-5ac^{-2})$
   (g) $\frac{(2d^2e)^2}{(4d^{-3}e^2)^{-1}}$

7. Solve the following equations:
   (a) $3 \times 3^x = 81$
   (b) $2^{x+1} = 0,125$
   (c) $4^x + 10 = 74$

PATTERNS

1. Create a sequence that fits this description: the first term is negative, and each successive term is obtained by squaring the previous term and then subtracting 10. Write down the first four terms of your created sequence.

2. For each of the following sequences, (i) write in words the rule that describes the relationship between the terms in the sequence, and (ii) use the rule to extend the sequence by three more terms:

   (a) –5; –2; 10; –20; ...

   ....

   ....

   ....

   (b) –4,5; –6,25; –8; ...

   ....

   ....

   ....

3. In this question you are given the rule by which each term of the sequence can be found. In all cases, n is the position of the term. Determine the first three terms of each of the sequences:

   (a) $3 - 5n$

   ....

   ....

   (b) $2n^2 - 3n + 1$

   ....

   ....

4. (a) Write down the rule by which each term of the sequence can be found (in a similar format to those given in question 3, where n is the position of the term): –15; –12; –9; ...

   ....

   ....

   (b) Use this rule to find the value of the 150th term of the sequence.

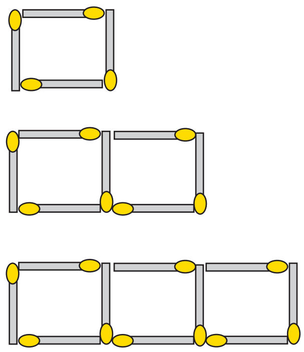
   ....

   ....

5. Determine the pattern and then write the missing values in the table below:

Position in sequence	1	2	3	4	5		10		
Value of the term	2	5	10	17					226

6. The picture below shows a pattern created by matchsticks.



(a) Draw your own series of matchstick patterns in which there is a common difference between each pattern. It must be different to all the matchstick patterns shown in Chapter 6 and this chapter, and should contain the first three matchstick patterns in the series.

(b) Write in words the rule that describes the number of matchsticks needed for each new pattern.

....

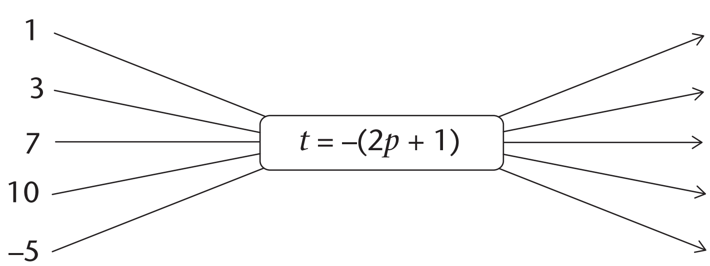
....

(c) Use the rule to determine the missing values in the table below, and fill them in:

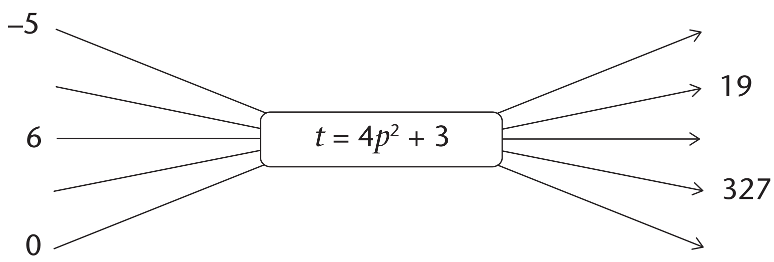
Number of the pattern	4	5	6	7		50
Number of matches needed						

FUNCTIONS AND RELATIONSHIPS

1. (a) Use the given formula to calculate the values of t, given the values of p:



(b) Use the given formula to calculate the missing input values, p, and output values, t.



2. Consider the values in the table below:

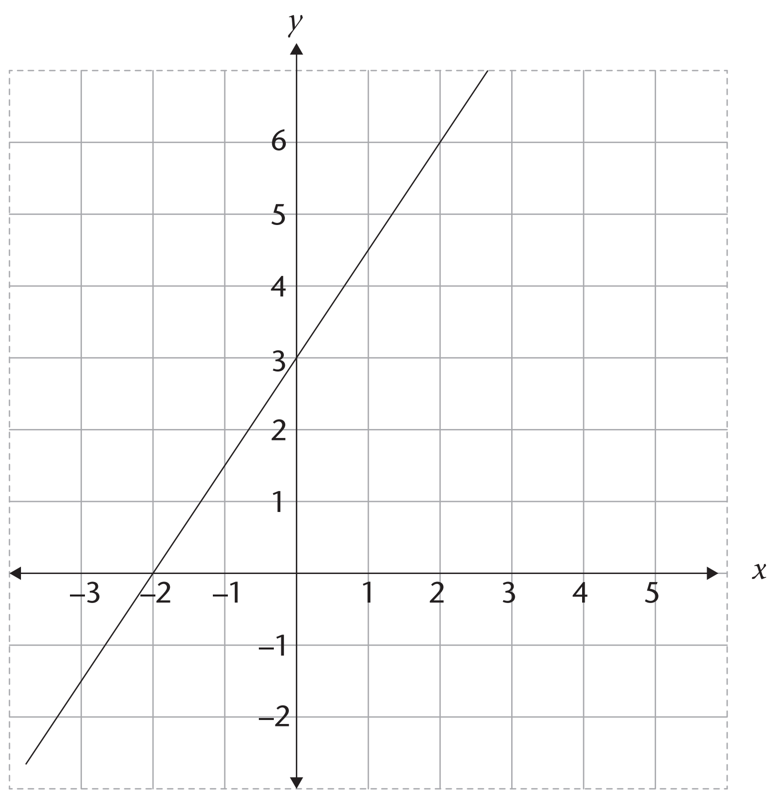
x	–2	–1	0	1		4		12		
y	–4	–1	2	5						65

(a) Write, as an algebraic formula, the rule for finding the y-values in the table. The formula is in the form $y = ax + b$, where a and b are integers.

....

(b) Use the rule to determine the missing values in the table, and fill them in.

3. Consider the graph shown below:



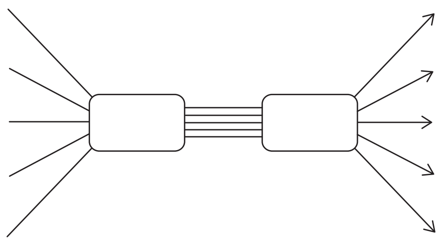
(a) Complete the following table by reading off the coordinates of points on the graph:

x	-3	-2	-1	0	1	2
y						

(b) Write down an algebraic formula for the graph, in the form $y = \ldots$

....

(c) Complete the flow diagram below to represent the relationship shown on the graph:



ALGEBRAIC EXPRESSIONS

1. Simplify as far as possible:

(a) $(2x^2 - 4x^2)^3$

....

....

....

(b) $-2x^2(5x^3 - 3x^2 + 2x - 5)$

....

....

....

(c) $(4b^2 - 7b^2)(5b^{-2} + 3b^{-1} - 7)$

....

....

....

(d) $\dfrac{18x^2 - 12x + 2}{6x}$

....

....

....

(e) $(2x + 5)(3x - 1)$

....

....

(f) $(4a - 3)^2$

....

....

(g) $\dfrac{6x^3 - 2(3x)(4x) + x}{4x^2}$

....

....

....

2. Simplify as far as possible:

(a) $4(a - 2b) - 5(3b + a)$

....

....

(b) $5 + 2(x^2 + 5x + 3)$

....

....

(c) $3x(2x^2 - 3x + 4) - 3(5 - 2x)$

....

....

(d) $(a + 3b - 2c) - (4a + b - c) - (2b - c + 3a)$

....

....

(e) $4(3x^2 + x - 2) - (x + 3)^2$

....

....

EQUATIONS

1. Solve the following equations:

   (a) $4 - 3x = -2$

   ....

   ....

   ....

   (b) $4(2x - 1) = -8$

   ....

   ....

   ....

   (c) $2x + 1 = 3(2x - 1)$

   ....

   ....

   ....

   (d) $(x + 2)(x - 4) = x^2 + 5x - 1$

   ....

   ....

   ....

2. Thomas is z years old and Tshilidzi is twice as old as Thomas. The sum of their ages is 42.

   (a) Write this information in an equation using the variable z.

   ....

   (b) Solve the equation to find Tshilidzi's age.

   ....

3. The base of a triangle is $(1{,}5x + 6)$ cm and the height is 4 cm. The area of the triangle is 24 cm^2.

   (a) Write this information in an equation in x.

   ....

   (b) Solve the equation to determine the value of x.

   ....

   ....

   ....

   (c) What is the length of the base of the triangle?

   ....

   ....

4. Solve for x:

   (a) $3^x = 9$

   ....

   ....

   ....

   (b) $2^{x+1} = 16$

   ....

   ....

   ....

Assessment

In this section, the numbers in brackets at the end of a question indicate the number of marks the question is worth. Use this information to help you determine how much working is needed. The total number of marks allocated to the assessment is 75.

1. Gareth completed the following number classification:

Number value	Number system				
	Real numbers	Natural numbers	Integers	Rational numbers	Irrational numbers
−1,5	✓		✓	✓	
$\sqrt{2}$	✓			✓	

(a) Gareth has made some errors. Complete the following table by putting the ticks in the correct boxes: (2)

Number value	Number system				
	Real numbers	Natural numbers	Integers	Rational numbers	Irrational numbers
−1,5					
$\sqrt{2}$					

(b) Explain why you have made the changes you have. (2)

....

....

2. Pheto invested R1 500 for 2 years in a bank account. At the end of this period, the initial investment had grown to R1 717,50. What simple interest rate did the bank give him? (Assume that the rate remained unchanged for the entire period.) Give your answer as a percentage. (3)

....

....

....

....

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3. A Benthian changed 2 500 Bendollars to Darsek when he visited the Klingon Empire, and received 2 000 Darsek after the 3% commission had been charged. Determine the Bendollars : Dasek exchange rate and then copy and complete the following sentence: "1 Klingon Darsek = ___ Bendollars". The missing value should be written correct to three decimal places. (3)

....

....

4. What is the difference in height between the highest point on the Earth's surface (Mt Everest: 8 848 m above sea level) and the deepest point of the sea (the bottom of the Marianas Trench, 10 994 m below sea level)? (1)

....

....

5. Write down two numbers that subtract to give an answer of 21. One of the numbers must be positive and the other negative. (2)

....

6. (a) What is the value of $(-1)^{1\,000\,001}$? (1)

....

(b) Explain how you can know the answer in (a) without needing a calculator. (1)

....

....

7. Simplify the following, without using a calculator. Show all steps of your working:

(a) $\frac{5}{2}x - \frac{11}{4}x + 1{,}125x$ (2)

....

....

(b) $\sqrt[3]{\frac{0{,}027x^7}{216x}}$ (4)

....

....

(c) $\frac{0{,}4x}{10} \times \frac{20x}{0{,}03} \div \frac{8x^2}{5}$ (4)

....

....

(d) $\frac{x}{4} + [8x(x + 1) \times \frac{0,5}{x+1}]$ (5)

....

....

8. The diameter of a carbon atom is 0,000000000154 metres. Write this in scientific notation. (2)

....

9. Simplify the following, leaving all answers with positive exponents:

(a) $3^{-9} \times 3^4$

(b) $\frac{(3d^3e^2)^3}{(2d^{-4}e)^{-1}}$ (5)

....

10. Solve for x: $9^{2x-3} = 3^x$ (3)

....

....

....

....

....

11. Consider the following sequence: 6 000; –1 500; 375; ...

(a) Extend the sequence by two more terms. (2)

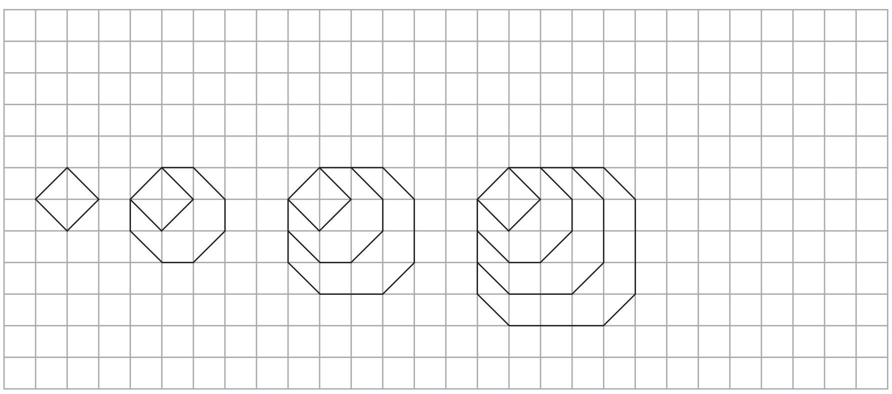
....

(b) Is this the correct rule for the sequence (where n is the position of the term in the sequence): $6\ 000(0,25)^{n-1}$? Explain your answer. (2)

....

....

12. The following figure shows a pattern created by matchsticks.



(a) Draw the 5th diagram in the pattern alongside the picture above. (2)

(b) The first two terms in the sequence created by the number of matchsticks in each pattern is 4; 11. Write down the next three terms in the sequence. (2)

....

....

(c) Write in words the rule that describes the relationship between the terms in the sequence. (2)

....

....

13. Consider the values in the table below:

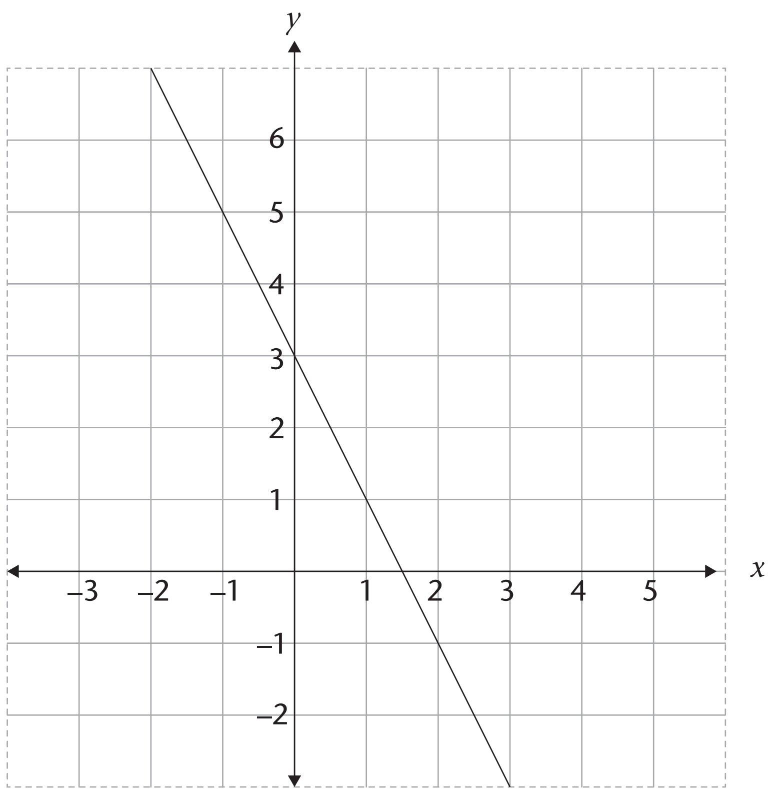
x	–2	–1	0	1		5		16		
y	–10	–3	–2	–1						7 998

(a) Write the rule for finding the y-values in the table as an algebraic formula. (Hint: Look at the cubes of the numbers.) (2)

....

(b) Use the rule to determine the missing values in the table, and fill them in. (3)

14. Consider the following graph:



(a) Complete the following table by reading off the coordinates of points on the graph: (2)

x	-2	-1	0	1	2	3
y						

(b) Write down an algebraic formula for the graph in the form $y = ...$ (2)

....

....

15. Simplify:

(a) $\frac{15 + x - 5x^2}{5x^2}$ (3)

....

....

(b) $(3x + 1)(3x - 1)$ (2)

....

....

(c) $4 - 3(2x + 3)^2$ (3)

....

....

16. Solve the following equations:

(a) $x^2 + 5x - 1 - x^2 - x + 3 = 3(x - 4)$ (4)

....

....

....

(b) $2(2x + 3) = (3x - 1)(-2)$ (4)

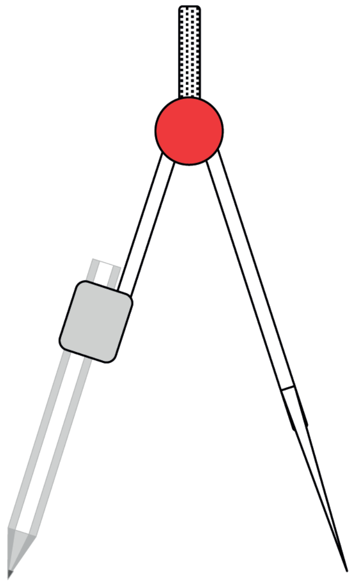
....

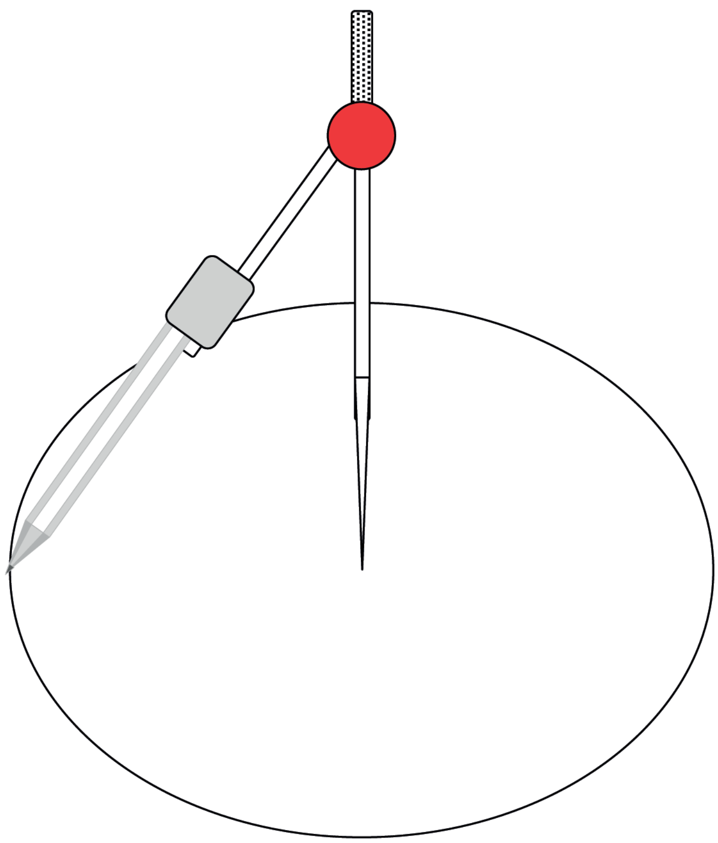
....

....

Chapter 10
Construction of geometric figures

This chapter revises the construction of geometric figures using only a compass and a ruler. You will revise and further explore constructions such as perpendicular lines, angle bisectors and special angles, because these constructions can help you to understand other constructions and properties of 2D shapes. You will investigate the relationships between angles inside and outside of a triangle, as well as congruency of triangles. Furthermore, you will find out about the diagonals of quadrilaterals, and the interior angles of different polygons.





10 Construction of geometric figures

10.1 Constructing perpendicular lines

REVISING PERPENDICULAR LINES

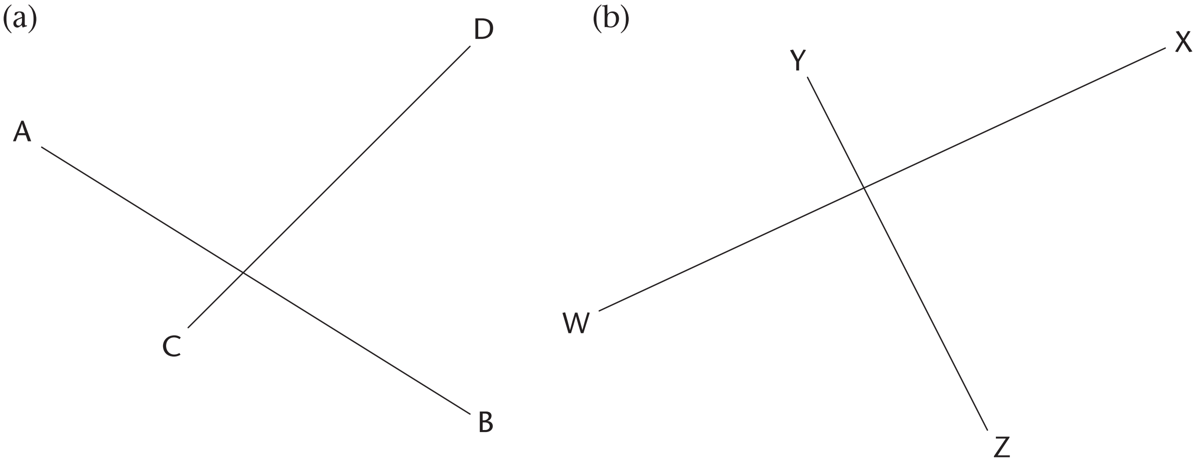
In Grade 8, you learnt about **perpendicular lines**.

1. What does it mean if we say 'two lines are perpendicular'?

   ....

2. Use your protractor to measure the angles between the following pairs of lines. Then state whether they are perpendicular or not.

(a) (b)

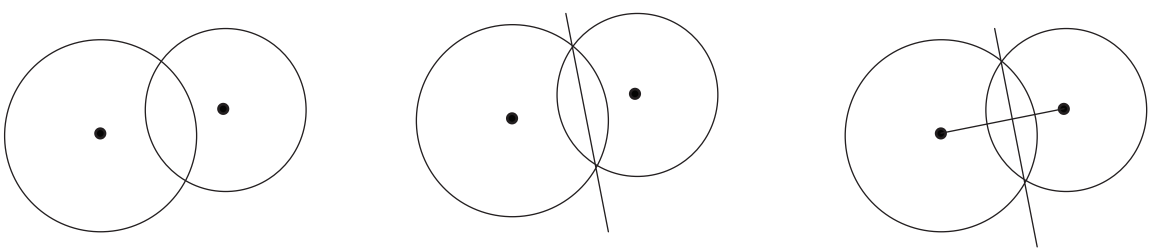


....

....

LINES THAT FORM WHEN CIRCLES INTERSECT

1. Do the following in your exercise book:
   (a) Use a compass to draw two overlapping circles of different sizes.
   (b) Draw a line through the points where the circles intersect (overlap).
   (c) Draw a line to join the centres of the circles.



Step (a) *Step (b)* *Step (c)*

(d) Use your protractor to measure the angles between the intersecting lines. ......

(e) What can you say about the intersecting lines? ....

....

....

2. Repeat questions 1(a) to (e) with circles that are the same size.
3. What conclusion can you make about a line drawn between the intersection points of two overlapping circles and a line through their centres?

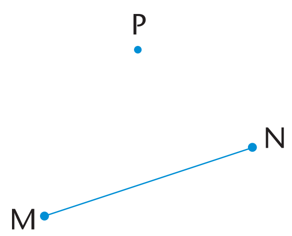
....

USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

Case 1: A perpendicular through a point that is not on the line segment

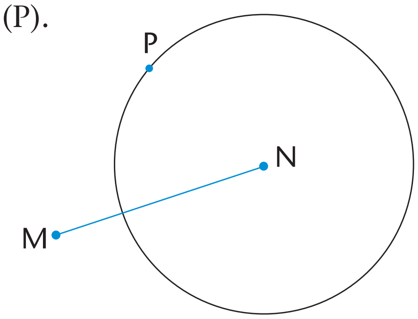
Copy the steps below in your exercise book.

You are given line segment MN with point P at a distance from it. You must construct a line that is perpendicular to MN, so that the perpendicular passes through point P.



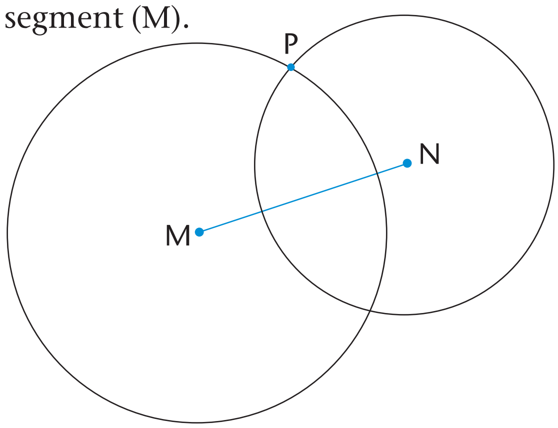
Step 1

Use your compass to draw a circle whose centre is the one end point of the line segment (N) and passes through the point (P).



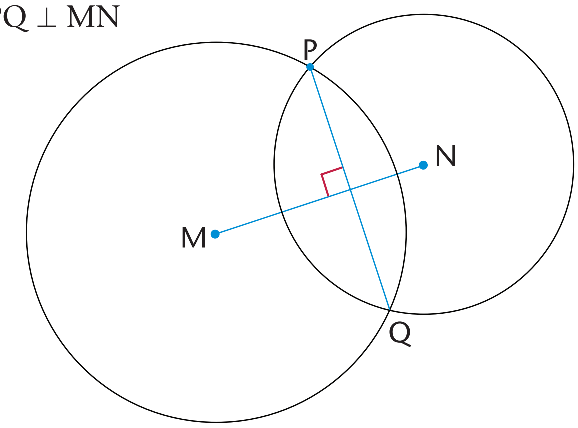
Step 2

Repeat step 1, but make the centre of your circle the other end point of the line segment (M).



Step 3

Join the points where the circles intersect: $PQ \perp MN$



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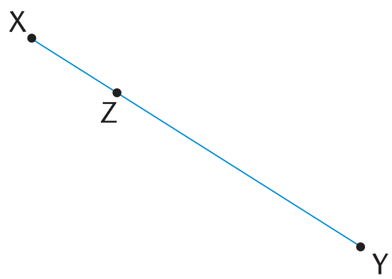
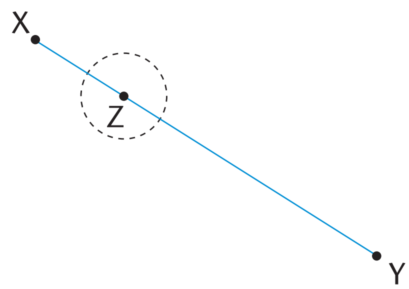
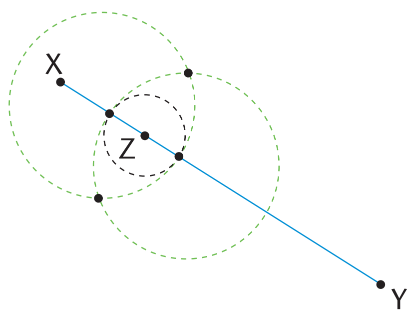
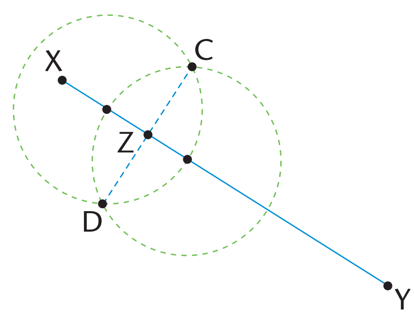
Case 2: A perpendicular at a point that is on the line segment

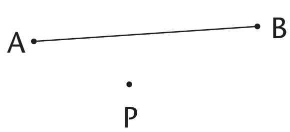
Copy the steps below in your exercise book.

You are given line segment XY with point Z on it. You must construct a perpendicular line passing through Z.  	**Step 1** Use your compass to draw a circle whose centre is Z. Make its radius smaller than ZX. Note the two points where the circle intersects XY. 
Step 2 Set your compass wider than it was for the circle with centre Z. Draw two circles of the same size whose centres are at the two points where the first (black) circle intersects XY. The two circles (green) will overlap.  	**Step 3** Join the intersection points of the two overlapping circles. Mark these points C and D: CD ⊥ XY and passes through point Z. 

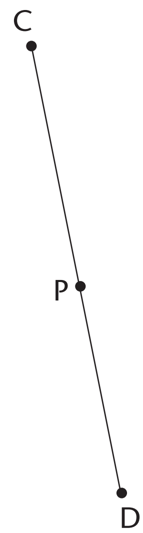
PRACTISE USING CIRCLES TO CONSTRUCT PERPENDICULAR LINES

In each of the following two cases, draw a line that is perpendicular to the segment, and passes through the point P.

1.



2.

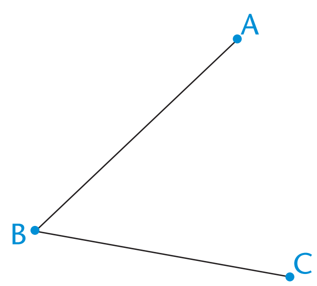
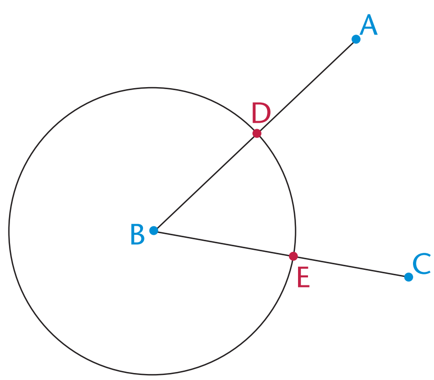


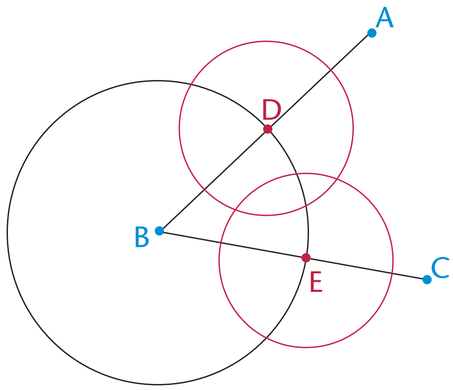
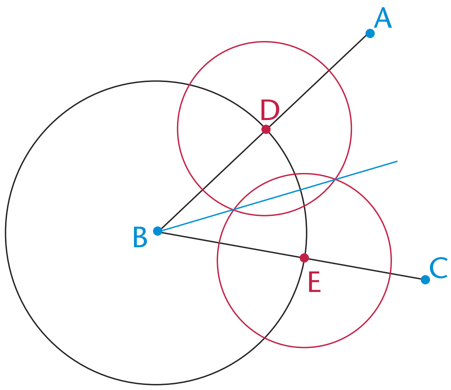
10.2 Bisecting angles

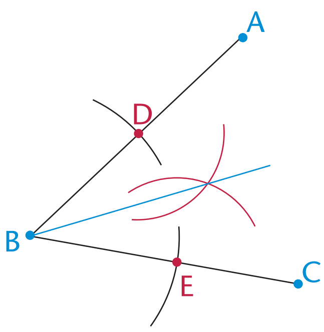
USING CIRCLES TO BISECT ANGLES

Work through the following example of using intersecting circles to **bisect** an angle. Do the steps yourself in your exercise book.

The word **bisect** means 'to cut in half'.

You are given $A\hat{B}C$. You must bisect the angle. 	**Step 1** Draw a circle with centre B to mark off an equal length on both arms of the angle. Label the points of intersection D and E: DB = BE. 

Step 2	**Step 3**
Draw two equal circles with centres at D and at E. Make sure the circles overlap.	Draw a line from B through the points where the two equal circles intersect. This line will bisect the angle.
	



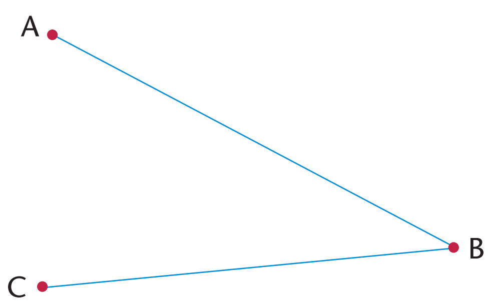
Same construction as in step 3 above

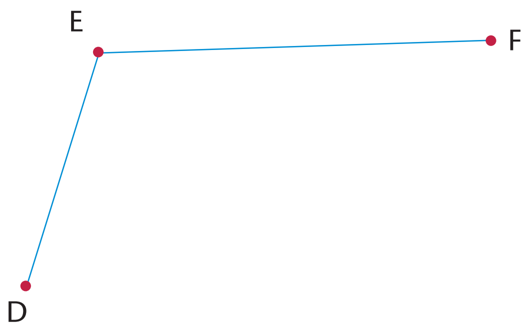
Can you explain why the method above works to bisect an angle?

Can you also see that we need not draw full circles, but can merely use parts of circles (arcs) to do the above construction?

PRACTISE BISECTING ANGLES

Bisect the angles below without using a protractor.





10.3 Constructing special angles without a protractor

Angles of 30°, 45°, 60° and 90° are known as **special angles**. You must be able to construct these angles without using a protractor.

CONSTRUCTING A 45° ANGLE

You have learnt how to draw a 90° angle, and how to bisect an angle, without using a protractor. Use this information to draw a 45° angle at point X on the line below.

Hint: Extend the line to the left of X.

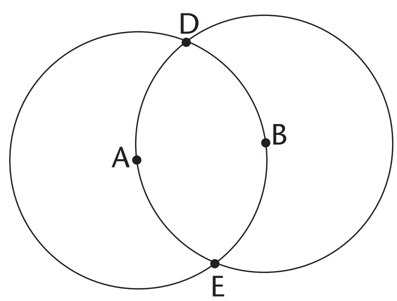
CONSTRUCTING 60° AND 30° ANGLES

1. What do you know about the sides and angles in an equilateral triangle?

    ....

2. In your exercise book, draw two circles with the following properties:
    - The circles are the same size.
    - Each circle passes through the other circle's centre.
    - The centres of the circles are labelled A and B.
    - The points of intersection of the circles are labelled D and E.

    An example is shown on the right.



3. Draw in the following line segments: AB, AD and DB.
4. What can you say about the lengths of AB, AD and DB?

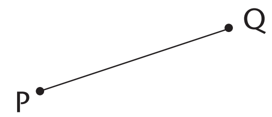
    ....

5. What kind of triangle is ABD? ....

6. Therefore, what do you know about $\hat{A}$, $\hat{B}$ and $\hat{D}$? ....

....

7. Use your knowledge of bisecting angles to create an angle of 30° on the construction you made in question 2.
8. Use what you have learnt above to construct an angle of 60° at point P on the following line segment:



CONSTRUCTING THE MULTIPLES OF SPECIAL ANGLES

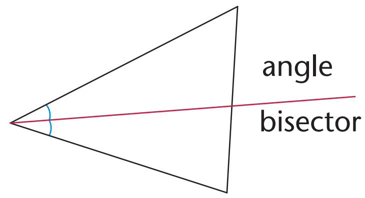
1. Complete the table below. The first one has been done for you:

Angle	Multiples below 360°	Angle	Multiples below 360°
30°	30°; 60°; 90°; 120°; 150°; 180°; 210°; 240°; 270°; 300°; 330°	45°	
60°		90°	

2. In your exercise book, construct the following angles without using a protractor. You will need to do more than one construction to create each angle.
   (a) 120° (b) 135° (c) 270° (d) 240° (e) 150°

10.4 Angle bisectors in triangles

You learnt how to bisect an angle in Section 10.2.
Now you will investigate the angle bisectors in a triangle.
An **angle bisector** is a line that cuts an angle in half.



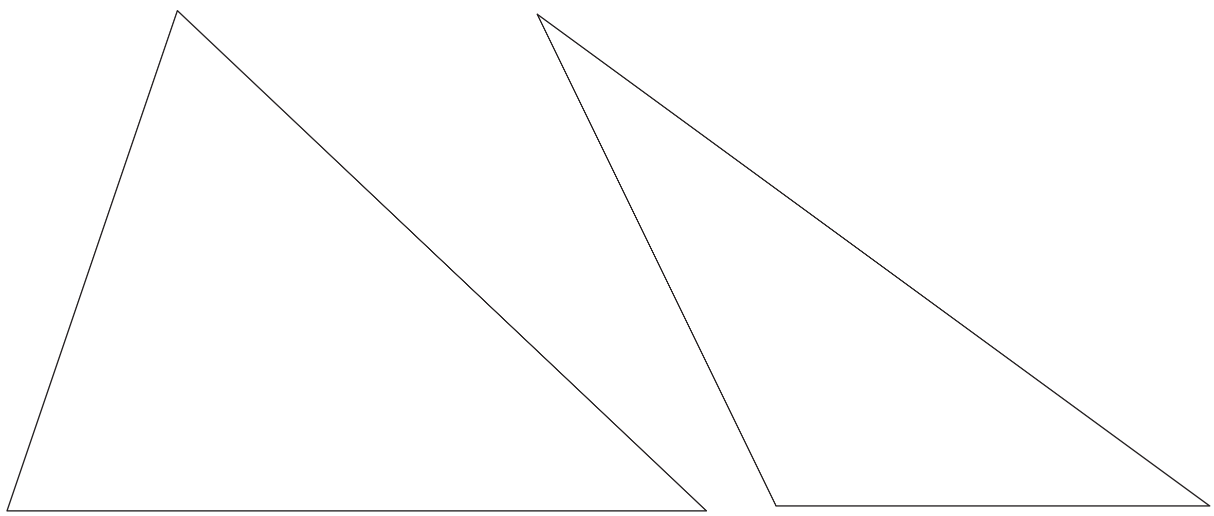
1. (a) Bisect each of the angles of the acute triangle below.
   (b) Extend each of the bisectors to the opposite side of the triangle.
   (c) What do you notice?

   ....

2. (a) Do the same with the obtuse triangle.
   (b) What do you notice?

   ....

   ....



3. Compare your triangles with those of two classmates. You should have the same results.

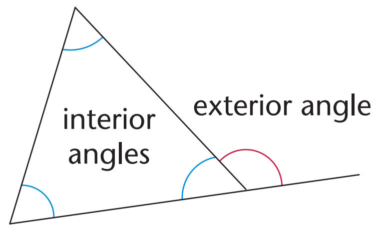
You should have found that the three **angle bisectors** of a triangle **intersect at one point**. This point is the same distance away from each side of the triangle.

10.5 Interior and exterior angles in triangles

WHAT ARE INTERIOR AND EXTERIOR ANGLES?

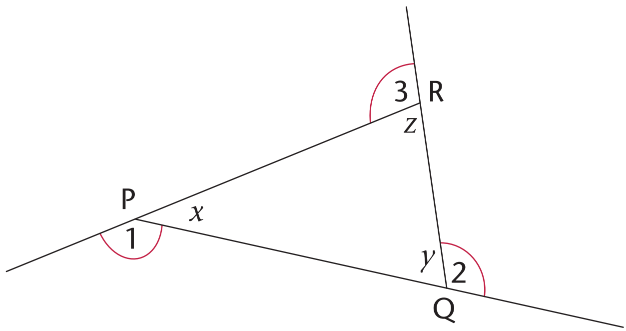
An **interior angle** is an angle that lies between two sides of a triangle. It is inside the triangle. A triangle has three interior angles.

An **exterior angle** is an angle between a side of a triangle and another side that is extended. It is outside the triangle.



Look at ΔPQR. Its three sides are extended to create three exterior angles.

Each exterior angle has one interior adjacent angle (next to it) and two **interior opposite angles**, as described in the following table.

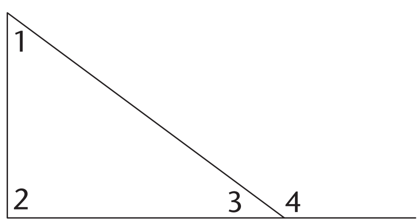


Exterior angle	Interior adjacent angle	Interior opposite angles
1	x	z and y
2	y	x and z
3	z	x and y

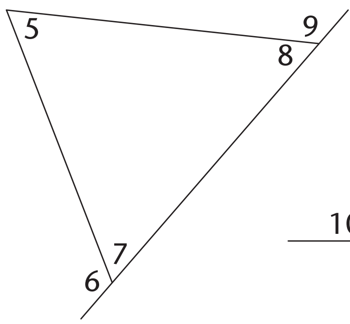
IDENTIFYING EXTERIOR ANGLES AND INTERIOR OPPOSITE ANGLES

1. Name each exterior angle and its two interior opposite angles below.

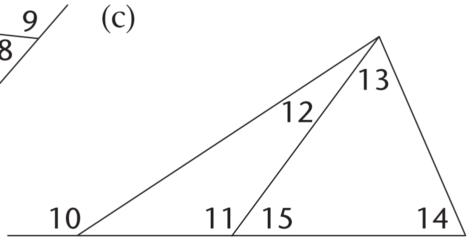
(a)



(b)

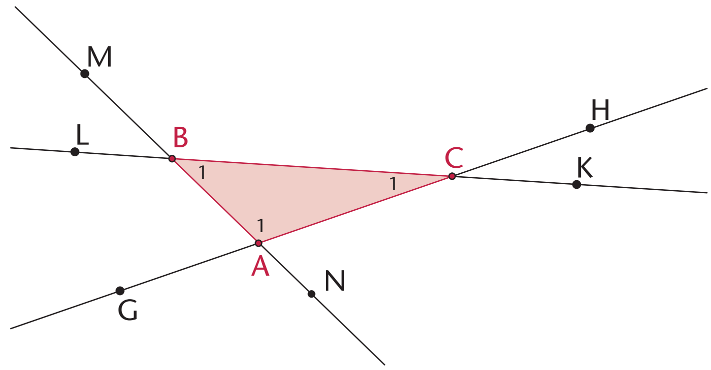


(c)



Ext. ∠					
Int. opp. ∠s					

2. ΔABC below has each side extended in both directions to create six exterior angles.



(a) Write down the names of the interior angles of the triangle. ....

....

(b) Since a triangle has three sides that can be extended in both directions, there are two exterior angles at each vertex. Write down the names of all the exterior angles of the triangle.

....

(c) Explain why $M\hat{B}L$ is not an exterior angle of ΔABC. ....

....

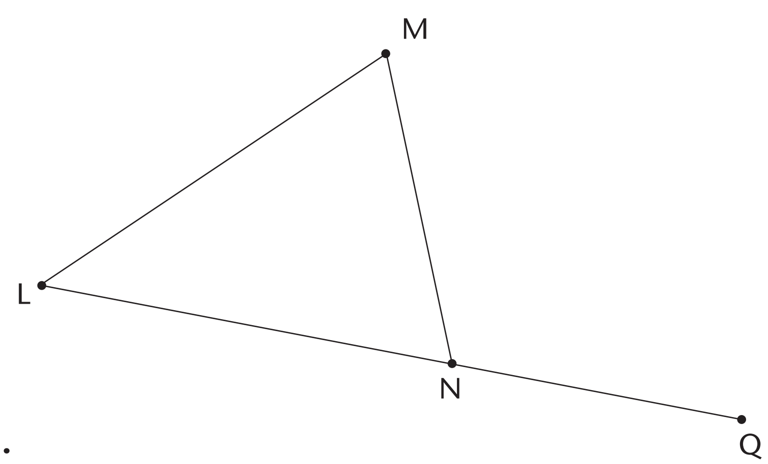
(d) Write down two other angles that are neither interior nor exterior.

....

INVESTIGATING THE EXTERIOR AND INTERIOR ANGLES IN A TRIANGLE

1. Consider ΔLMN. Write down the name of the exterior angle. ....
2. Use a protractor to measure the interior angles and the exterior angle. Write the measurements on the drawing.
3. Use your findings in question 2 to complete this sum:

   $L\widehat{M}N + M\hat{L}N =$ ....

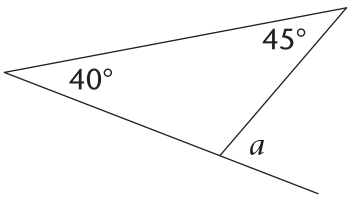


4. What is the relationship between the exterior angle of a triangle and the sum of the interior opposite angles? ....

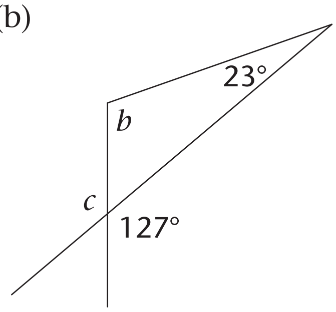
The **exterior angle** of a triangle is equal to the sum of the interior opposite angles.

5. Work out the sizes of angles *a* to *f* below, without using a protractor. Give reasons for the statements you make as you work out the answers.

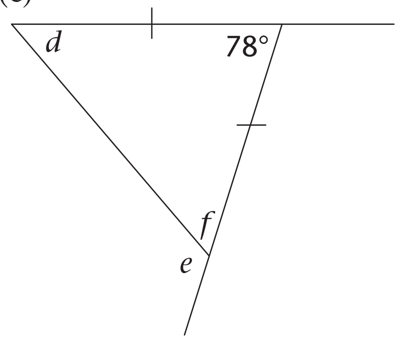
(a)



(b)

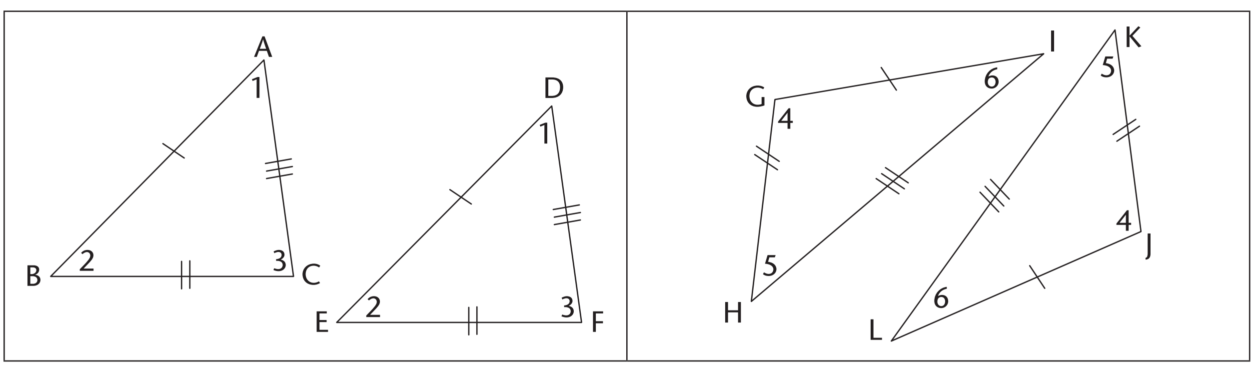


(c)



10.6 Constructing congruent triangles

Two triangles are **congruent** if they have exactly the **same shape and size**: they are able to fit exactly on top of each other. This means that all three corresponding sides and three corresponding angles are equal, as shown in the following two pairs.



ΔABC ≡ ΔDEF and ΔGHI ≡ ΔJKL. In each pair, the corresponding sides and angles are equal.

MINIMUM CONDITIONS FOR CONGRUENCY

To determine whether two triangles are congruent, we need a certain number of measurements, but not all of these. Let's investigate which measurements give us only one possible triangle.

1. Use a ruler, compass and protractor to construct the following triangles. Each time minimum measurements are given.
   (a) Given three sides: side, side, side (SSS):
   ΔDEF with DE = 7 cm, DF = 6 cm and EF = 5 cm.

   (b) Given three angles: angle, angle, angle (AAA):
   ΔABC with $\hat{A} = 80°$, $\hat{B} = 60°$ and $\hat{C} = 40°$.

(c) Given one side and two angles: side, angle, angle (SAA):
ΔGHI with GH = 8 cm, $\hat{G} = 60°$ and $\hat{H} = 30°$.

(d) Given two sides and an included angle: side, angle, side (SAS):
ΔJKL with JK = 9 cm, $\hat{K} = 130°$ and KL = 7 cm.

(e) Given two sides and an angle that is not included: side, side, angle (SSA):
ΔMNP with MN = 10 cm, $\hat{M} = 50°$ and PN = 8 cm.

(f) Given a right angle, the hypotenuse and a side (RHS):
ΔTRS with $TR \perp RS$, RS = 7 cm and TS = 8 cm.

(g) Triangle UVW with UV = 6 cm and VW = 4 cm.

2. Compare your triangles with those of three classmates. Which of your triangles are congruent to theirs? Which are not congruent?

....

....

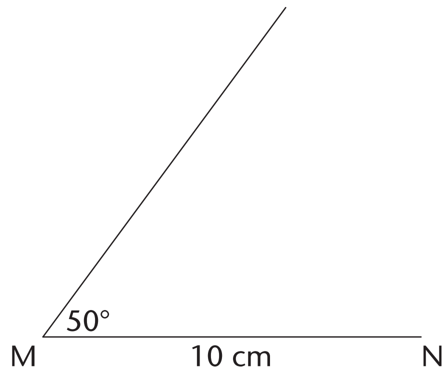
....

....

3. Go back to ΔMNP (1e). Did you find that you can draw two different triangles that both meet the given measurements? One of the triangles will be obtuse and the other acute. Follow the construction steps below to see why this is so.

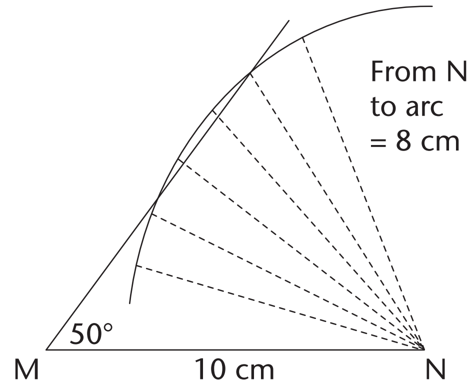
Step 1

Construct MN = 10 cm and the 50° angle at M, even though you do not know the length of unknown side (MP).



Step 2

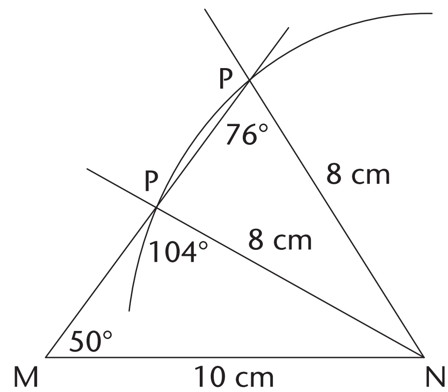
$\hat{N}$ is unknown, but NP = 8 cm. So construct an arc 8 cm from N. Every point on the arc is 8 cm from N.



Step 3

Point P must be 8 cm from N and fall on the unknown side of the triangle. The arc intersects the third side at two points, so P can be at either point.

So two triangles are possible, each meeting the conditions given, i.e. MN = 10 cm, NP = 8 cm and $\hat{M}$ = 50°.



4. Complete the table. Write down whether we can construct a congruent triangle when the following conditions are given.

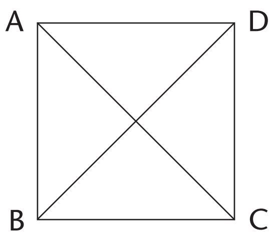
Conditions	**Congruent?**
3 sides (SSS)	
2 sides (SS)	
3 angles (AAA)	
2 angles and a side (AAS)	
2 sides and an angle not between the sides (SSA)	
2 sides and an angle between the sides (SAS)	
Right-angled with the hypotenuse and a side (RHS)	

10.7 Diagonals of quadrilaterals

DRAWING DIAGONALS

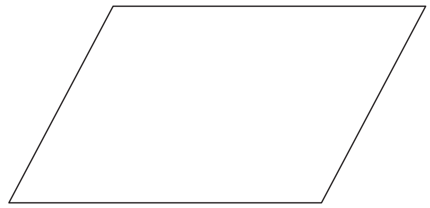
A **diagonal** is a straight line inside a figure that joins two vertices of the figure, where the vertices are not next to each other.

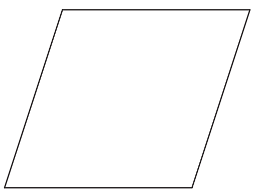
1. Look at the quadrilaterals below. The two diagonals of the square have been drawn in: AC and BD.
2. Draw in the diagonals of the other quadrilaterals below.

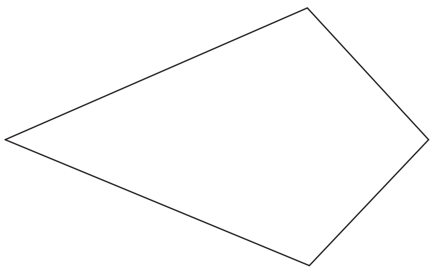


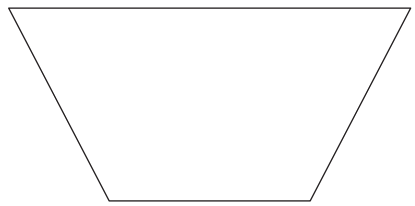
Square


Rectangle


Parallelogram


Rhombus


Kite


Trapezium

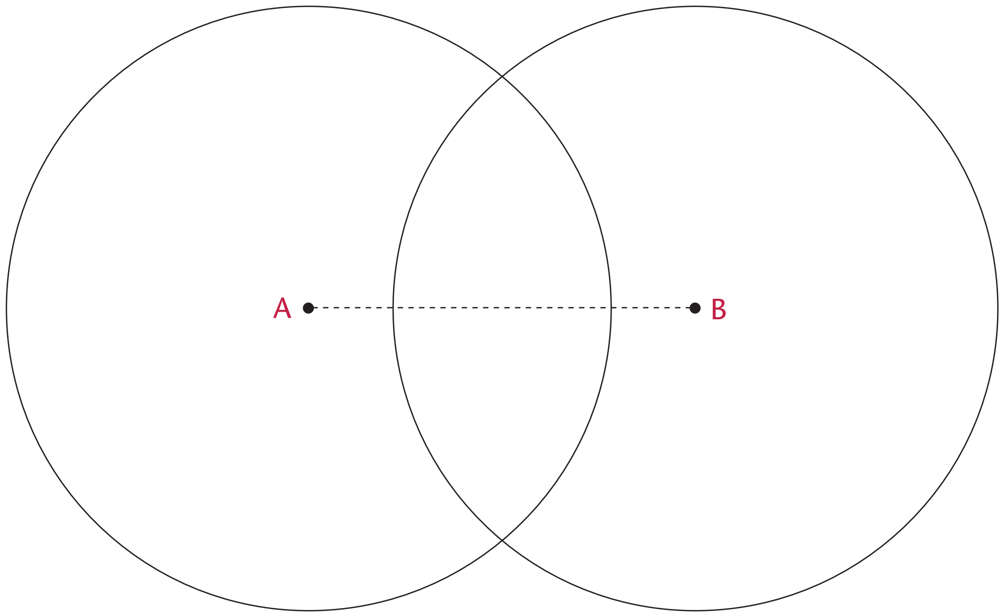
3. How many sides does a quadrilateral have? ....
4. How many angles does a quadrilateral have? ....
5. How many diagonals does a quadrilateral have? ....

DIAGONALS OF A RHOMBUS

On the next page are two overlapping circles with centres A and B. The circles are the same size.

1. Construct a rhombus inside the circles by joining the centre of each circle with the intersection points of the circles. Join AB.
2. Construct the perpendicular bisector of AB. (Go back to Section 10.1 if you need help.) What do you find?

A **perpendicular bisector** is a line that cuts another line in half at a right angle (90°).



3. Do the diagonals bisect each other? ....

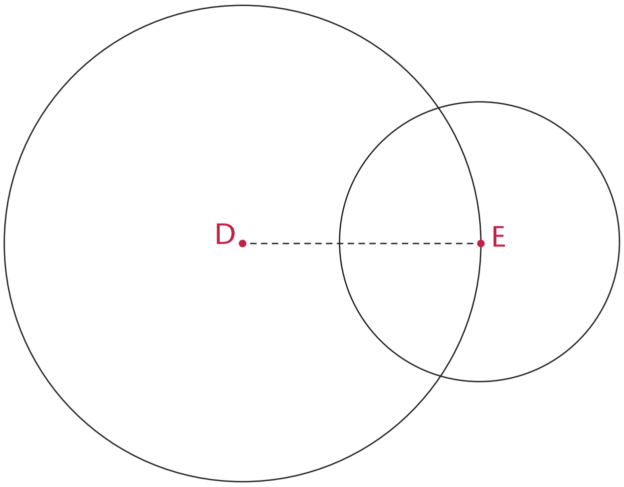
4. Complete the sentence: The diagonals of a rhombus will always ....

....

DIAGONALS OF A KITE

Below are two overlapping circles with centres D and E. The circles are different sizes.

1. Construct a kite by joining the centre points of the circles to the intersection points of the circles.
2. Draw in the diagonals of the kite.
3. Mark all lines that are the same length.



4. Are the diagonals of the kite perpendicular?

....

5. Do the diagonals of the kite bisect each other?

6. What is the difference between the diagonals of a rhombus and those of a kite?

DIAGONALS OF PARALLELOGRAMS, RECTANGLES AND SQUARES

1. Use the grid to draw a parallelogram, rectangle and square.

2. Draw in the diagonals of the quadrilaterals above.
3. Indicate on each shape all the lengths in the diagonals that are equal. (Use a ruler.)
4. Use the information you have found to complete the table below. Fill in 'yes' or 'no'.

Quadrilateral	Diagonals equal	Diagonals bisect	Diagonals meet at 90°
Parallelogram			
Rectangle			
Square			

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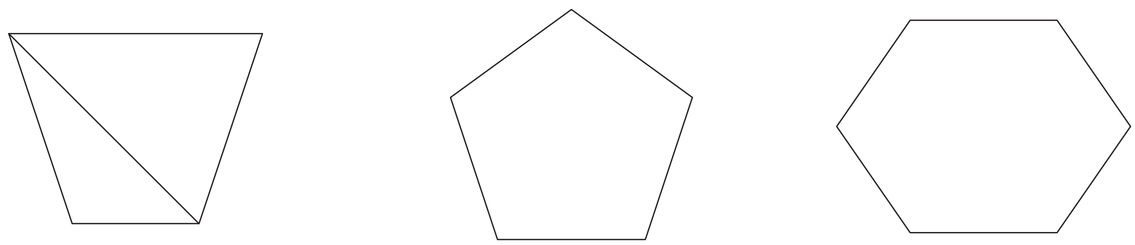
10.8 Angles in polygons

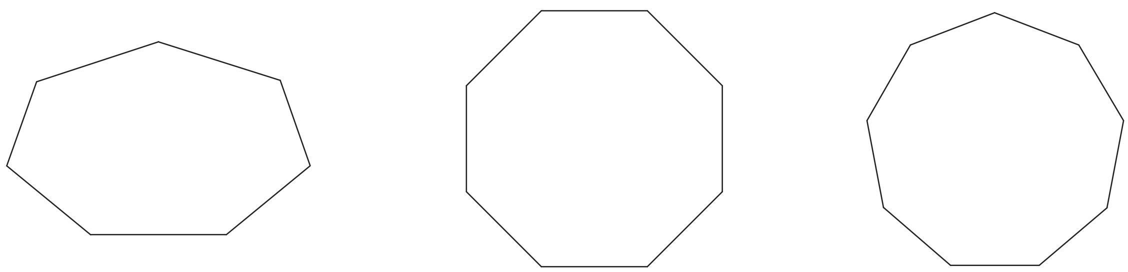
USING DIAGONALS TO INVESTIGATE THE SUM OF THE ANGLES IN POLYGONS

1. We can divide a quadrilateral into two triangles by drawing in one diagonal.
   (a) Draw in diagonals to divide each of the other polygons below into as few triangles as possible.
   (b) Write down the number of triangles in each polygon.



Quadrilateral *Pentagon* *Hexagon*

No. of Δs	2		
Sum of ∠s	$2 \times 180° = 360°$		



Heptagon *Octagon* *Nonagon*

No. of Δs			
Sum of ∠s			

2. The sum of the angles of one triangle = 180°. A quadrilateral is made up of two triangles, so the sum of the angles in a quadrilateral = $2 \times 180° = 360°$. Work out the sum of the interior angles of each of the other polygons above.

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WORKSHEET

1. Match the words in the column on the right with the definitions on the left. Write the letter of the definition next to the matching word.

(a) A quadrilateral that has diagonals that are perpendicular and they bisect each other	Kite
(b) A quadrilateral that has diagonals that are perpendicular to each other, and only one diagonal bisects the other	Congruent
(c) A quadrilateral that has equal diagonals that bisect each other	Exterior angle
(d) Figures that have exactly the same size and shape	Rhombus
(e) Divides into two equal parts	Perpendicular
(f) An angle that is formed outside a closed shape: it is between the side of the shape and a side that has been extended	Bisect
(g) Lines that intersect at 90 degrees	Special angles
(h) 90°, 45°, 30°, 60°	Rectangle

2. Complete the sentence: The exterior angle in a triangle is equal to ....

....

3. (a) Construct $\triangle PQR$ below with angles of 30° and 60°. The side between the angles must be 8 cm. You may use only a ruler and a compass.

(b) Will all triangles with the same measurements above be congruent to $\triangle PQR$? Explain your answer.

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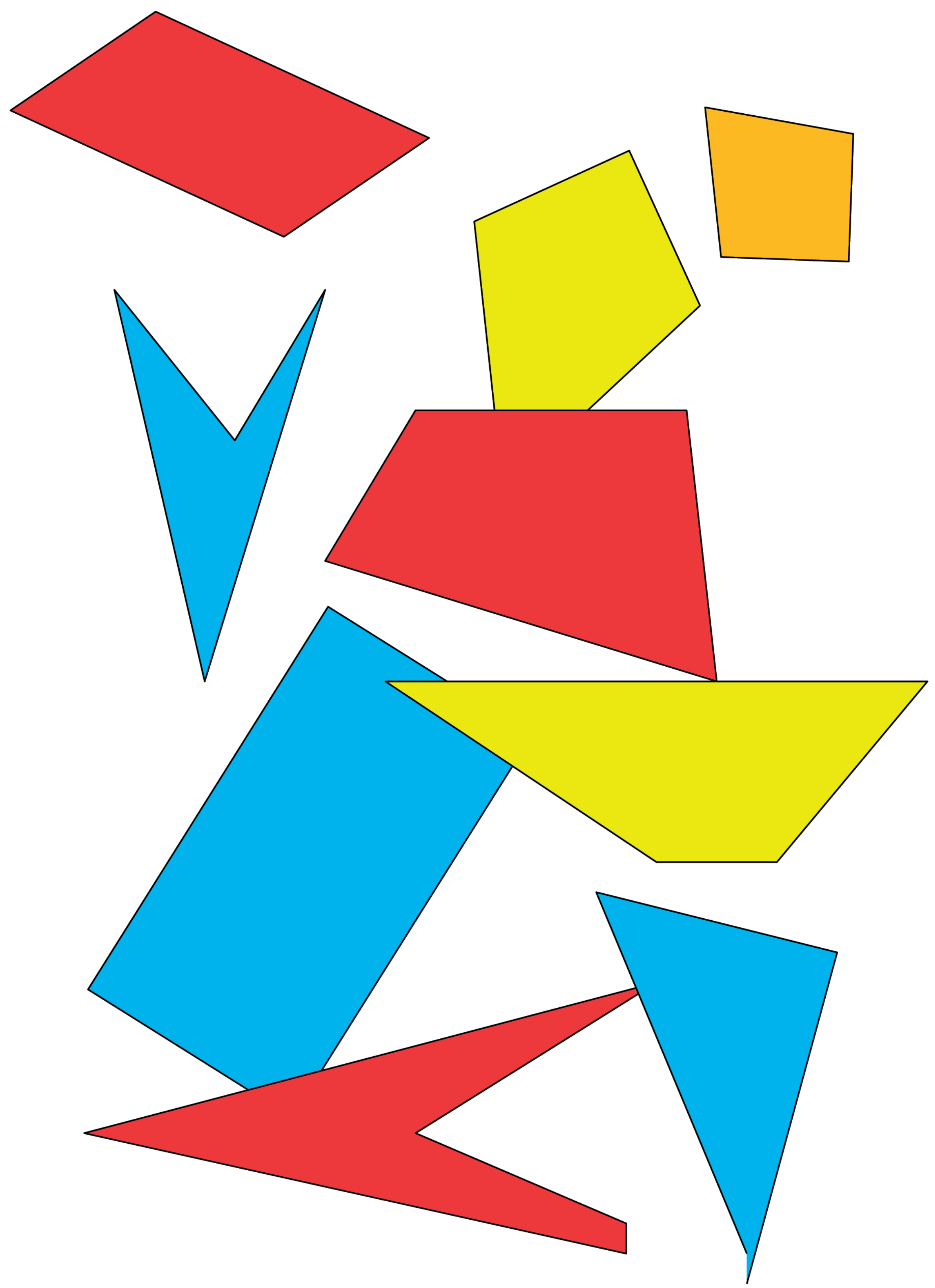
Chapter 11
Geometry of 2D shapes

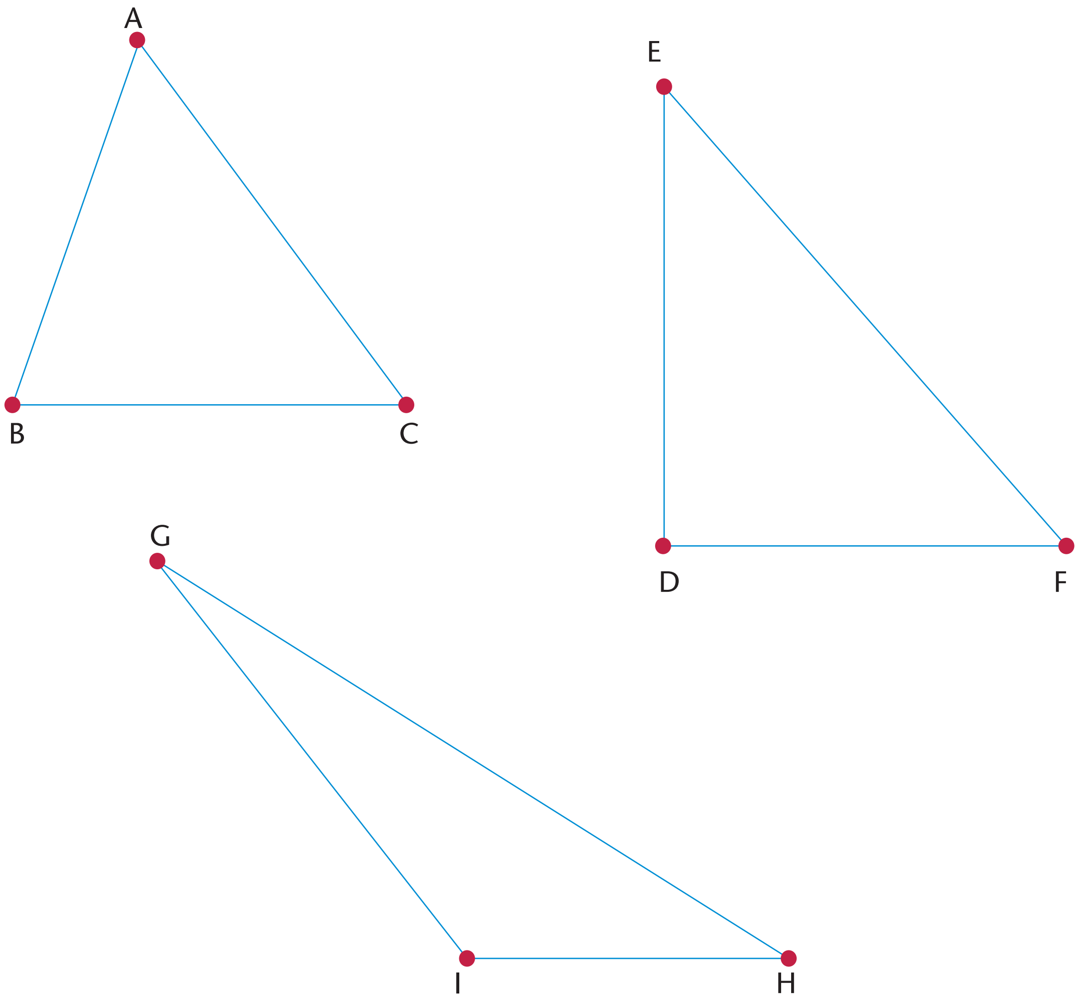
You have learnt to distinguish between equilateral, isosceles and right-angled triangles, and between the following quadrilaterals: parallelograms, rectangles, squares, rhombi, trapeziums and kites. You have investigated the properties of these figures to classify them, such as which sides are equal or parallel, or which angles are equal. In this chapter, you will use your knowledge of the properties of these figures, as well as general properties of triangles and quadrilaterals, to work out further information about the figures. You will also learn more about congruency and similarity in triangles.



11 Geometry of 2D shapes

11.1 Revision: Classification of triangles

1. Use a protractor to measure the interior angles of each of the following triangles. Write the sizes of the angles on the diagrams.



2. Classify the triangles in question 1 according to their angle properties. Choose from the following types of triangles: acute-angled, obtuse-angled and right-angled.

   (a) ΔABC is an .... triangle, because ....
   ....

   (b) ΔEDF is a .... triangle, because ....

   (c) ΔGHI is an obtuse-angled triangle, because ....
   ....

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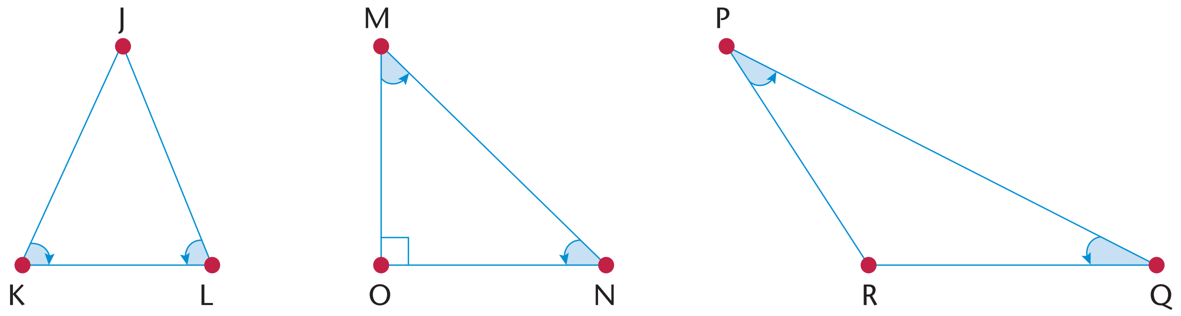
3. The marked angles in each triangle below are equal. Classify the triangles according to angle and side properties.

(a) Δ....... is an acute isosceles triangle, because .... and ....

(b) Δ....... is a right-angled isosceles triangle, because .... and ....

(c) Δ....... is an obtuse isosceles triangle, because .... and ....

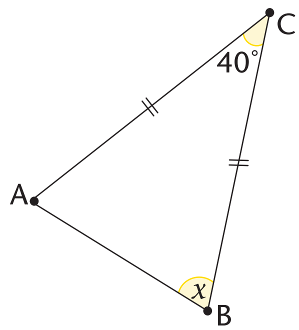


4. Say for what kind of triangle each statement is true. If it is true for all triangles, then write 'All triangles'.

	Statement	True for
(a)	Two sides of the triangle are equal.	
(b)	One angle of the triangle is obtuse.	
(c)	Two angles of the triangle are equal.	
(d)	All three angles of the triangle are equal to 60°.	
(e)	The size of an exterior angle is equal to the sum of the opposite interior angles.	
(f)	The longest side of the triangle is opposite the biggest angle.	
(g)	The sum of the two shorter sides of the triangle is bigger than the length of the longest side.	
(h)	The square of the length of one side is equal to the sum of the squares of the other sides.	
(i)	The square of the length of one side is bigger than the sum of the squares of the other sides.	
(j)	The sum of the interior angles of the triangle is 180°.	

11.2 Finding unknown angles in triangles

When you have to determine the size of an unknown angle or length of a shape in geometry, you must give a reason for each statement you make.
Complete the example below:

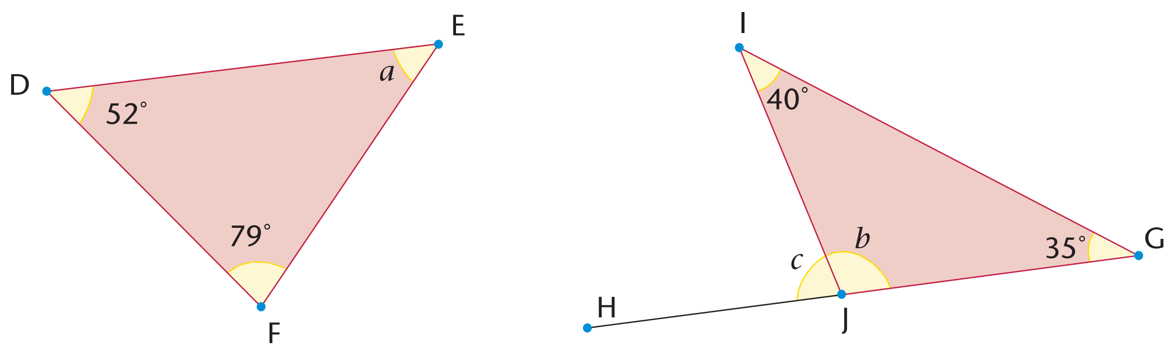


In ΔABC, AC = BC and $\hat{C}$ = 40°. Find the size of $\hat{B}$ (shown in the diagram as x).

Statement	Reason
AC = BC	Given
∴ $\hat{A} = \hat{B}$	
180° = 40° + x + x	Sum ∠s Δ
180° – 40° = 2x	
∴ x =	

FINDING UNKNOWN LENGTHS AND ANGLES

1. Calculate the sizes of the unknown angles.



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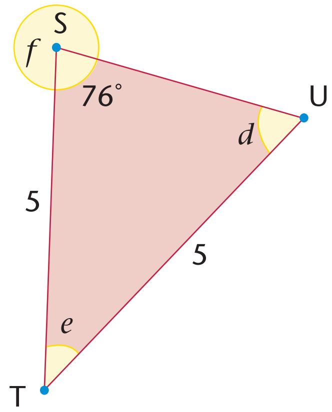
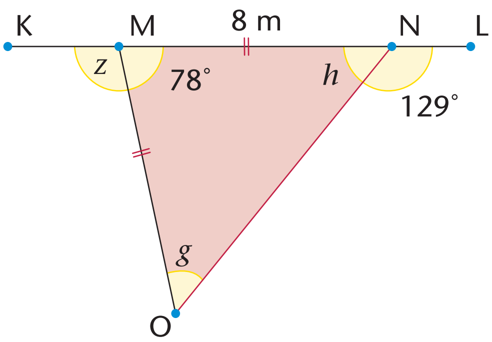
2. Determine the sizes of the unknown angles and the length of MO.





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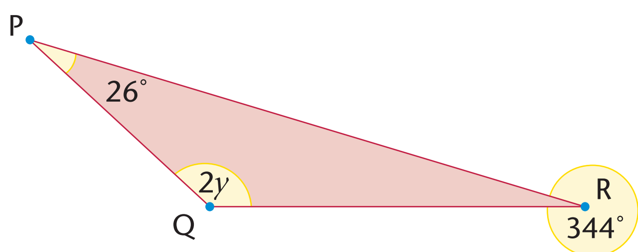
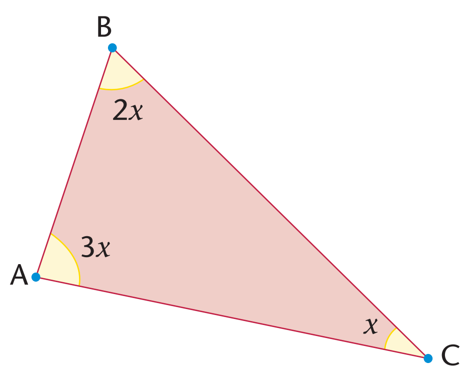
3. Calculate the sizes of y and x.





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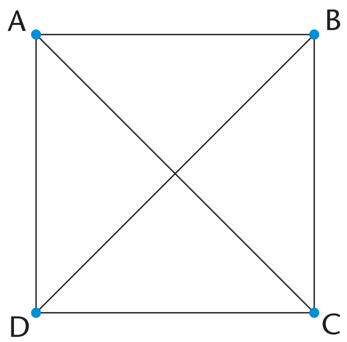
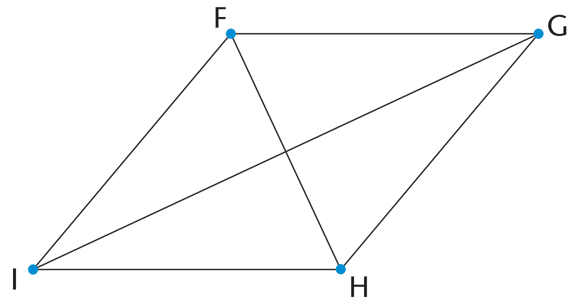
11.3 Quadrilaterals

PROPERTIES OF QUADRILATERALS

1. Name the following quadrilaterals. Mark equal angles and equal sides in each figure. Use your ruler and protractor to measure angle sizes and lengths where necessary.





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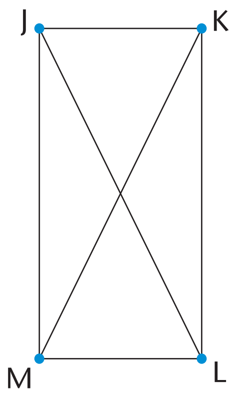

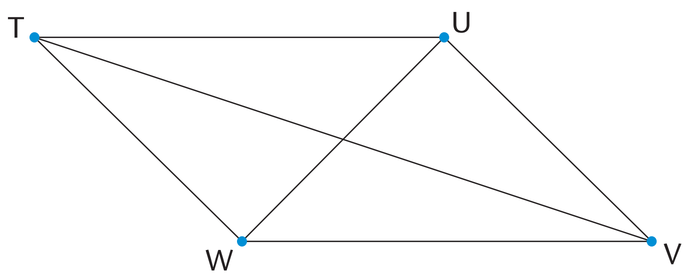

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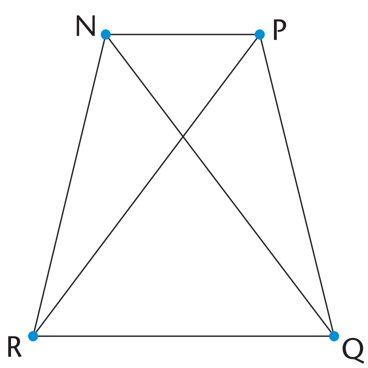

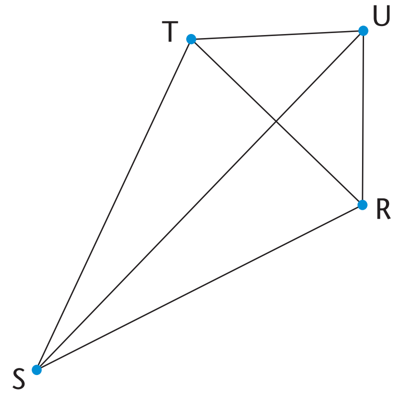

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2. Complete the following table:

Properties	True for the following quadrilaterals					
	Square	Rhombus	Rectangle	Parallelogram	Kite	Trapezium
At least one pair of opposite angles is equal.	yes	yes	yes	yes	yes	no
Both pairs of opposite angles are equal.						
At least one pair of adjacent angles is equal.						
All four angles are equal.						
Any two opposite sides are equal.						
Two adjacent sides are equal, and the other two adjacent sides are also equal.						
All four sides are equal.						
At least one pair of opposite sides is parallel.						
Any two opposite sides are parallel.						
The two diagonals are perpendicular.						
At least one diagonal bisects the other one.						
The two diagonals bisect each other.						
The two diagonals are equal.						
At least one diagonal bisects a pair of opposite angles.						
Both diagonals bisect a pair of opposite angles.						
The sum of the interior angles is 360°.						

3. Look at the properties of a square and a rhombus.
   (a) Are all the properties of a square also the properties of a rhombus? Explain.

   ....

   ....

   ....

   ....

   ....

   (b) Are all the properties of a rhombus also the properties of a square? Explain.

   ....

   (c) Which statement is true?

   A square is a special kind of rhombus. ....

   A rhombus is a special kind of square. ....

4. Look at the properties of rectangles and squares.
   (a) Are all the properties of a square also the properties of a rectangle? Explain.

   ....

   ....

   (b) Are all the properties of a rectangle also the properties of a square? Explain.

   ....

   (c) Which statement is true?

   A square is a special kind of rectangle. ....

   A rectangle is a special kind of square. ....

5. Look at the properties of parallelograms and rectangles.
   (a) Are all the properties of a parallelogram also the properties of a rectangle? Explain.

   ....

   (b) Are all the properties of a rectangle also the properties of a parallelogram? Explain.

   ....

   (c) Which statement is true?

   A rectangle is a special parallelogram. ....

   A parallelogram is a special rectangle. ....

6. Look at the properties of a rhombus and a parallelogram. Is a rhombus a special kind of parallelogram? Explain.

....

7. Compare the properties of a kite and a parallelogram. Why is a kite not a special kind of parallelogram?

....

....

....

8. Compare the properties of a trapezium and a parallelogram. Why is a trapezium not a special kind of parallelogram?

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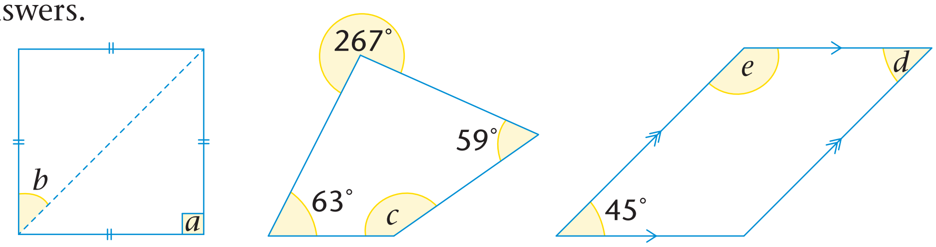
UNKNOWN SIDES AND ANGLES IN QUADRILATERALS

1. Determine the sizes of angles a to e in the quadrilaterals below. Give reasons for your answers.



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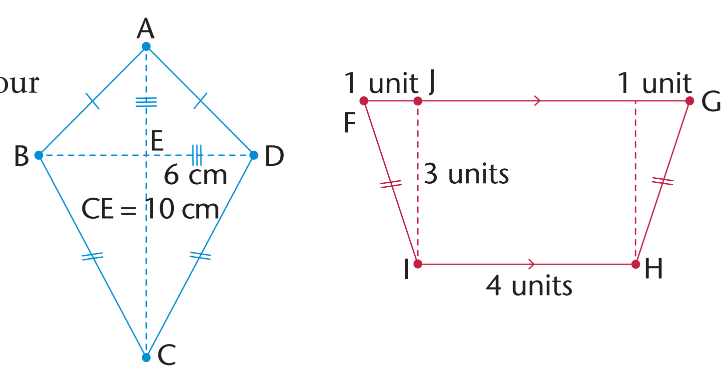
2. Calculate the perimeters of the quadrilaterals on the right. Give your answers to two decimal places.



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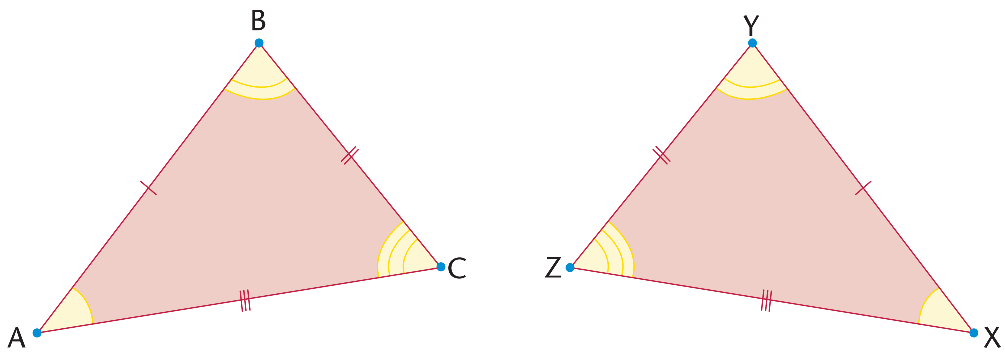
11.4 Congruent triangles

DEFINITION AND NOTATION OF CONGRUENT TRIANGLES

If two triangles are congruent, then they have exactly the same size and shape. In other words, if you cut out one of the triangles and place it on the other, they will match exactly.

> If you know that two triangles are congruent, then each side in the one triangle will be equal to each corresponding side in the second triangle. Also, each angle in the one triangle will be equal to each corresponding angle in the second triangle.



In the triangles above, you can see that $\Delta ABC \equiv \Delta XYZ$.

Congruency symbol

$\equiv$ means 'is congruent to'

The order in which you write the letters when stating that two triangles are congruent is very important. The letters of the corresponding vertices between the two triangles must appear in the same position in the notation. For example, the notation for the triangles above should be: $\Delta ABC \equiv \Delta XYZ$, because it indicates that $\hat{A} = \hat{X}$, $\hat{B} = \hat{Y}$, $\hat{C} = \hat{Z}$, $AB = XY$, $BC = YZ$ and $AC = XZ$.

It is incorrect to write $\Delta ABC \equiv \Delta ZYX$. Although the letters refer to the same triangles, this notation indicates that $\hat{A} = \hat{Z}$, $\hat{C} = \hat{X}$, $AB = ZY$ and $BC = YX$, and these statements are not true.

Write down the equal angles and sides according to the following notations:

1. $\Delta KLM \equiv \Delta PQR$: ....
2. $\Delta FGH \equiv \Delta CST$: ....

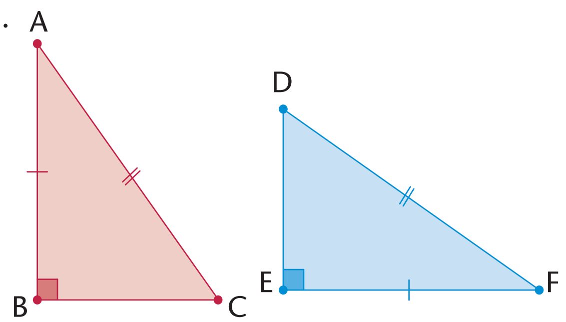
MINIMUM CONDITIONS FOR CONGRUENT TRIANGLES

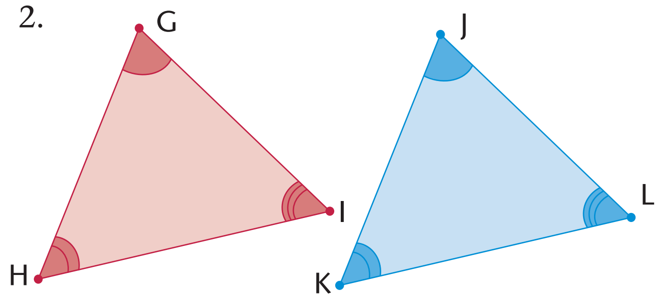
In the previous chapter, you investigated the minimum conditions that must be satisfied in order to establish that two triangles are congruent.

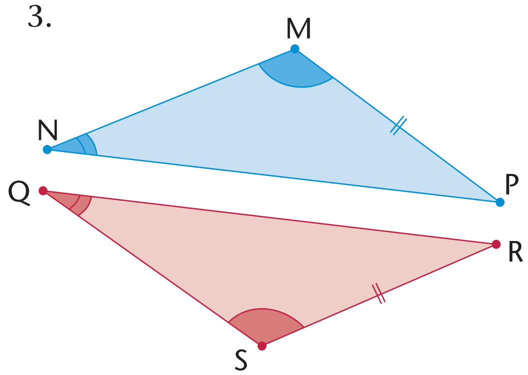
The conditions for congruency:

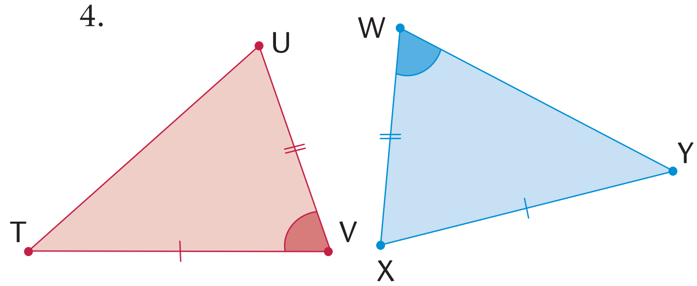
- SSS (all corresponding sides are equal)
- SAS (two corresponding sides and the angle between the two sides are equal)
- AAS (two corresponding angles and any corresponding side are equal)
- RHS (both triangles have a 90° angle and have equal hypotenuses and one other side equal).

Decide whether or not the triangles in each pair below are congruent. For each congruent pair, write the notation correctly and give a reason for congruency.

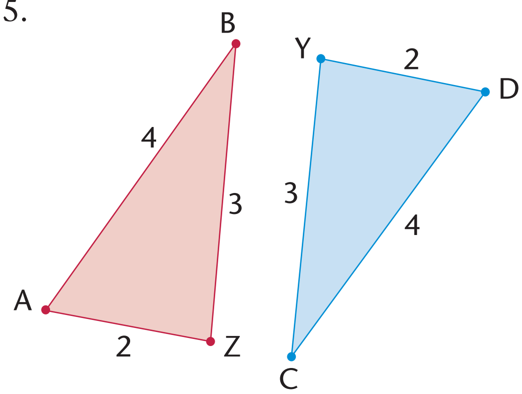
1.


2.


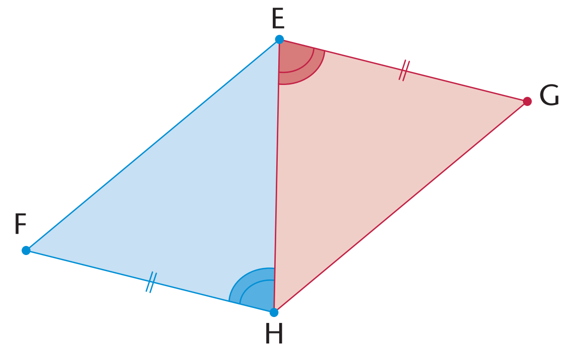
3.


4.


5.



6.



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PROVING THAT TRIANGLES ARE CONGRUENT

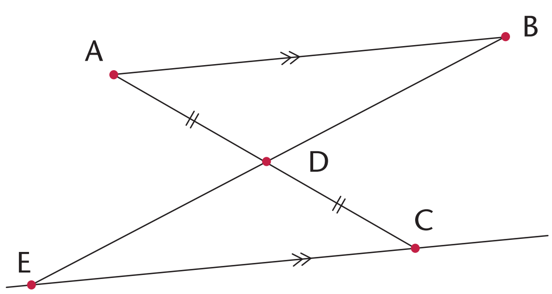
You can use what you know about the minimum conditions for congruency to prove that two triangles are congruent.

When giving a proof for congruency, remember the following:

- Each statement you make needs a reason.
- You must give three statements to prove any two triangles congruent.
- Give the reason for congruency.

Example:

In the sketch on the right: AB || EC and AD = DC.
Prove that the triangles are congruent.



Solution:

Statement	Reason
In ΔABD and ΔCED:	
1) AD = DC	Given
2) $A\hat{D}B = C\hat{D}E$	Vert. opp. ∠s
3) $B\hat{A}D = E\hat{C}D$	Alt. ∠s (AB || EC)
∴ ΔABD ≡ ΔCED	AAS

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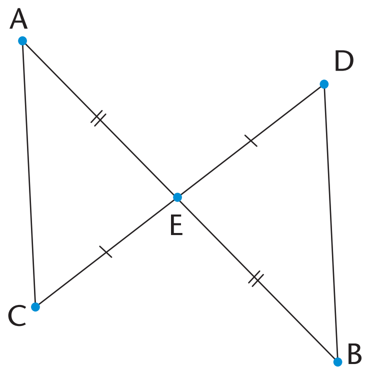
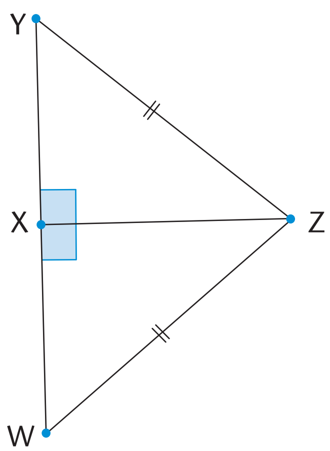
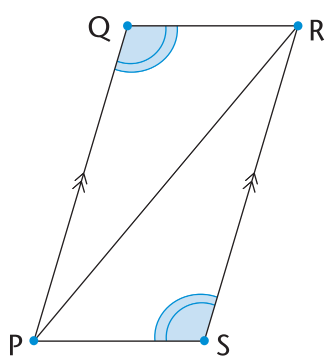
1. Prove that $\Delta ACE \equiv \Delta BDE$.

	Statement	Reason
	....	....
	....	....
	....	....
	....	....

2. Prove that $\Delta WXZ \equiv \Delta YXZ$.

	Statement	Reason
	....	....
	....	....
	....	....
	....	....
	....	....

3. Prove that QR = SP. (Hint: First prove that the triangles are congruent.)

	Statement	Reason
	....	....
	....	....
	....	....
	....	....
	....	....

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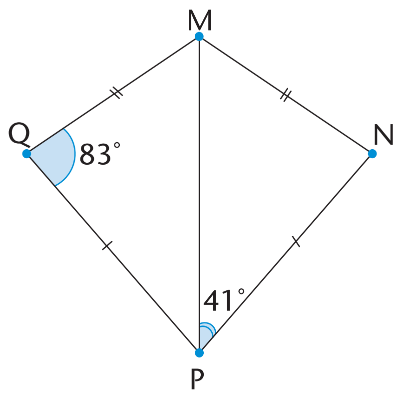
4. Prove that the triangles below are congruent. Then find the size of $Q\hat{M}P$.

	Statement	Reason
	....	....
	....	....
	....	....
	....	....
	....	....
	....	....
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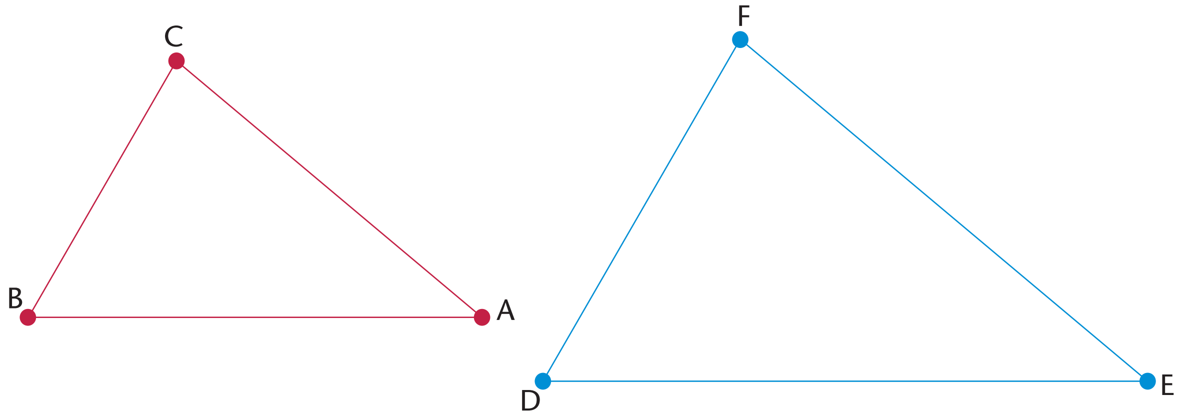
11.5 Similar triangles

PROPERTIES OF SIMILAR TRIANGLES

ΔBAC and ΔDEF below are similar to each other. Similar figures have the same shape, but their sizes can be different.



1. (a) Use a protractor to measure the angles in each triangle above. Then complete the table below.

Angle	Angle	What do you notice?
$\hat{B}$ =	$\hat{D}$ =	
$\hat{A}$ =	$\hat{E}$ =	
$\hat{C}$ =	$\hat{F}$ =	

(b) What can you say about the sizes of the angles in similar triangles?

....

2. (a) Use a ruler to measure the lengths of the sides in each triangle in question 1. Then complete the table below.

Length (cm)	Length (cm)	Ratio
BA =	DE =	BA : DE = $= 1 : 1\frac{1}{3}$
BC =	DF =	BC : DF = =
CA =	FE =	CA : FE = =

(b) What can you say about the relationship between the sides in similar triangles?

Ratio reminder

You read 2 : 1 as 'two to one'.

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3. The following notation shows that the triangles are similar: ΔBAC ||| ΔDEF. Why do you think we write the first triangle as ΔBAC and not as ΔABC?

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The properties of similar triangles:

- The corresponding angles are equal.
- The corresponding sides are in proportion.

Notation for similar triangles:

If ΔXYZ is similar to ΔPQR, then we write: ΔXYZ ||| ΔPQR.

As for the notation of congruent figures, the order of the letters in the notation of similar triangles indicates which angles and sides are equal. For ΔXYZ ||| ΔPQR:

Angles: $\hat{X} = \hat{P}$ and $\hat{Y} = \hat{Q}$ and $\hat{Z} = \hat{R}$

Sides: XY : PQ = XZ : PR = YZ : QR

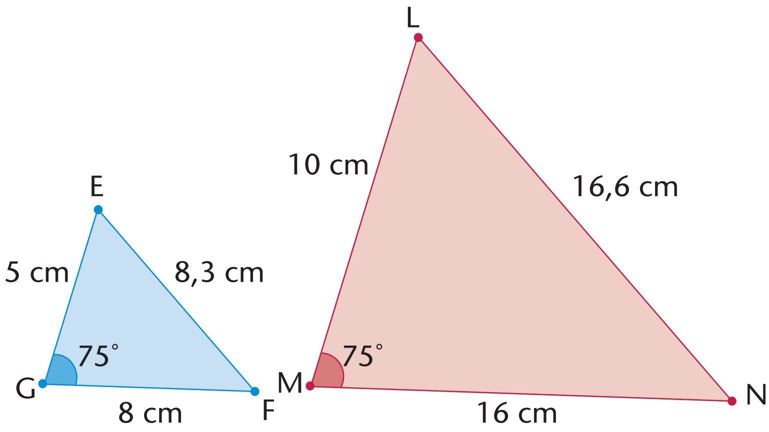
If the triangles' vertices were written in a different order, then the statements above would not be true.

When proving that triangles are similar, you either need to show that the corresponding angles are equal or you must show that the sides are in proportion.

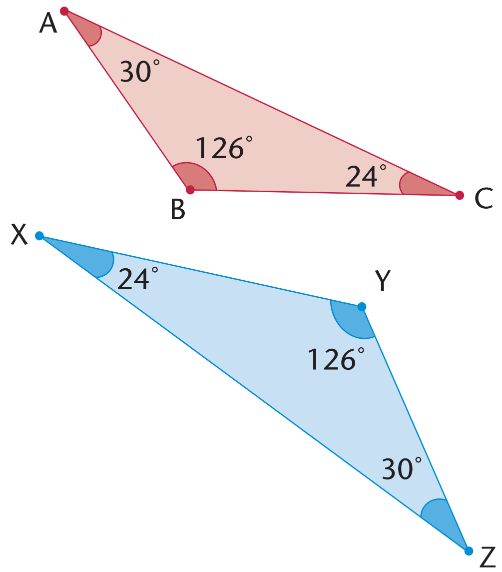
WORKING WITH PROPERTIES OF SIMILAR TRIANGLES

1. Decide if the following triangles are similar to each other.

(a)



(b)



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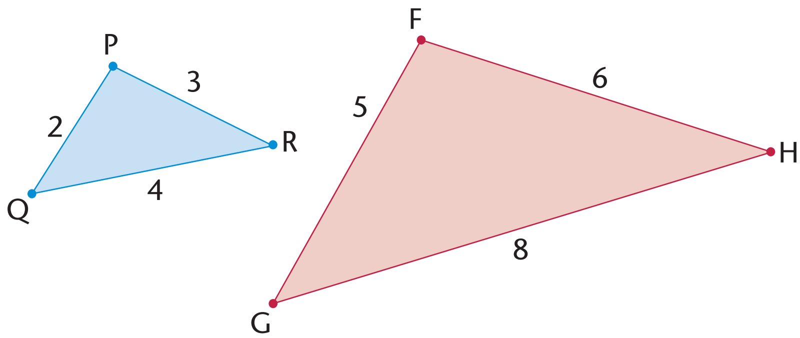
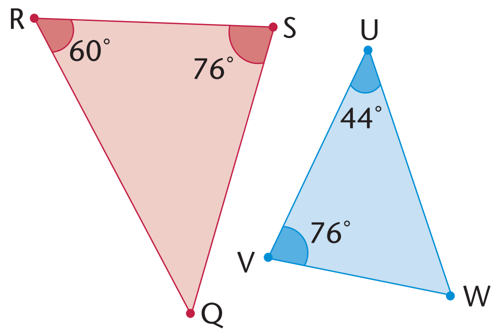
(c)



(d)



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2. Do the following task in your exercise book.
   - Use a ruler and protractor to construct the triangles described in (a) to (d) below.
   - Use your knowledge of similarity to draw the second triangle in each question.
   - Indicate the sizes of the corresponding sides and angles on the second triangle.

   (a) In ΔEFG, $\hat{G} = 75°$, EG = 4 cm and GF = 5 cm.
   ΔABC is an enlargement of ΔEFG, with its sides three times longer.

   (b) In ΔMNO, $\hat{M} = 45°$, $\hat{N} = 30°$ and MN = 5 cm.
   ΔPQR is similar to ΔMNO. The sides of ΔMNO to ΔPQR are in proportion 1 : 3.

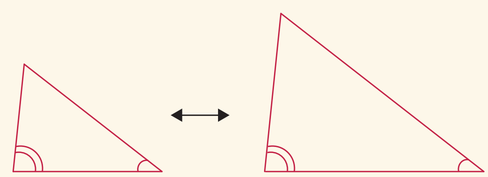
   (c) ΔRST is an isosceles triangle. $\hat{R} = 40°$, RS is 10 cm and RS = RT.
   ΔVWX is similar to ΔRST. The sides of ΔRST to ΔVWX are in proportion $1 : \frac{1}{2}$.

   (d) ΔKLM is right-angled at $\hat{L}$, LM is 7 cm and the hypotenuse is 12 cm.
   ΔXYZ is similar to ΔKLM, so that the sides are a third of the length of ΔKLM.

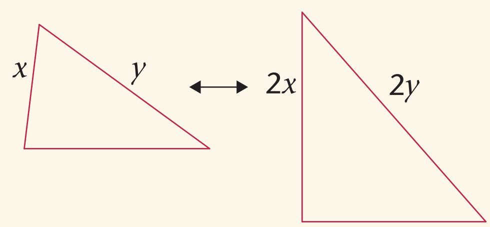
INVESTIGATION: MINIMUM CONDITIONS FOR SIMILARITY

Which of the following are minimum conditions for similar triangles?

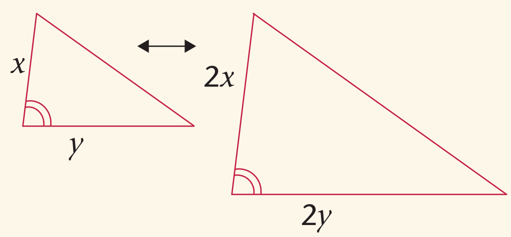
(a) Two angles in one triangle are equal to two angles in another triangle. ....



(b) Two sides of one triangle are in the same proportion as two sides in another triangle. ....



(c) Two sides of one triangle are in the same proportion as two sides in another triangle, and the angle between the two sides is equal to the angle between the corresponding sides. ....



(d) Two sides of one triangle are in the same proportion as two sides in another triangle, and one angle not between the two sides is equal to the corresponding angle in the other triangle. ....

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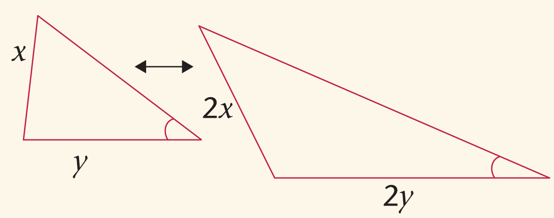
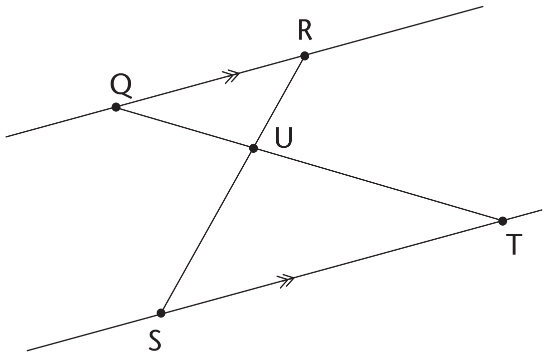


SOLVING PROBLEMS WITH SIMILAR TRIANGLES

1. Line segment QR is parallel to line segment ST.

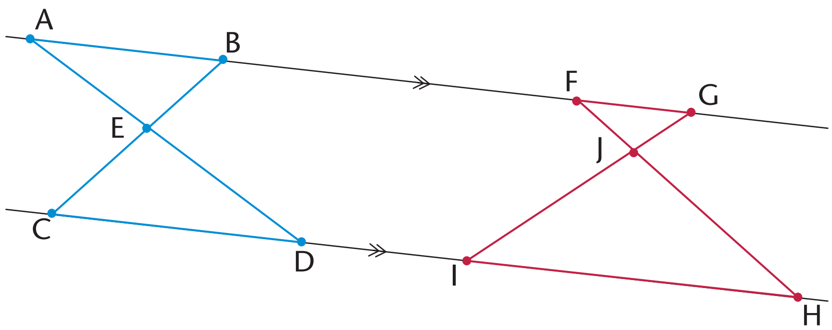


Parallel lines never meet. Two lines are parallel to each other if the distance between them is the same along the whole length of the lines.

Complete the following proof that ΔQRU ||| ΔTSU:

Statement	Reason
$R\hat{Q}T = Q\hat{T}S$	Alt. ∠s
$Q\hat{R}S =$	
=	Vert. opp. ∠s
∴ ΔQRU ||| ΔTSU	Equal ∠s (or AAA)

2. The following intersecting line segments form triangle pairs between parallel lines.



(a) Are the triangles in each pair similar? Explain.

....

(b) Write down pairs of similar triangles. ....

(c) Are triangles like these always similar? Explain how you can be sure without measuring every possible triangle pair.

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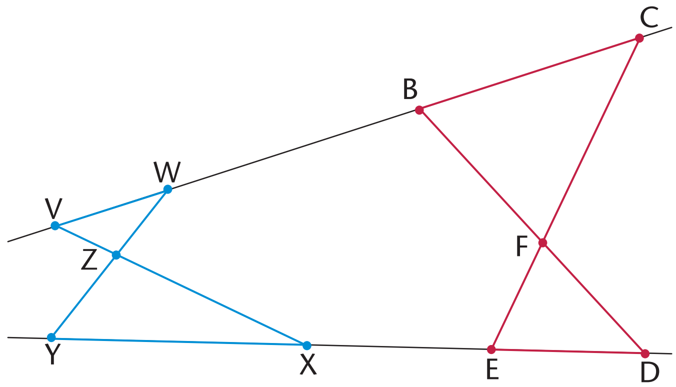
3. The intersecting lines on the right form triangle pairs between the line segments that are not parallel. Are these triangle pairs similar? Explain why or why not.



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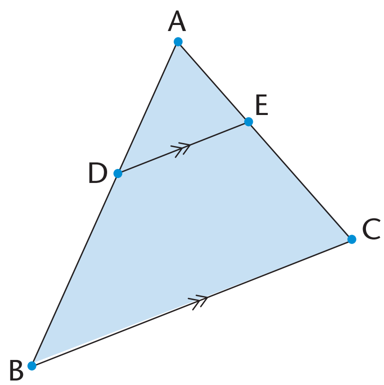
4. Consider the triangles below. DE || BC. Prove that ΔABC ||| ΔADE.

	Statement	Reason
	....	....
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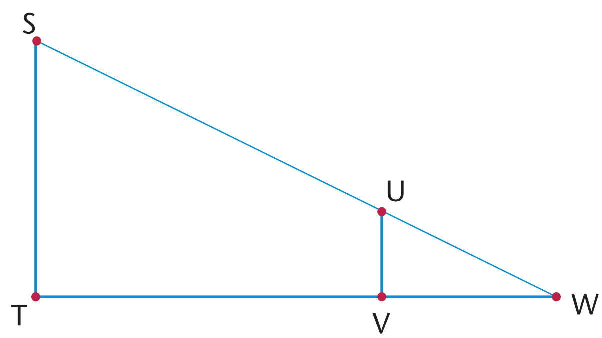
5. In the diagram on the right, ST is a telephone pole and UV is a vertical stick. The stick is 1 m high and it casts a shadow of 1,7 m (VW). The telephone pole casts a shadow of 5,1 m (TW). Use similar triangles to calculate the height of the telephone pole.



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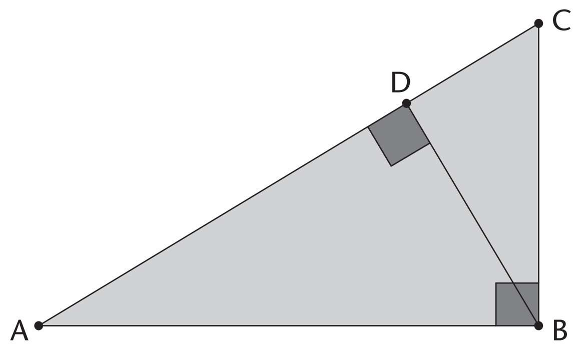
6. How many similar triangles are there in the diagram below? Explain your answer.



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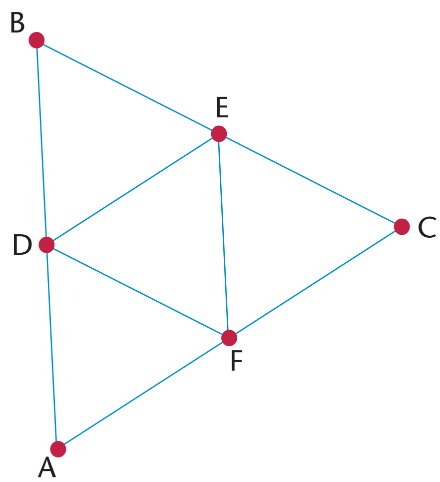
11.6 Extension questions

1. ΔABC on the right is equilateral. D is the midpoint of AB, E is the midpoint of BC and F is the midpoint of AC.



   (a) Prove that ΔBDE is an equilateral triangle.

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   (b) Find all the congruent triangles. Give a proof for each.

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(c) Name as many similar triangles as you can. Explain how you know they are similar.

....

(d) What is the proportion of the corresponding sides of the similar triangles?

....

(e) Prove that DE is parallel to AC.

....

....

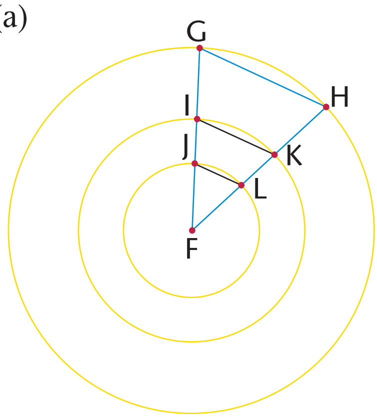
....

(f) Is DF parallel to BC? Is EF parallel to BA? Explain.

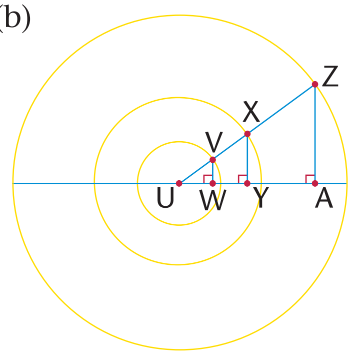
....

2. Consider the similar triangles drawn below using concentric circles. Explain why the triangles are similar in each diagram.

(a)



(b)



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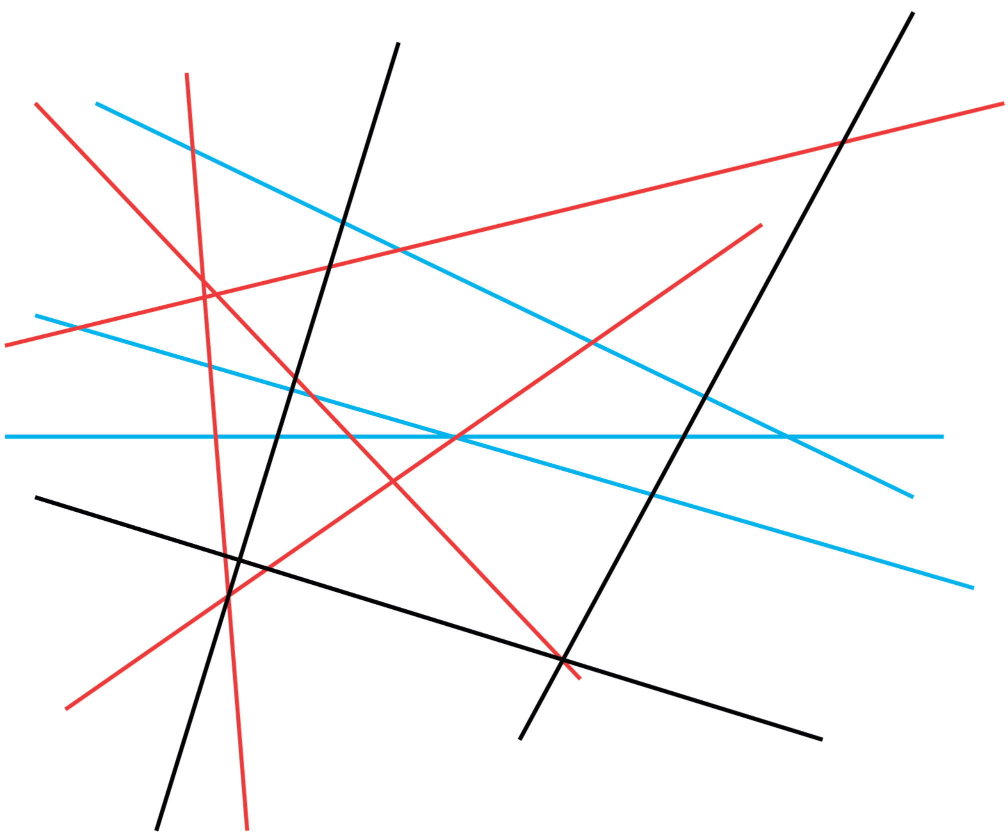
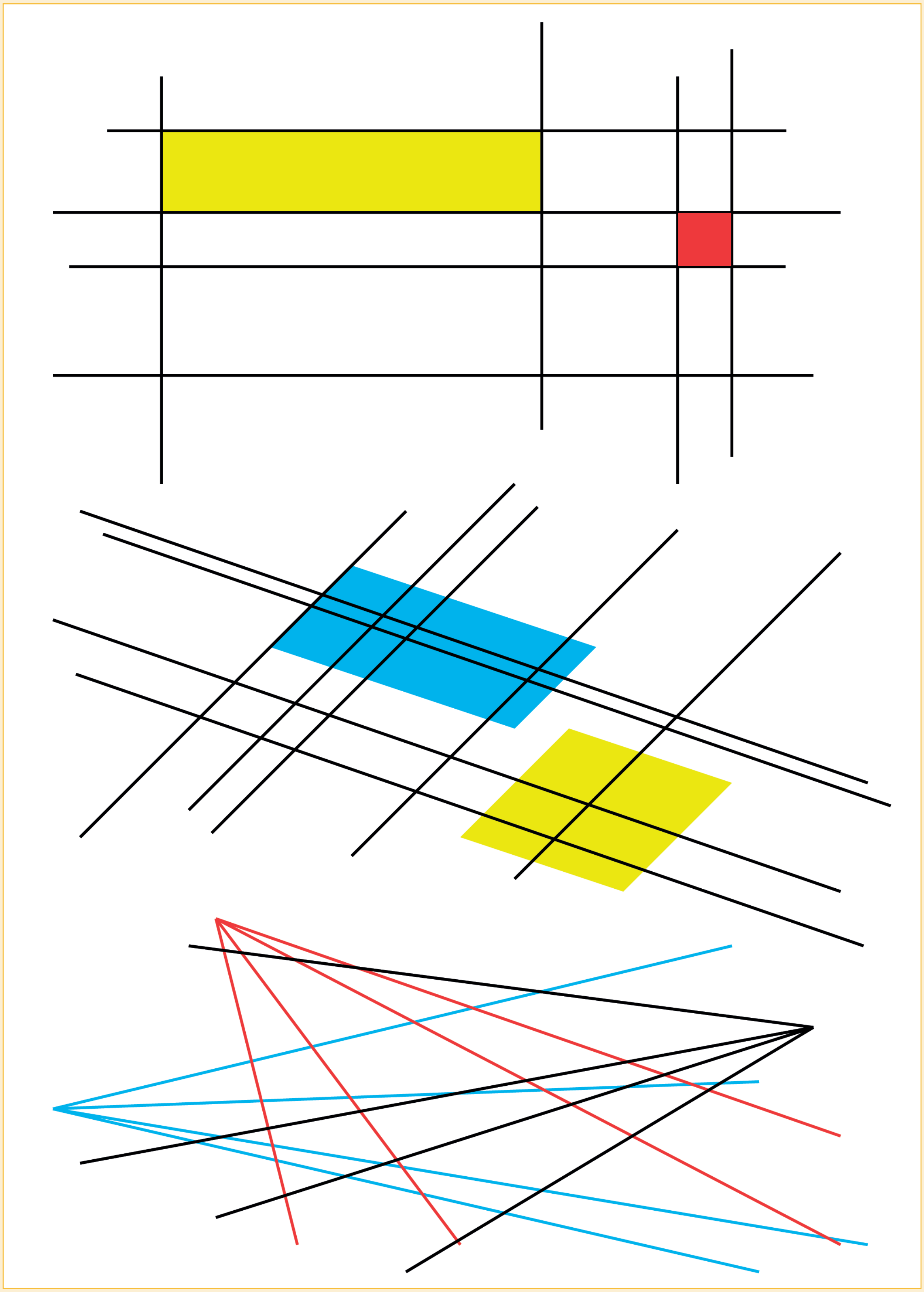
Chapter 12
Geometry of straight lines

In Grade 8 you identified relationships between angles on straight lines. In this chapter, you will revise all of the angle relationships and write clear descriptions of them.





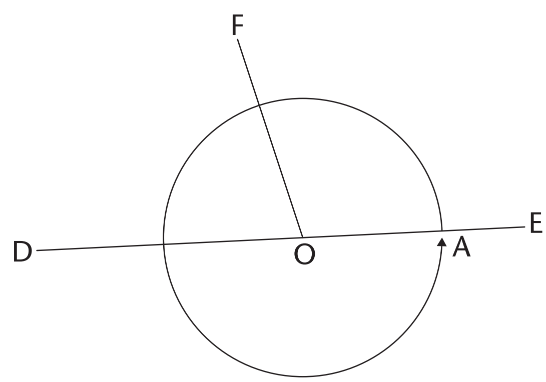
12 Geometry of straight lines

12.1 Angle relationships

Remember that 360° is one full revolution.

If you look at something and then turn all the way around so that you are looking at it again, you have turned through an angle of 360°. If you turn only halfway around, so that you look at something that was right behind your back, you have turned through an angle of 180°.

1. Answer the questions about the figure below.



(a) Is angle FOD in the figure smaller or bigger than a right angle?

....

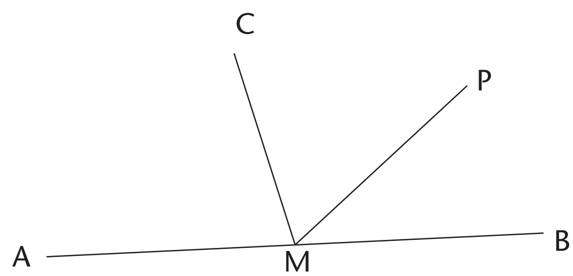
(b) Is angle FOE in the above figure smaller or bigger than a right angle?

....

On the figure above, $F\hat{O}D + F\hat{O}E$ = half of a revolution = 180°.

The sum of the angles on a straight line is 180°.
When the sum of angles is 180°, the angles are called **supplementary**.

2. $C\hat{M}A$ in the figure below is 75°.
AMB is a straight line.



(a) How big is $C\hat{M}B$? ....

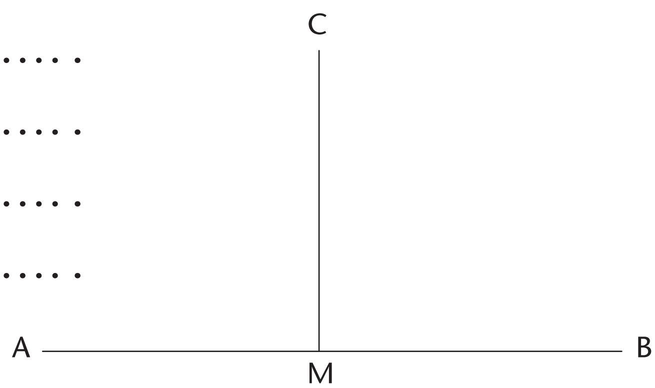
(b) Why do you say so? ....

3. $P\hat{M}B$ in the figure in question 2 is 40°.
   How big is $C\hat{M}P$? ....
   Explain your reasoning.

   ....

4. In the figure below, AMB is a straight line and $A\hat{M}C$ and $B\hat{M}C$ are equal angles.
   (a) How big are these angles? ....
   (b) How do you know this?

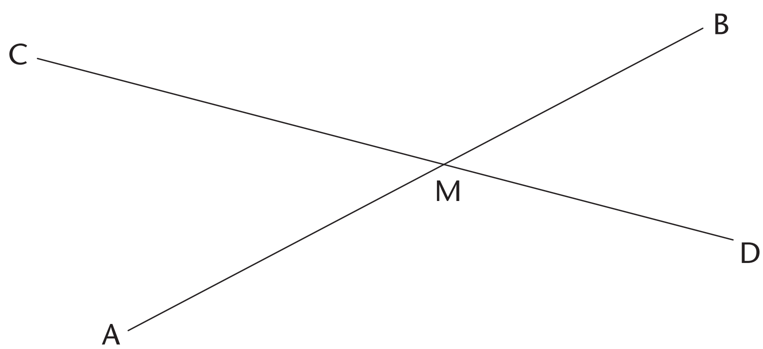
   ....
   ....
   ....
   ....



When one line forms two equal angles where it meets another line, the two lines are said to be **perpendicular**.

Because the two equal angles are angles on a straight line, their sum is 180°, hence each angle is 90°.

5. In the figure below, lines AB and CD intersect at point M.



   (a) Does it look as if $C\hat{M}A$ and $B\hat{M}D$ are equal? ....

In this chapter, you are required to give good reasons for every statement you make.

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(b) Can you explain why they are equal?

....

....

....

(c) What does $C\hat{M}A + D\hat{M}A$ equal? .... Why do you say so?

....

(d) What is $C\hat{M}A + C\hat{M}B$? .... Why do you say so?

....

(e) Is it true that $C\hat{M}A + D\hat{M}A = C\hat{M}A + C\hat{M}B$? ....

(f) Which angle occurs on both sides of the equation in (e)? ....

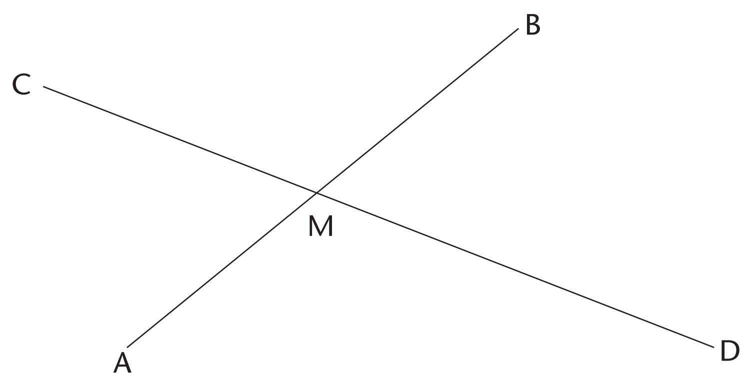
6. Look carefully at your answers to questions 5(c) to 5(e).
Now try to explain your observation in question 5(a).

....

....

7. In the figure below, AB and CD intersect in M. Four angles are formed. Angle CMB and angle AMD are called **vertically opposite** angles. Angle CMA and angle BMD are also **vertically opposite**.

When two straight lines intersect, the vertically opposite angles are equal.

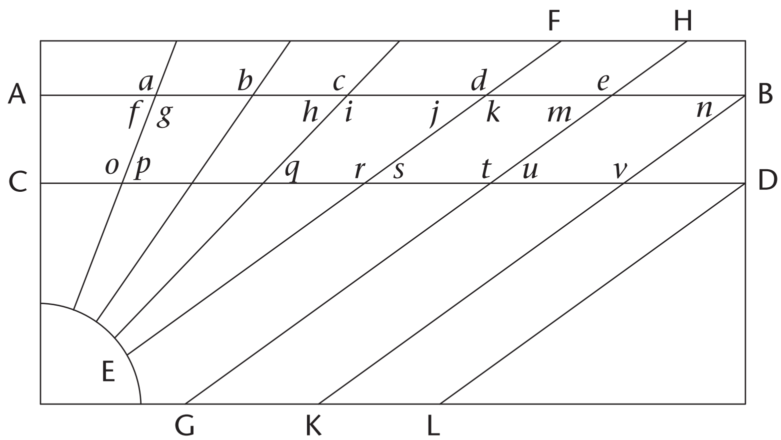


(a) If angle BMC = 125°, how big is angle AMD? ....

(b) Why do say so? ....

LINES AND ANGLES

A line that intersects other lines is called a **transversal**.



In the above pattern, AB is parallel to CD and EF || GH || KB || LD.

1. Angles *a*, *b*, *c*, *d*, and *e* are **corresponding angles**. Do the corresponding angles look appear to be equal?

 ....

2. Investigate whether the corresponding angles are equal by using tracing paper. Trace the angle you want to compare to other angles and place it on top of the other angle to find out if they are equal. What do you notice?

 ....

3. Angles *f*, *h*, *j*, *m* and *n* are also corresponding angles. Identify all the other groups of corresponding angles in the pattern.

 ....

 ....

4. Describe the position of corresponding angles that are formed when a transversal intersects other lines.

 ....

 ....

 ....

5. The following are pairs of **alternate angles**: *g* and *o*; *j* and *s*; and *k* and *r*. Do these angles appear to be equal?

....

6. Investigate whether the alternate angles are equal by using tracing paper. Trace the angle you want to compare and place it on top of the other angle to find out if they are equal. What do you notice?

....

....

7. Identify two more pairs of alternate angles.

....

8. Clearly describe the relative position of alternate angles that are formed when a transversal intersects other lines.

....

....

9. Did you notice something about some of the pairs of corresponding angles when you did the investigation in question 6? Describe your finding.

....

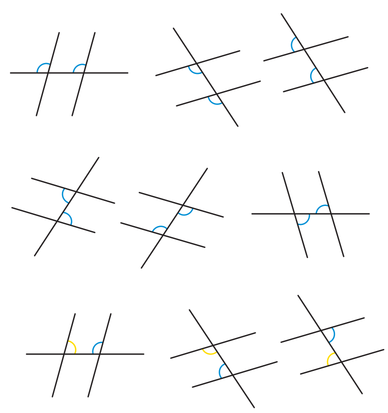
10. Angles *f* and *o*; *i* and *q* and *k* and *s* are all pairs of **co-interior angles**. Identify three more pairs of co-interior angles in the pattern.

....

The angles in the same relative position at each intersection where a straight line crosses two others are called **corresponding angles**.

Angles on different sides of a transversal and between two other lines are called **alternate angles**.

Angles on the same side of the transversal and between two other lines are called **co-interior angles**.

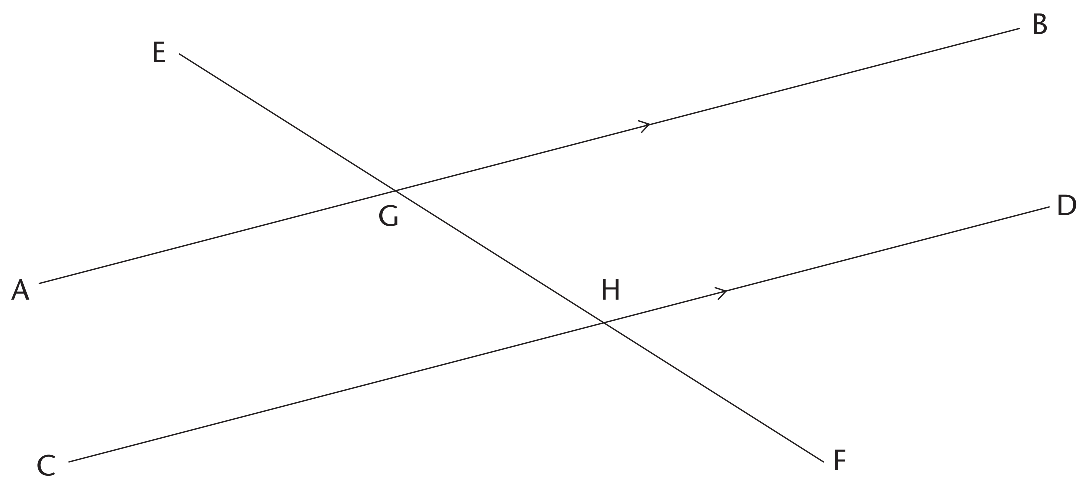


ANGLES FORMED BY PARALLEL LINES

Corresponding angles

The lines AB and CD below never meet. Lines that never meet and are at a fixed distance from one another are called parallel lines. We write AB || CD.

> Parallel lines have the same direction, i.e. they form **equal corresponding angles** with any line that intersects them.



The line EF cuts AB at G and CD at H.
EF is a transversal that cuts parallel lines AB and CD.

1. (a) Look carefully at the angles EGA and EHC in the above figure. They are called **corresponding angles**. Do they appear to be equal?

   ....

   (b) Measure the two angles to check whether they are equal. What do you notice?

   ....

2. Suppose $E\hat{G}A$ and $E\hat{H}C$ are really equal. Would $E\hat{G}B$ and $E\hat{H}D$ then also be equal? Give reasons to support your answer.

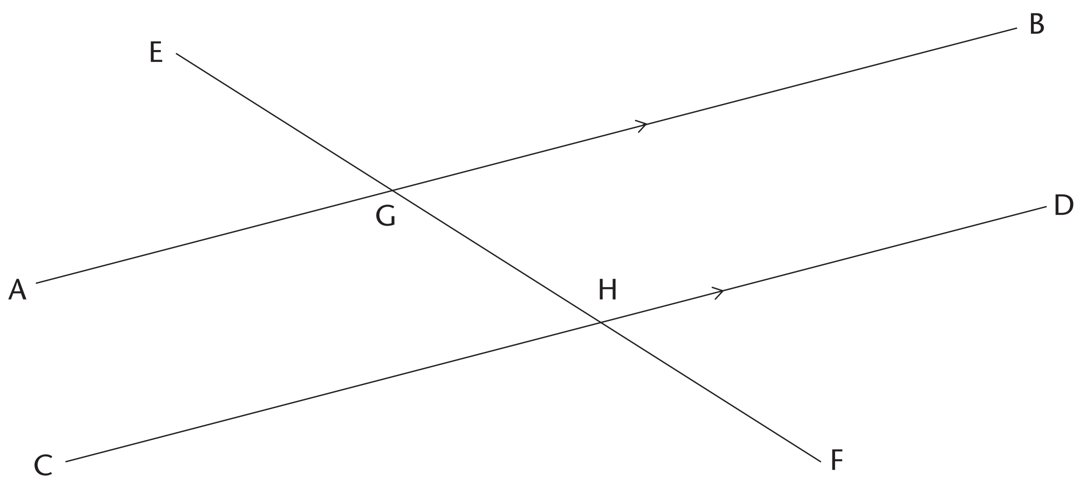
   ....

   ....

When two parallel lines are cut by a transversal, the corresponding angles are equal.

Alternate angles

The angles $B\hat{G}F$ and $C\hat{H}E$ below are called **alternate angles**. They are on opposite sides of the transversal.



3. Do you think angles AGF and DHE should also be called alternate angles?

....

....

4. Do you think alternate angles are equal? Investigate by using the tracing paper like you did previously, or measure the angles accurately with your protractor.
   What do you notice?

....

....

When parallel lines are cut by a transversal, the alternate angles are equal.

5. Try to explain why alternate angles are equal when the lines that are cut by a transversal are parallel, keeping in mind that corresponding angles are equal.

....

....

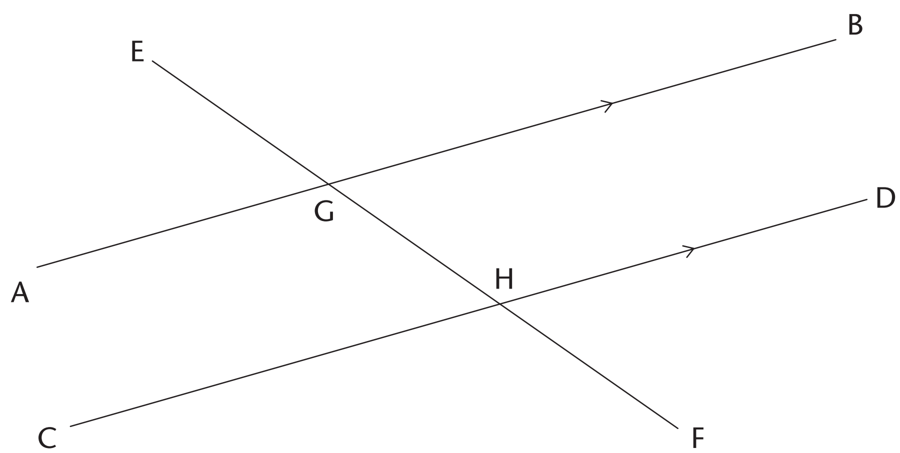
By answering the following questions, you should be able to see how you can explain why alternate angles are equal when parallel lines are cut by a transversal.

6. Are angles $B\hat{G}H$ and $D\hat{H}F$ in the figure corresponding angles? ....

   What do you know about corresponding angles?

   ....

   ....



7. (a) What can you say about $B\hat{G}H + A\hat{G}H$? Give a reason.

   ....

   (b) What can you say about $D\hat{H}G + C\hat{H}G$? Give a reason.

   ....

   (c) Is it true that $B\hat{G}H + A\hat{G}H = D\hat{H}G + C\hat{H}G$? Explain.

   ....

   (d) Will the equation in (c) still be true if you replace angle $B\hat{G}H$ on the left-hand side with angle $C\hat{H}G$?

   ....

   ....

8. Look carefully at your work in question 7 and write an explanation why alternate angles are equal, when two parallel lines are cut by a transversal.

   ....

   ....

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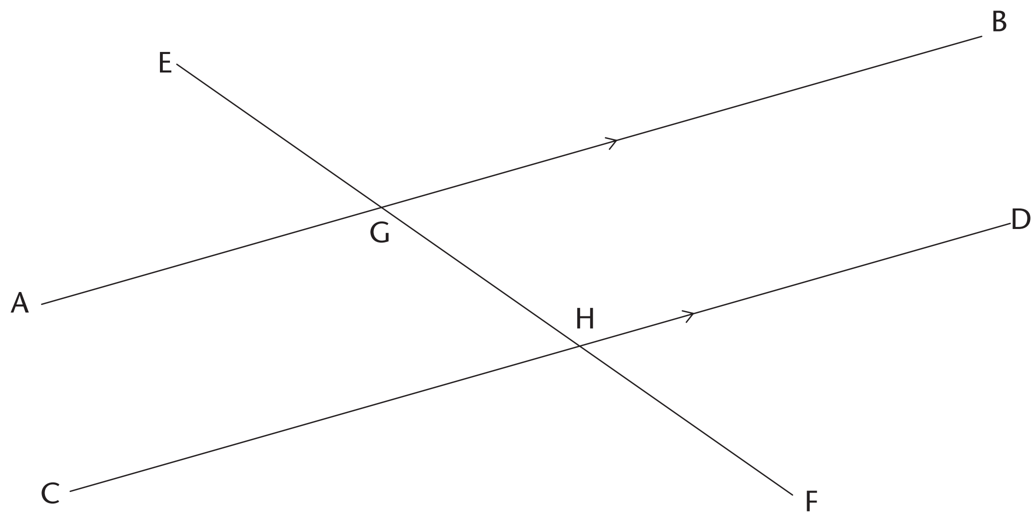
Co-interior angles

"co-" means together.
"co-interior" means on the same side.

The angles $A\hat{G}H$ and $C\hat{H}G$ in the figure below are called **co-interior angles**. They are on the same side of the transversal.



9. (a) What do you know about $C\hat{H}G + D\hat{H}G$? Explain.

....

(b) What do you know about $B\hat{G}H + A\hat{G}H$? Explain.

....

(c) What do you know about $B\hat{G}H + C\hat{H}G$? Explain.

....

(d) What conclusion can you draw about $A\hat{G}H + C\hat{H}G$? Give detailed reasons for your conclusion.

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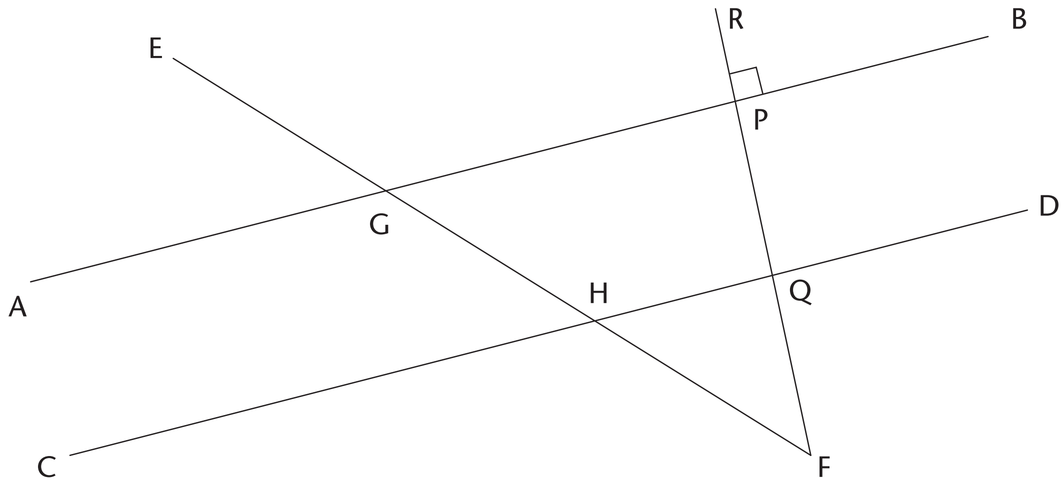
When two parallel lines are cut by a transversal, the sum of two co-interior angles is 180°.

Another way of saying this is to say that the two co-interior angles are **supplementary.**

12.2 Identify and name angles

1. In the figure below, the line RF is perpendicular to AB.



(a) Is RF also perpendicular to CD? Justify your answer.

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(b) Name four pairs of supplementary angles in the figure. In each case say how you know that the angles are supplementary.

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(c) Name four pairs of co-interior angles in the figure.

....

(d) Name four pairs of corresponding angles in the figure.

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....

(e) Name four pairs of alternate angles in the figure.

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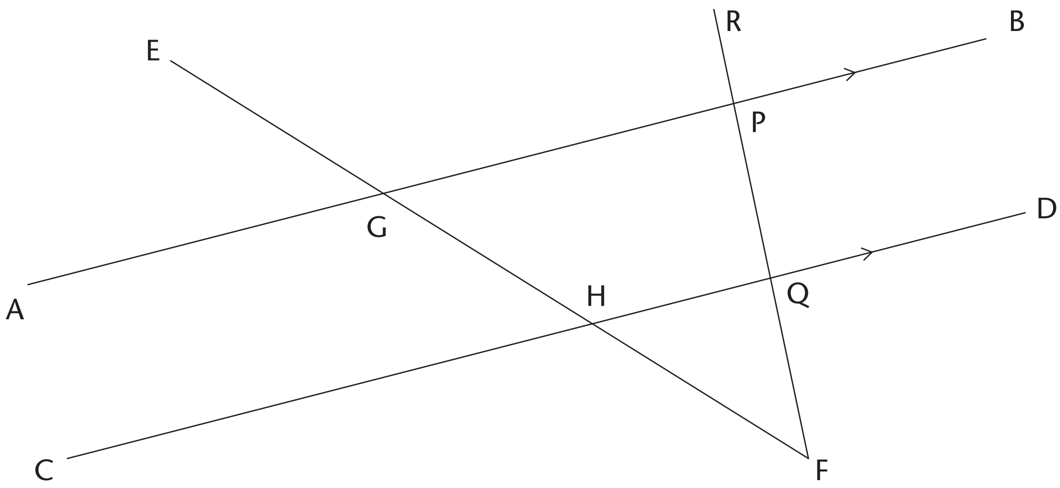
2. Now you are given that AB and CD in the figure below are parallel.



(a) If it is also given that RF is perpendicular to AB, will RF also be perpendicular to CD? Justify your answer.

....

....

(b) Name all pairs of supplementary angles in the figure. In each case say how you know that the angles are supplementary.

....

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(c) Suppose $E\hat{G}A = x$. Give the size of as many angles in the figure as you can, in terms of x. Each time give a reason for your answer.

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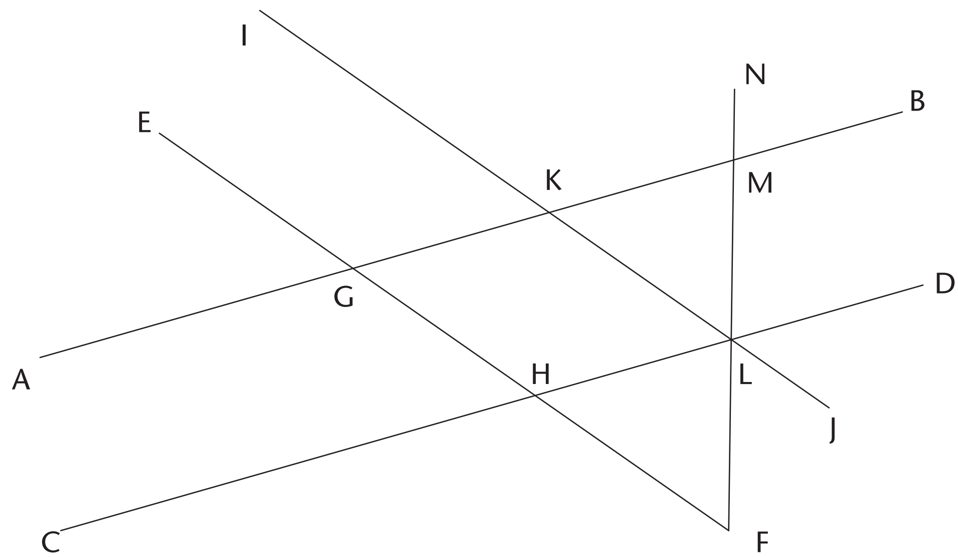
12.3 Solving problems

When you solve problems in geometry you can use a shorthand way to write your reasons. For example, if two angles are equal because they are corresponding angles, then you can write (corr ∠s, AB || CD) as the reason.

1. Line segments AB and CD in the figure below are parallel. EF and IJ are also parallel. Mark these facts on the figure, and then answer the questions.



(a) Name five angles in the figure that are equal to $G\hat{H}D$. Give a reason for each of your answers.

....

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(b) Name all the angles in the figure that are equal to $A\hat{G}H$. Give a reason for each of your answers.

....

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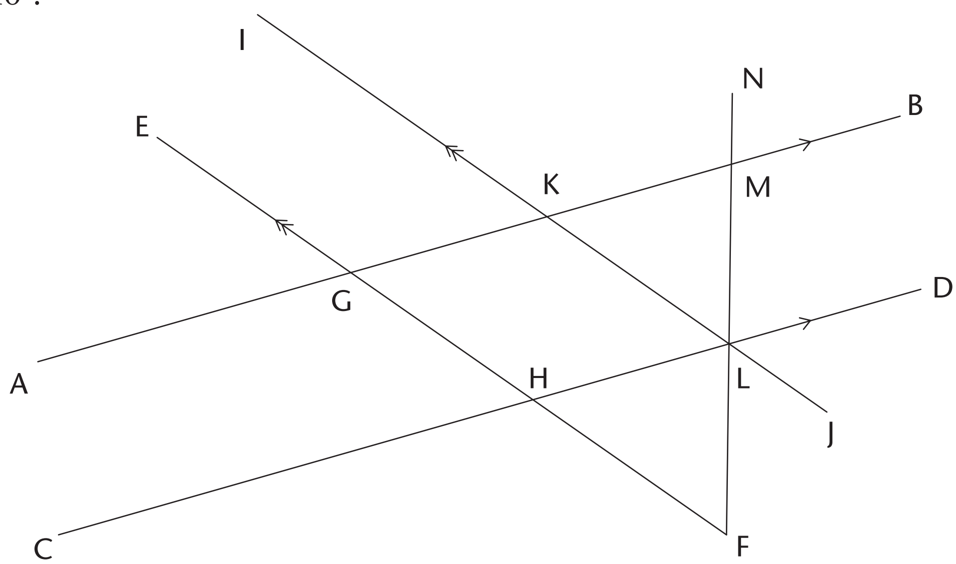
2. AB and CD in the figure below are parallel. EF and IJ are also parallel. $N\widehat{M}B = 80°$ and $J\hat{L}F = 40°$.



Find the sizes of as many angles in the figure as you can, giving reasons.

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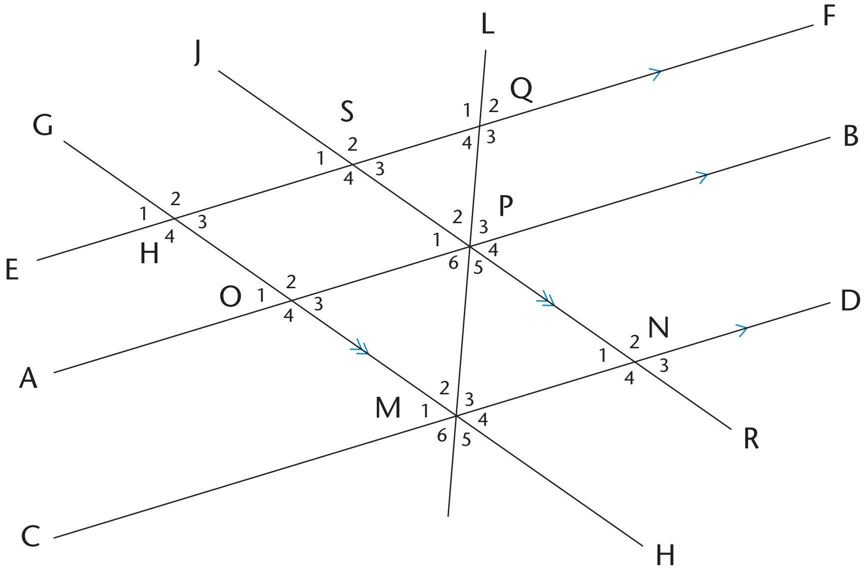
3. In the figure below, AB || CD; EF || AB; JR || GH. You are also given that $P\hat{M}N = 60°$, $R\hat{N}D = 50°$.



(a) Find the sizes of as many angles in the figure as you can, giving reasons.

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(b) Are EF and CD parallel? Give reasons for your answers.

....

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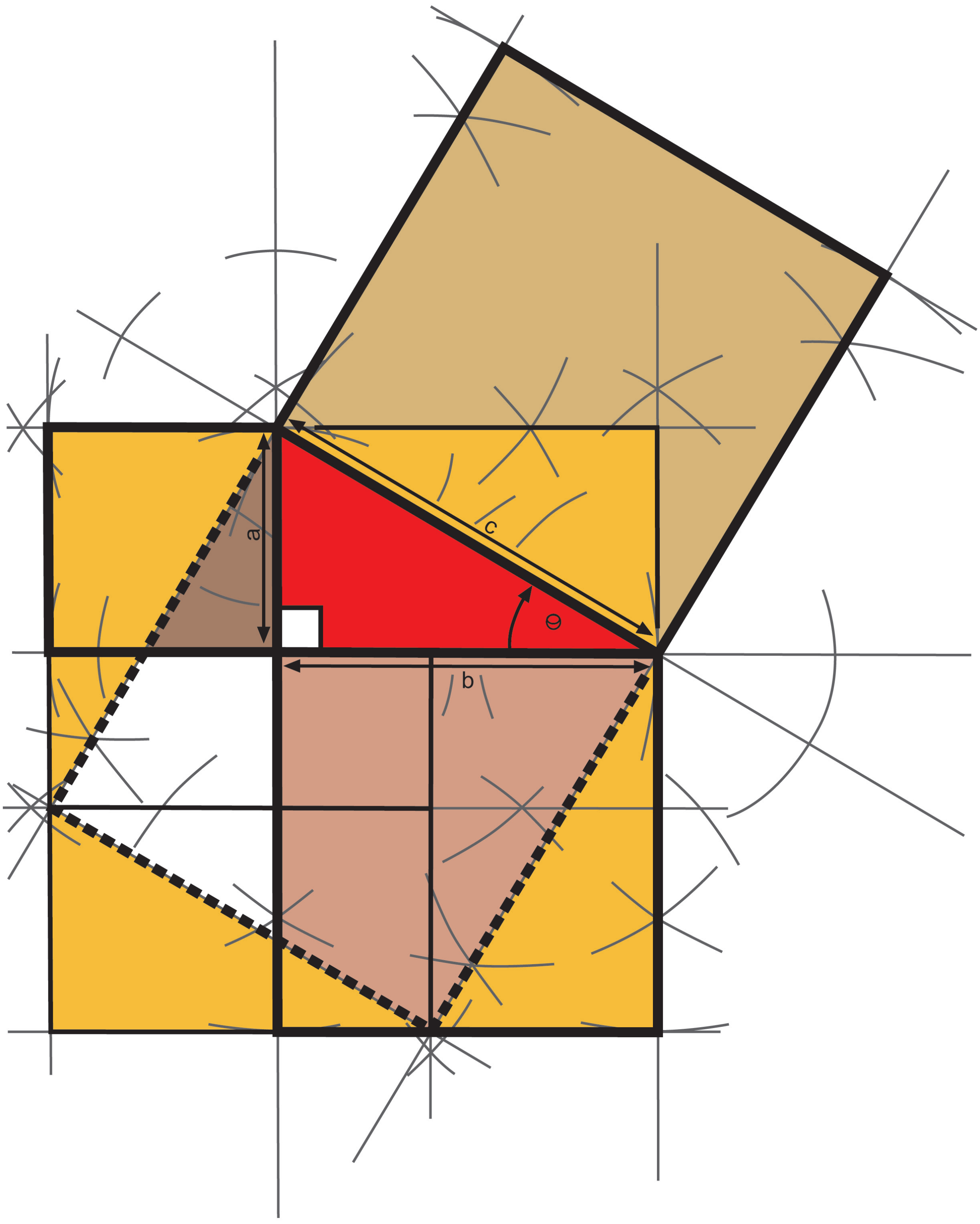
Chapter 13
Pythagoras' Theorem

In this chapter, you will revise what you have learnt about the Theorem of Pythagoras in Grade 8. You will investigate how the theorem is proved, what it means, and how to apply it in order to work out unknown lengths in right-angled triangles and other geometric figures.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c
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b

13 Pythagoras' Theorem

13.1 Investigating the sides of a right-angled triangle

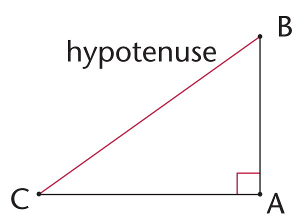
A **theorem** is a rule or a statement that has been proved through reasoning. Pythagoras' Theorem is a rule that applies only to **right-angled triangles**. The theorem is named after the Greek mathematician, Pythagoras.

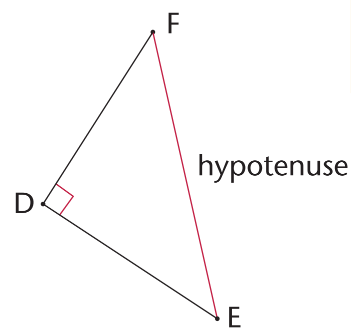
A right-angled triangle has one 90° angle. The longest side of the right-angled triangle is called the **hypotenuse**.

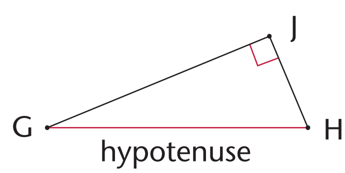
Pythagoras (569–475 BC)

Pythagoras was an influential mathematician. Like many Greek mathematicians of 2 500 years ago, he was also a philosopher and a scientist. He formulated the best-known theorem, today known as Pythagoras' Theorem.

However, the theorem had already been in use 1 000 years earlier, by the Chinese and the Babylonians.







The **hypotenuse** is the side opposite the 90° angle in a right-angled triangle. It is always the longest side.

How to say it:
'high - *pot* - eh - news'

INVESTIGATING SQUARES ON THE SIDES OF RIGHT-ANGLED TRIANGLES

1. The figure shows a right-angled triangle with squares on each of the sides.
   (a) Write down the areas of the following:

   Square A: ....

   Square B: ....

   Square C: ....

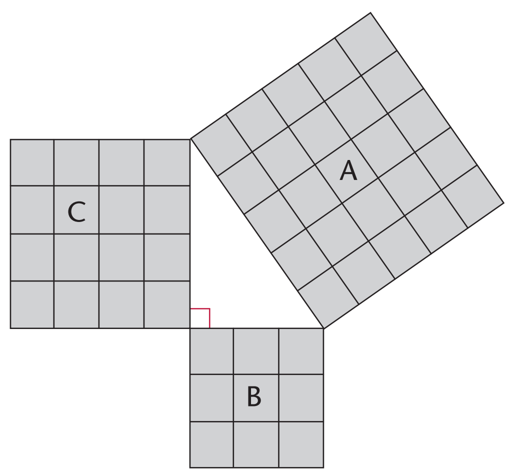
   (b) Add Area of square B + Area of square C:

   ....

   (c) What do you notice about the areas?

   ....

   ....



2. The figure below is similar to the one in question 1. The lengths of the sides of the right-angled triangle are 5 cm and 12 cm.
   (a) What is the length of the hypotenuse? Count the squares.

   ....

   (b) Use the squares to find the following:
   Area of A:

   ....

   Area of B:

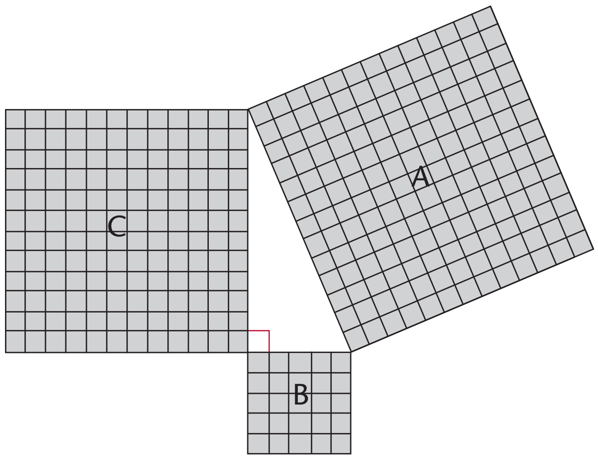
   ....

   Area of C:

   ....

   Area of B + Area of C:

   ....



   (c) What do you notice about the areas? Is it similar to your answer in 1(c)?

   ....

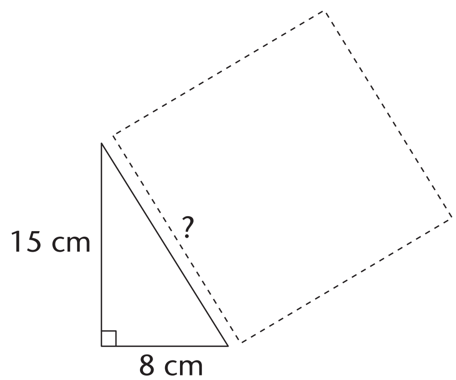
   ....

   ....

3. A right-angled triangle has side lengths of 8 cm and 15 cm. Use your findings in the previous questions to answer the following questions:
   (a) What is the area of the square drawn along the hypotenuse?

   ....

   ....



   ....

   ....

   ....

   (b) What is the length of the triangle's hypotenuse?

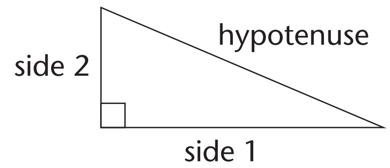
   ....

   ....

In the previous activity, you should have discovered Pythagoras' Theorem for right-angled triangles.

Pythagoras' Theorem says:
In a right-angled triangle, a square formed on the hypotenuse will have the same area as the sum of the area of the two squares formed on the other sides of the triangle. Therefore:

$(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$



13.2 Checking for right-angled triangles

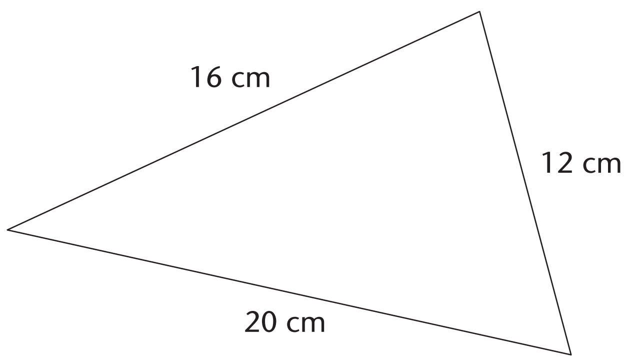
Pythagoras' Theorem applies in two ways:

- If a triangle is right-angled, the sides will have the following relationship: $(\text{Hypotenuse})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$
- If the sides have the relationship: $(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$, then the triangle is a right-angled triangle.

So we can test if any triangle is right-angled without using a protractor.

Example:

Is a triangle with sides 12 cm, 16 cm and 20 cm right-angled?



$(\text{Longest side})^2 = 20^2 = 400 \text{ cm}^2$

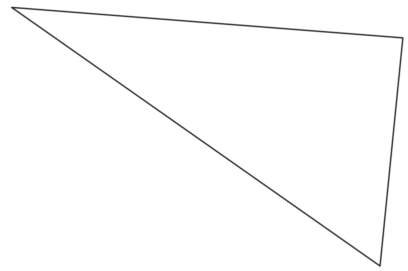
$(\text{Side } 1)^2 + (\text{Side } 2)^2 = 12^2 + 16^2 = 144 + 256 = 400 \text{ cm}^2$

$(\text{Longest side})^2 = (\text{Side } 1)^2 + (\text{Side } 2)^2$

$\therefore$ The triangle is right-angled.

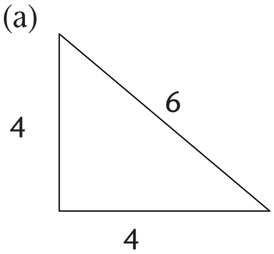
ARE THESE RIGHT-ANGLED TRIANGLES?

1. This triangle's side lengths are 29 mm, 20 mm and 21 mm. Prove that it is a right-angled triangle.

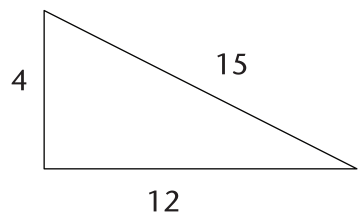


2. Use Pythagoras' Theorem to determine whether these triangles are right-angled. All values are in the same units.

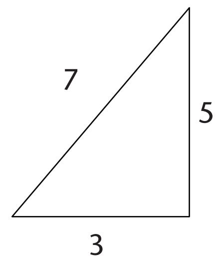
(a)



(b)



(c)



3. Determine whether the following side lengths would form right-angled triangles. All values are in the same units.

(a) 7, 9 and 12

(b) 7, 12 and 14

(c) 16, 8 and 10

(d) 6, 8 and 10

(e) 8, 15 and 17

(f) 16, 21 and 25

13.3 Finding missing sides

You can use the Theorem of Pythagoras to find the lengths of missing sides if you know that a triangle is right-angled.

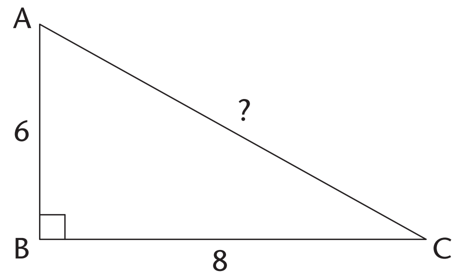
FINDING THE MISSING HYPOTENUSE

Example:

Calculate the length of the hypotenuse if the lengths of the other two sides are 6 units and 8 units.

ΔABC is right-angled, so:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 6^2 + 8^2 \text{ units}^2 \\ &= 36 + 64 \text{ units}^2 \\ &= 100 \text{ units}^2 \\ AC &= \sqrt{100} \text{ units} \\ &= 10 \text{ units} \end{aligned}$$



Sometimes the square root of a number is not a whole number or a simple fraction. In these cases, you can leave the answer under the square root sign. This form of the number is called a **surd**.

Surd form

You pronounce *surd* so that it rhymes with *word*.

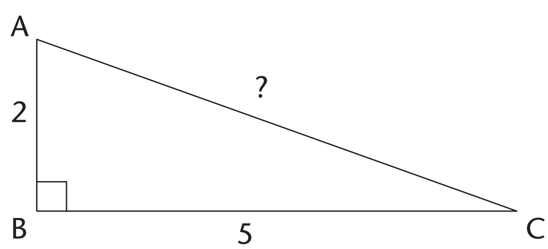
$\sqrt{5}$ is an example of a number in surd form.

$\sqrt{9}$ is not a surd because you can simplify it:

$\sqrt{9} = 3$

Example:

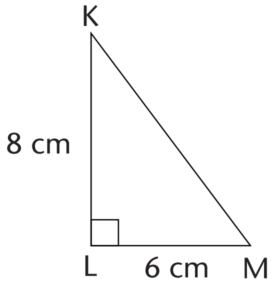
Calculate the length of the hypotenuse of ΔABC if $\hat{B} = 90°$, $AB = 2$ units and $BC = 5$ units. Leave your answer in surd form, where applicable. Remember when taking the square root that length is always positive.



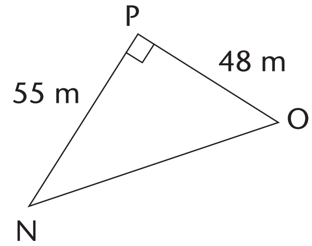
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 2^2 + 5^2 \text{ units}^2 \\ &= 4 + 25 \text{ units}^2 \\ &= 29 \text{ units}^2 \\ AC &= \sqrt{29} \text{ units} \end{aligned}$$

1. Find the length of the hypotenuse in each of the triangles below. Leave the answers in surd form where applicable.

(a)



(b)

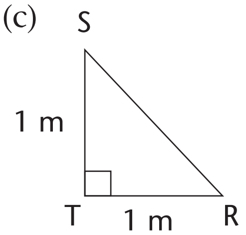


....

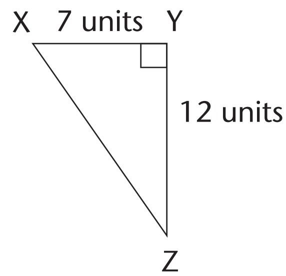
....

....

(c)



(d)



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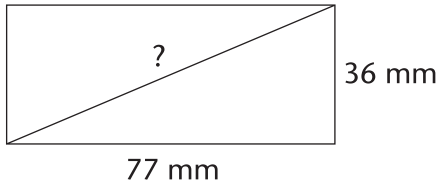
....

....

2. A rectangle has sides with lengths 36 mm and 77 mm. Find the length of the rectangle's diagonal.



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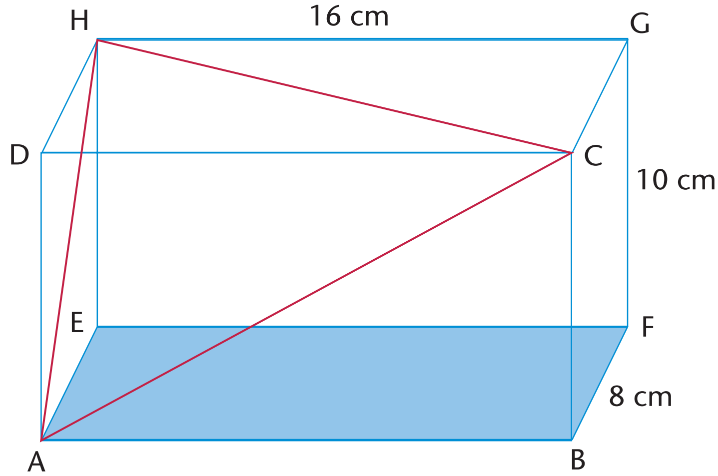
3. ΔABC has $\hat{A} = 90°$, AB = 3 cm and AC = 5 cm. Make a rough sketch of the triangle, and then calculate the length of BC.

....

....

....

4. A rectangular prism is made of glass. It has a length of 16 cm, a height of 10 cm and a breadth of 8 cm. ABCD and EFGH are two of its faces. ΔACH has been drawn inside the prism. Is ΔACH right-angled? Answer the questions to find out.



(a) Calculate the length of the sides of ΔACH. Note that all three sides of the triangles are diagonals of rectangles. AC is in rectangle ABCD, AH is in ADHE and HC is in HDCG.

....

....

....

(b) Is ΔACH right-angled? Explain your answer.

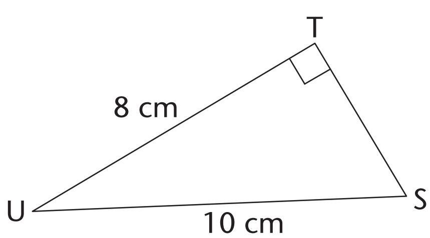
....

....

FINDING ANY MISSING SIDE IN A RIGHT-ANGLED TRIANGLE

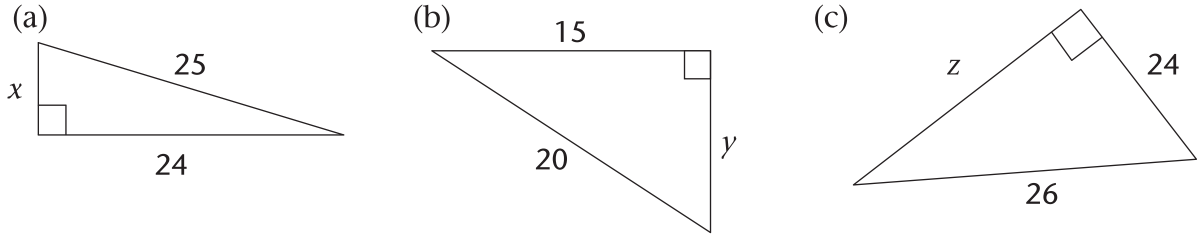
Example:

Find the length of TS in the triangle below.



$$US^2 = TU^2 + TS^2$$
$$10^2 = 8^2 + TS^2$$
$$100 = 64 + TS^2$$
$$36 = TS^2$$
$$\sqrt{36} = TS$$
$$\therefore TS = 6 \text{ cm}$$

1. In the right-angled triangles below, calculate the length of the sides that have not been given. Leave your answers in surd form where applicable.



....

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2. Calculate the length of the third side of each of the following right-angled triangles. First draw a rough sketch of each of the triangles before you do any calculations. Round off your answers to two decimal places.
   (a) ΔABC has AB = 12 cm, BC = 18 cm and $\hat{A} = 90°$. Calculate AC.

   ....

   ....

   ....

   ....

   (b) ΔDEF has $\hat{F} = 90°$, DE = 58 cm and DF = 41 cm. Calculate EF.

   ....

   ....

   ....

   ....

   (c) ΔJKL has $\hat{K} = 90°$, JK = 119 m, KL = 167 m. Calculate JL.

   ....

   ....

   ....

   ....

(d) ΔPQR has PQ = 2 cm, QR = 8 cm and $\hat{Q} = 90°$. Calculate PR.

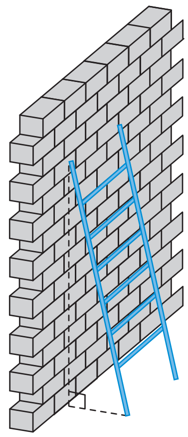
....

....

....

....

3. (a) A ladder of length 5 m is placed at an angle against a wall. The bottom of the ladder is 1 m away from the wall. How far up the wall will the ladder reach? Round off to two decimal places.



....

....

....

....

(b) If the ladder reaches a height of 4,5 m against the wall, how far away from the wall was it placed? Round off to two decimal places.

....

....

....

PYTHAGOREAN TRIPLES

Sets of **whole numbers** that can be used as the sides of a right-angled triangle are known as **Pythagorean triples**, for example:

3-4-5 5-12-13 7-24-25 16-30-34 20-21-29

You extend these triples by finding multiples of them. For examples, triples from the 3-4-5 set include the following:

3-4-5 6-8-10 9-12-15 12-16-20

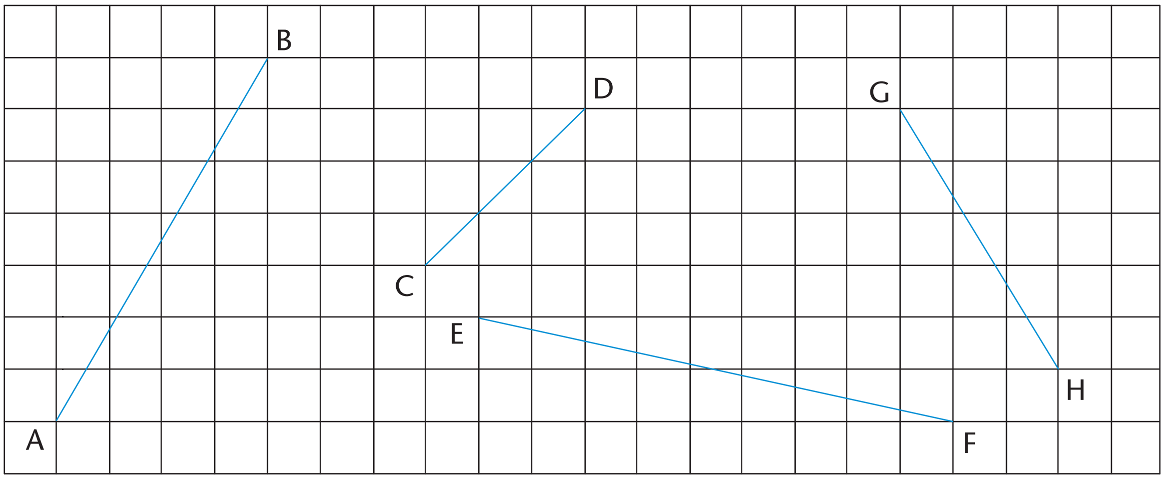
There are many old writings that record Pythagorean triples. For example, from 1900 to 1600 BC the Babylonians had already calculated very large Pythagorean triples, such as:

1 679-2 400-2 929.

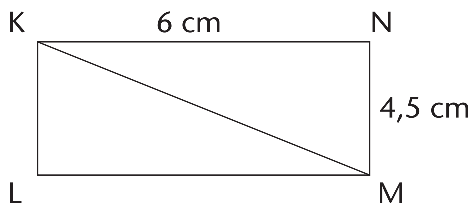
How many Pythagorean triples can you find? What is the largest one you can find that is not a multiple of another one?

13.4 More practice using Pythagoras' Theorem

1. Four lines have been drawn on the grid below. Each square is 1 unit long. Calculate the lengths of the lines: AB, CD, EF and GH. Do the calculations in your exercise book and write the answers below. Leave your answers in surd form.



2. (a) Calculate the area of rectangle KLMN.



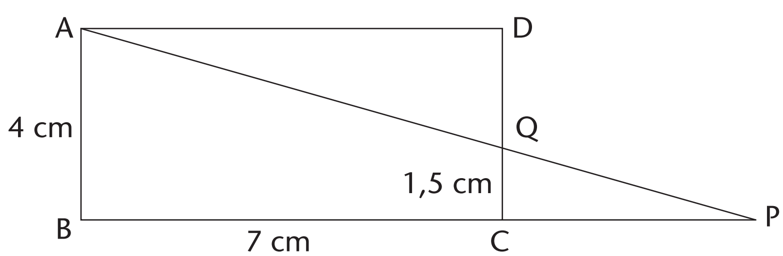
....

(b) Calculate the perimeter of ΔKLM.

....

....

3. ABCD is a rectangle with AB = 4 cm, BC = 7 cm and CQ = 1,5 cm. Round off your answers to two decimal places if the answers are not whole numbers.



(a) What is the length of QD?

....

(b) If CP = 4,2 cm, calculate the length of PQ.

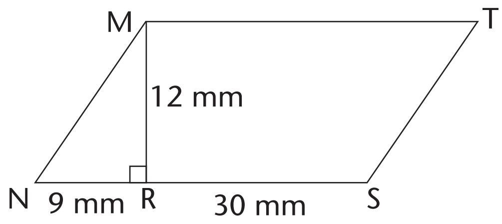
....

(c) Calculate the length of AQ and the area of ΔAQD.

....

....

4. MNST is a parallelogram. NR = 9 mm and MR = 12 mm.



(a) Calculate the area of ΔMNR.

....

(b) Calculate the perimeter of MNST.

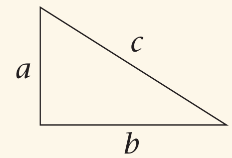
....

....

PYTHAGORAS' THEOREM AND OTHER TYPES OF TRIANGLES

Pythagoras' Theorem works only for right-angled triangles. But we can also use it to find out whether other triangles are acute or obtuse, as follows.

- **If the square of the longest side is *less* than the sum of the squares of the two shorter sides, the *biggest angle is acute*.**
  For example, in a 6-8-9 triangle: $6^2 + 8^2 = 100$ and $9^2 = 81$.
  81 is less than 100 ∴ the 6-8-9 triangle is acute.
- **If the square of the longest side is *more* than the sum of the squares of the two shorter sides, the *biggest angle is obtuse*.**
  For example, in a 6-8-11 triangle: $6^2 + 8^2 = 100$ and $11^2 = 121$.
  121 is more than 100 ∴ the 6-8-11 triangle is obtuse.



Complete the following table. It is based on the triangle on the right.
Decide whether each triangle described is right-angled, acute or obtuse.

a	b	c	$a^2 + b^2$	c^2	Fill in =, > or <	Type of triangle
3	5	6	$3^2 + 5^2 = 9 + 25 = 34$	$6^2 = 36$	$a^2 + b^2 < c^2$	Acute
2	4	6			$a^2 + b^2$ ..... c^2	
5	7	9			$a^2 + b^2$ ..... c^2	
12	5	13			$a^2 + b^2$ ..... c^2	
12	16	20	$12^2 + 16^2 = 144 + 256 = 400$	$20^2 = 400$	$a^2 + b^2 = c^2$	Right-angled
7	9	11			$a^2 + b^2$ ..... c^2	
8	12	13			$a^2 + b^2$ ..... c^2	

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WORKSHEET

1. Write down Pythagoras' Theorem in the way that you best understand it.

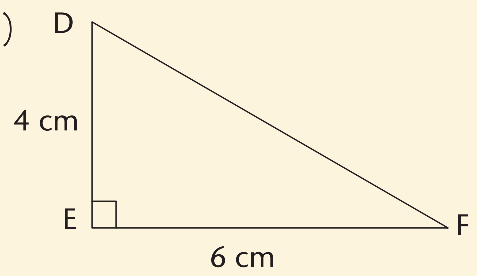
....

....

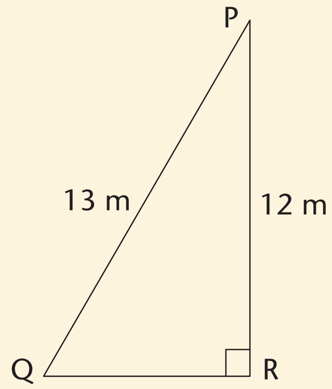
....

2. Calculate the lengths of the missing sides in the following triangles. Leave the answers in surd form if necessary.

(a)



(b)



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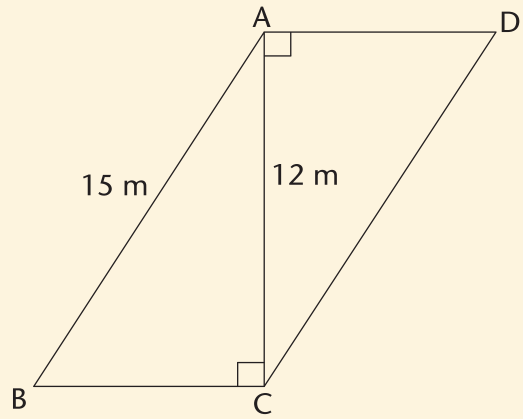
3. ABCD is a parallelogram.



(a) Calculate the perimeter of ABCD.

....

....

....

(b) Calculate the area of ABCD.

....

....

....

Chapter 14
Area and perimeter of 2D shapes

In this chapter, you will revise how to calculate the perimeter and area of squares, rectangles, triangles and circles. The perimeter of a shape is the distance all the way around the sides of the shape. The area of a shape is the flat space inside the shape. You will also learn how to calculate the areas of parallelograms, rhombi, kites and trapeziums, as well as investigate the effect on the perimeter and area of a shape when its dimensions are doubled.

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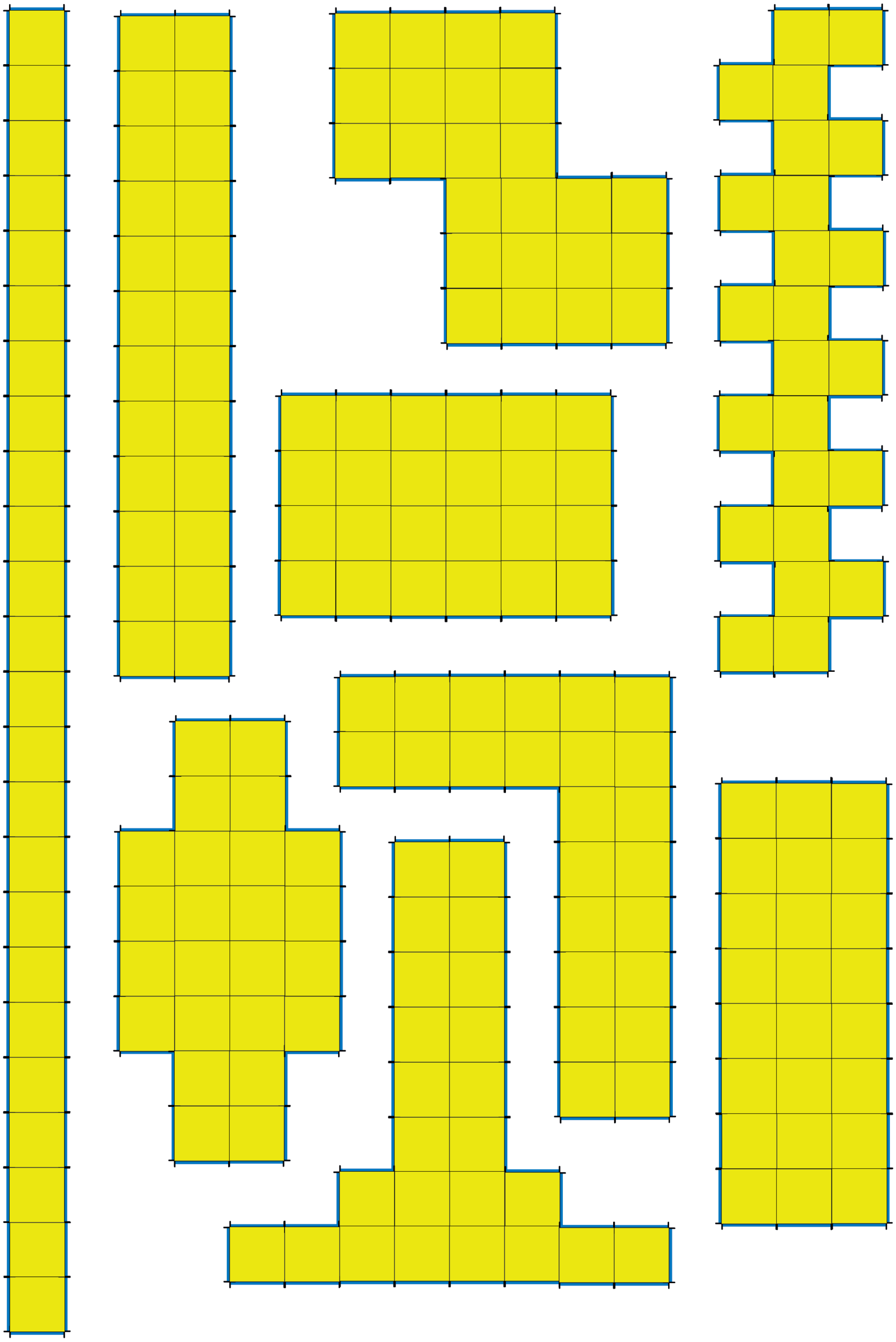


14 Area and perimeter of 2D shapes

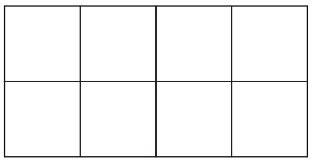
14.1 Area and perimeter of squares and rectangles

REVISING CONCEPTS

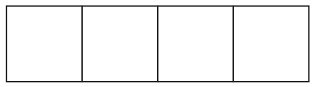
1. Each block in figures A to F below measures 1 cm × 1 cm. What is the perimeter and area of each of the figures?
   Complete the table below.

> The **perimeter** (*P*) of a shape is the distance along the sides of the shape. The **area** (*A*) of a figure is the size of the flat surface enclosed by the figure.

A

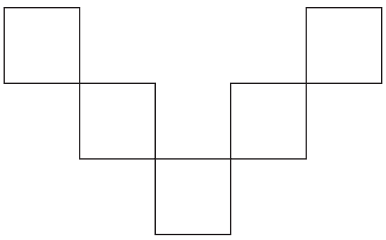


B

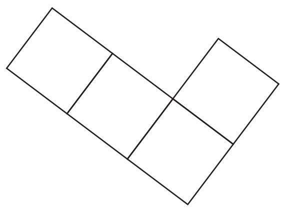


C

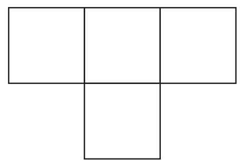
D



E



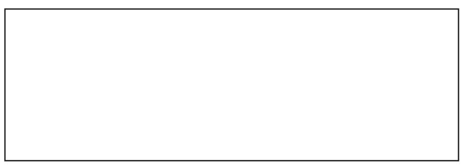
F



G

6 cm

2 cm



H

2 cm

2 cm

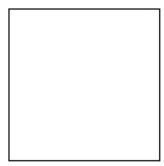


Figure	Perimeter	Area	Number of 1 cm × 1 cm squares
A			
B			
C			
D			
E			
F			
G			
H			

2. Consider the rectangle below. It is formed by tessellating identical squares that are 1 cm by 1 cm each. The white part has squares that are hidden.

   (a) Write down, without counting, the total number of squares that form this rectangle, including those that are hidden.
   Explain your reasoning.

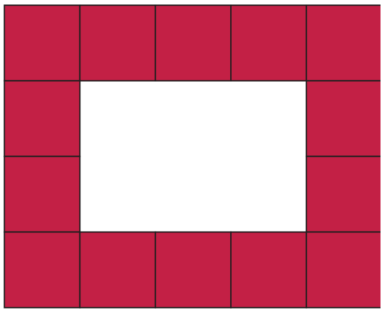
   ....

   ....

   (b) What is the area of the rectangle, including the white part?

   ....

To **tessellate** means to cover a surface with identical shapes in such a way that there are no gaps or overlaps. Another word for tessellating is **tiling**.



Area of a rectangle = length × breadth
= $l \times b$

Area of a square = $l \times l$
= l^2

Both length (l) and breadth (b) are expressed in the same unit.

3. Sipho and Theunis each paint a wall to earn some money during the school holidays. Sipho paints a wall 4 m high and 10 m long. Theunis's wall is 5 m high and 8 m long. Who should be paid more? Explain.

   ....

   ....

   ....

4. What is the area of a square with a length of 12 mm?

   ....

   ....

5. The area of a rectangle is 72 cm^2 and its length is 8 cm. What is its breadth?

   ....

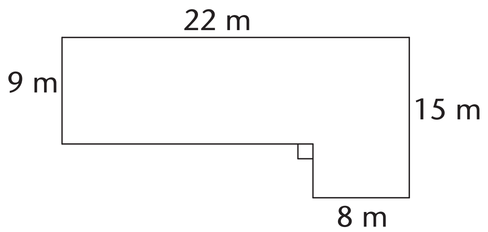
   ....

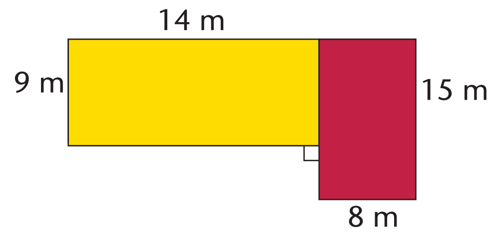
   ....

14.2 Area and perimeter of composite figures

BREAKING UP FIGURES AND PUTTING THEM BACK TOGETHER AGAIN

1. The diagram on the left below shows the floor plan of a room.
   (a) We can calculate the area of the room by dividing the floor into two rectangles, as shown in the diagram on the right below.





Area of the room = Area of yellow rectangle + Area of red rectangle
= $(l \times b) + (l \times b)$
= $(14 \times 9) + (15 \times 8)$
= $126 + 120$
= $246\ \text{m}^2$

   (b) The yellow part of the room has a wooden floor and the red part is carpeted. What is the area of the wooden floor? What is the area of the carpet?

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   (c) Calculate the area of the room using two different shapes. Draw a sketch.

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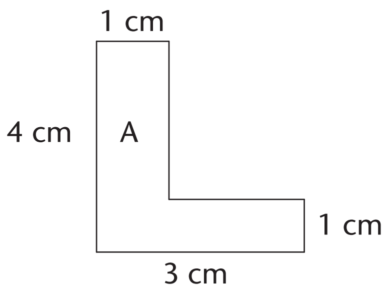
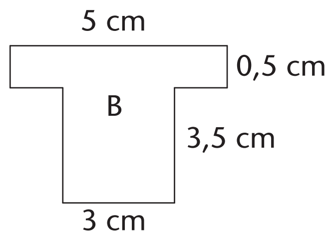
2. Calculate the area of the figures below.





3. Which of the following rules can be used to calculate the perimeter (P) of a rectangle? Explain.
   - Perimeter $= 2 \times (l + b)$
   - Perimeter $= l + b + l + b$
   - Perimeter $= 2l + 2b$
   - Perimeter $= l + b$

l and *b* refer to the length and the breadth of a rectangle.

The following are equivalent expressions for perimeter:
$P = 2l + 2b$ and $P = 2(l + b)$ and $P = l + b + l + b$

4. Check with two classmates that the rule or rules you have chosen above correct; then apply it to calculate the perimeter of figure A. Think carefully!

5. The perimeter of a rectangle is 28 cm and its breadth is 6 cm. What is its length?

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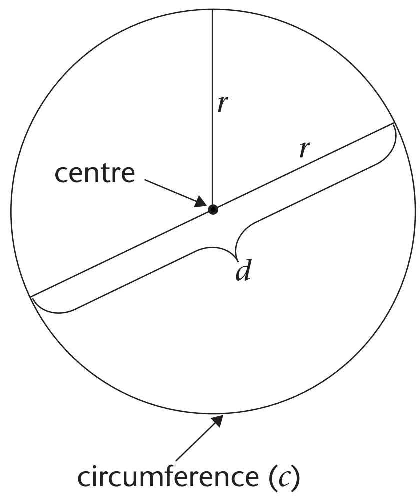
14.3 Area and perimeter of circles

REVISING CONCEPTS FROM PREVIOUS GRADES

The perimeter of a circle is called the **circumference** of a circle. You will remember the following about circles from previous grades:



- The distance across the circle through its centre is called the **diameter** (d) of the circle.
- The distance from the centre of the circle to any point on the circumference is called the **radius** (r).
- The circumference (c) of a circle divided by its diameter is equal to the irrational value we call **pi** (π). To simplify calculations, we often use the approximate values: $\pi \approx 3{,}14$ or $\frac{22}{7}$.

The following are important formulae to remember:

- $d = 2r$ and $r = \frac{1}{2}d$
- Circumference of a circle (c) = $2\pi r$
- Area of a circle (A) = πr^2

CIRCLE CALCULATIONS

In the following calculations, use $\pi = 3{,}14$ and round off your answers to two decimal places. If you take a square root, remember that length is always positive.

1. Calculate the perimeter and area of the following circles:

   (a) A circle with a radius of 5 m

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   (b) A circle with a diameter of 18 mm

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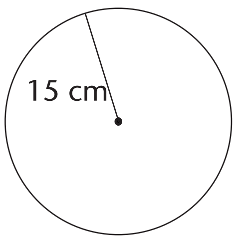
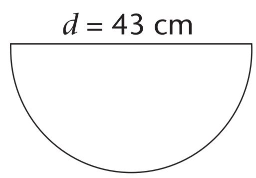
2. Calculate the radius of a circle with:
   (a) a circumference of 53 cm
   (b) a circumference of 206 mm

3. Work out the area of the following shapes:

A

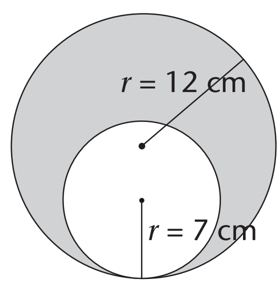


B



4. Calculate the radius and diameter of a circle with:
   (a) an area of 200 m^2
   (b) an area of 1 000 m^2

5. Calculate the area of the shaded part.



14.4 Converting between units

CONVERTING BETWEEN UNITS USED FOR PERIMETER AND AREA

Always make sure that you use the correct units in your calculations. Practise the conversions below.

Remember:
1 cm = 10 mm   1 mm = 0,1 cm
1 m = 100 cm   1 cm = 0,01 m
1 km = 1 000 m   1 m = 0,001 km

1. Convert the following:
   (a) 34 cm = ... mm
   (b) 501 m = ... km
   (c) 226 m = ... cm
   (d) 0,58 km = ... m
   (e) 1,9 cm = ... mm
   (f) 73 mm = ... cm
   (g) 924 mm = ... m
   (h) 32,23 km = ... m

Remember, to convert between square units, you can use method shown below:

To convert cm^2 to m^2:

$$1\ cm^2 = 1\ cm \times 1\ cm$$
$$= 0,01\ m \times 0,01\ m$$
$$= 0,0001\ m^2$$

Example

Convert 50 cm^2 to m^2

$$1\ cm^2 = 0,0001\ m^2$$
$$\therefore 50\ cm^2 = 50 \times 0,0001\ m^2$$
$$= 0,005\ m^2$$

2. Convert to cm^2:
   (a) 650 mm^2
   (b) 1 200 mm^2

   ....
   ....

   (c) 18 m^2
   (d) 0,045 m^2

   ....
   ....

   (e) 93 mm^2
   (f) 177 m^2

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3. (a) Convert 93 mm^2 to m^2.
   (b) Convert 0,017 km^2 to m^2.

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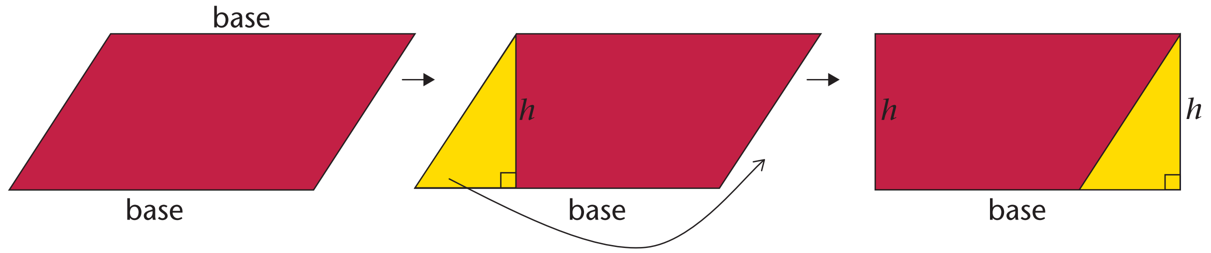
14.5 Area of other quadrilaterals

PARALLELOGRAMS

As shown below, a parallelogram can be made into a rectangle if a right-angled triangle from one side is cut off and moved to its other side.



So we can find the area of a parallelogram using the formula for the area of a rectangle:

Area of rectangle = $l \times b$

= (base of parallelogram) × (perpendicular height of parallelogram)

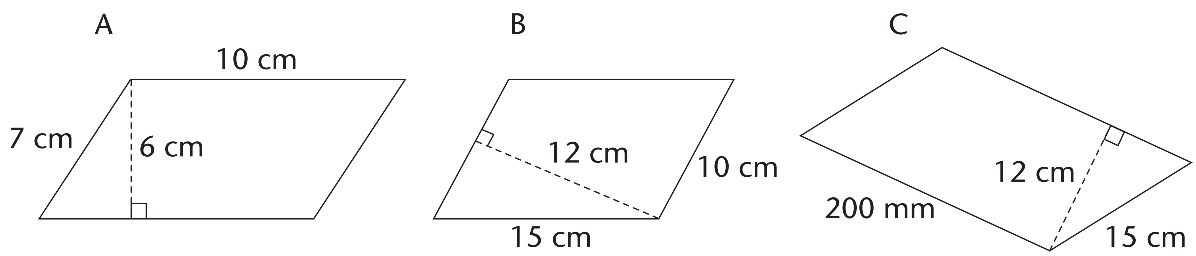
Area of parallelogram = Area of rectangle

∴ Area of parallelogram = base × perp. height

> We can use any side of the parallelogram as the base, but we must use the perpendicular height on the side we have chosen.

1. (a) Copy the parallelogram above into your exercise book.
   (b) Using the shorter side as the base of the parallelogram, follow the steps above to derive the formula for the area of a parallelogram.

2. Work out the area of the following parallelograms using the formula.



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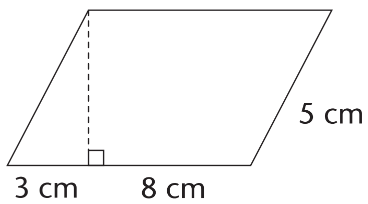
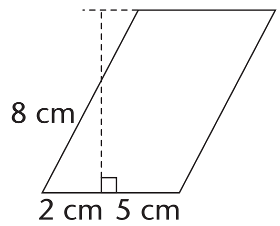
3. Work out the area of the parallelograms. Use the Theorem of Pythagoras to calculate the unknown sides you need. Remember to use the pre-rounded value for height and then round the final answer to two decimal places where necessary.





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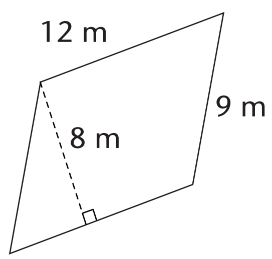

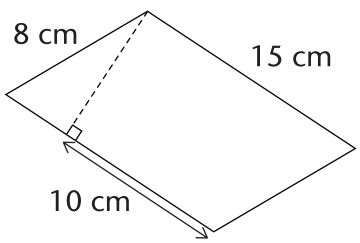

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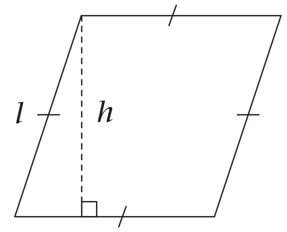
RHOMBI

A rhombus is a parallelogram with all sides equal.

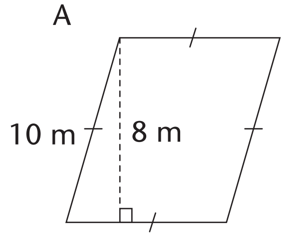
In the same way we derived the formula for the area of a parallelogram, we can show the following:



- Area of a rhombus = length × perp. height

1. Work in your exercise book. Show how to derive the formula for the area of a rhombus.
2. Calculate the areas of the following rhombi. Round off answers to two decimal places where necessary.

A



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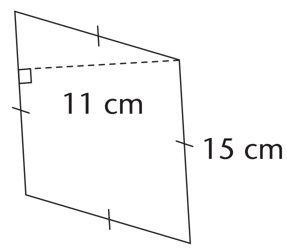

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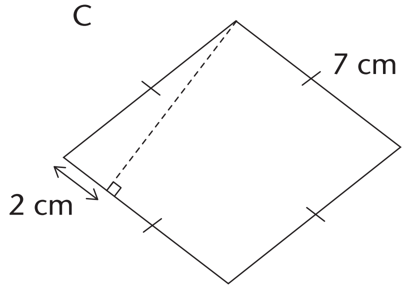

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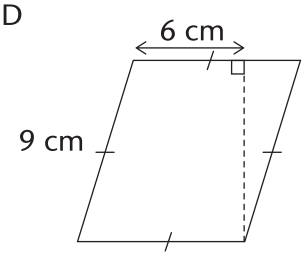

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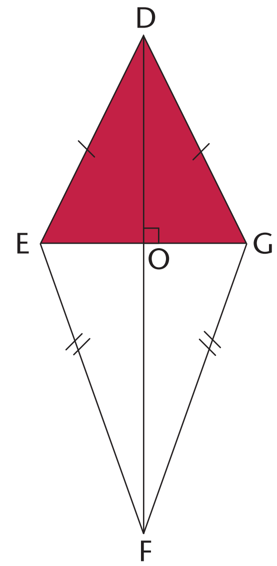
KITES

To calculate the area of a kite, you use one of its properties, namely that the diagonals of a kite are perpendicular.



Area of kite DEFG = Area of ΔDEG + Area of ΔEFG

$$= \frac{1}{2}(b \times h) + \frac{1}{2}(b \times h)$$

$$= \frac{1}{2}(EG \times OD) + \frac{1}{2}(EG \times OF)$$

$$= \frac{1}{2}EG(OD + OF)$$

$$= \frac{1}{2}EG \times DF$$

Notice that EG and DF are the diagonals of the kite.

$\therefore$ Area of a kite = $\frac{1}{2}$(diagonal 1 × diagonal 2)

1. Calculate the area of kites with the following diagonals. Give your answers in m^2.
   (a) 150 mm and 200 mm
   (b) 25 cm and 40 cm

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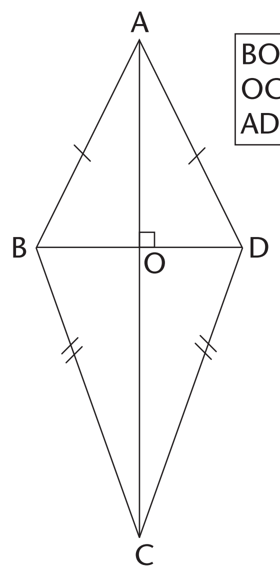
2. Calculate the area of the kite.



BO = OD = 6 cm
OC = 15 cm
AD = 10 cm

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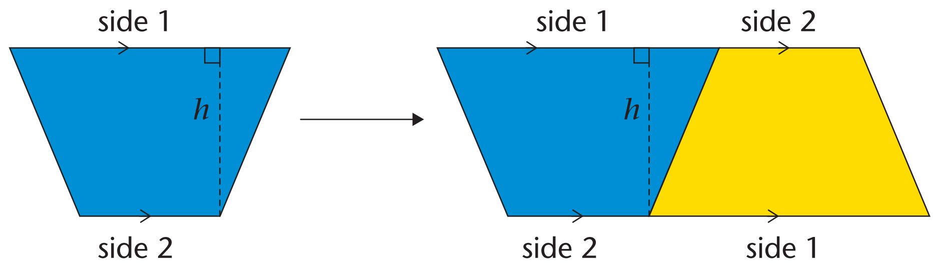
TRAPEZIUMS

A trapezium has two parallel sides. If we tessellate (tile) two trapeziums as shown in the diagram below, we form a parallelogram. (The yellow trapezium is the same size as the blue one. The base of the parallelogram is equal to the sum of the parallel sides of the trapezium.)



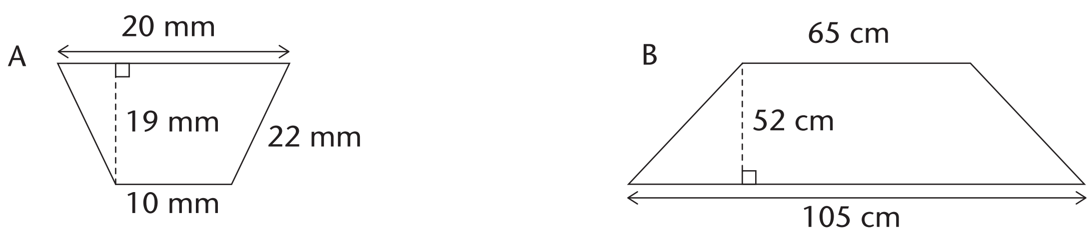
We can use the formula for the area of a parallelogram to work out the formula for the area of a trapezium as follows:

$$\begin{aligned}\text{Area of parallelogram} &= \text{base} \times \text{height}\\ &= (\text{side 1} + \text{side 2}) \times \text{height}\end{aligned}$$

$$\begin{aligned}\text{Area of trapezium} &= \frac{1}{2}\ \text{area of parallelogram}\\ &= \frac{1}{2}(\text{side 1} + \text{side 2}) \times \text{height}\end{aligned}$$

$\therefore$ Area of a trapezium = $\frac{1}{2}$ (sum of parallel sides) × perp. height

Calculate the area of the following trapeziums:



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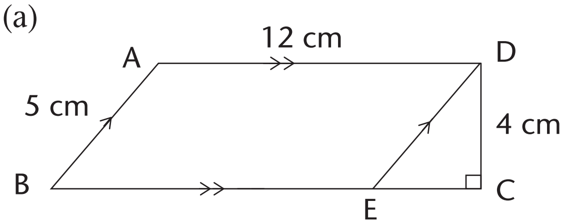
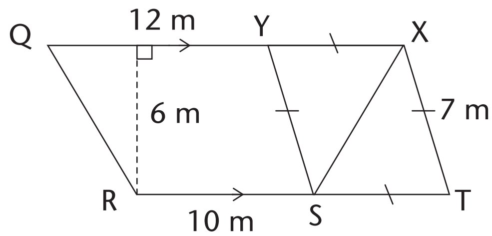
AREAS OF COMPOSITE SHAPES

Calculate the areas of the following 2D shapes. Round off your answers to two decimal places where necessary.

(a)



(b)



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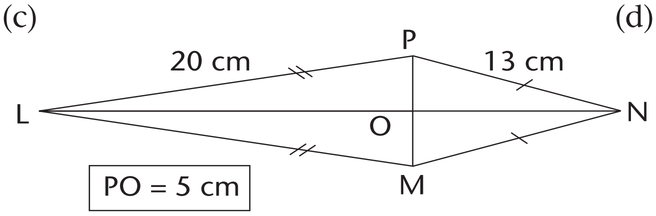
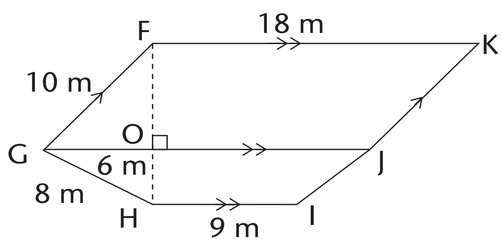
(c)



(d)



14.6 Doubling dimensions of a 2D shape

Remember that a 2D shape has two dimensions, namely length and breadth. You have used lengths and breadths in different forms, to work out the perimeters and areas of shapes, for example:

- length and breadth for rectangles and squares
- bases and perpendicular heights for triangles, rhombi and parallelograms
- two diagonals for kites.

But how does doubling one or both of the dimensions of a figure affect the figure's perimeter and area?

Doubling means to multiply by 2.

The four sets of figures on the next page are drawn on a grid of squares. Each row shows an original figure, the figure with one of its dimensions doubled, and the figure with both of its dimensions doubled. Each square has a side of 1 unit.

1. Work out the perimeter and area of each shape. Round off your answers to two decimal places where necessary.
2. Which figure in each set is congruent to the original figure?

   ....

3. Fill in the perimeter (*P*) and area (*A*) of each figure in the table below.

Figure	Original figure	Figure with both dimensions doubled
A	P = ... A = ...	P = ... A = ...
B	P = ... A = ...	P = ... A = ...
C	P = ... A = ...	P = ... A = ...
D	P = ... A = ...	P = ... A = ...

4. Look at the completed table above. What patterns do you notice? Choose one:
   - When both dimensions of a shape are doubled, its **perimeter is doubled** and its **area is doubled**.
   - When both dimensions of a shape are doubled, its **perimeter is doubled** and its area is **four times bigger**.

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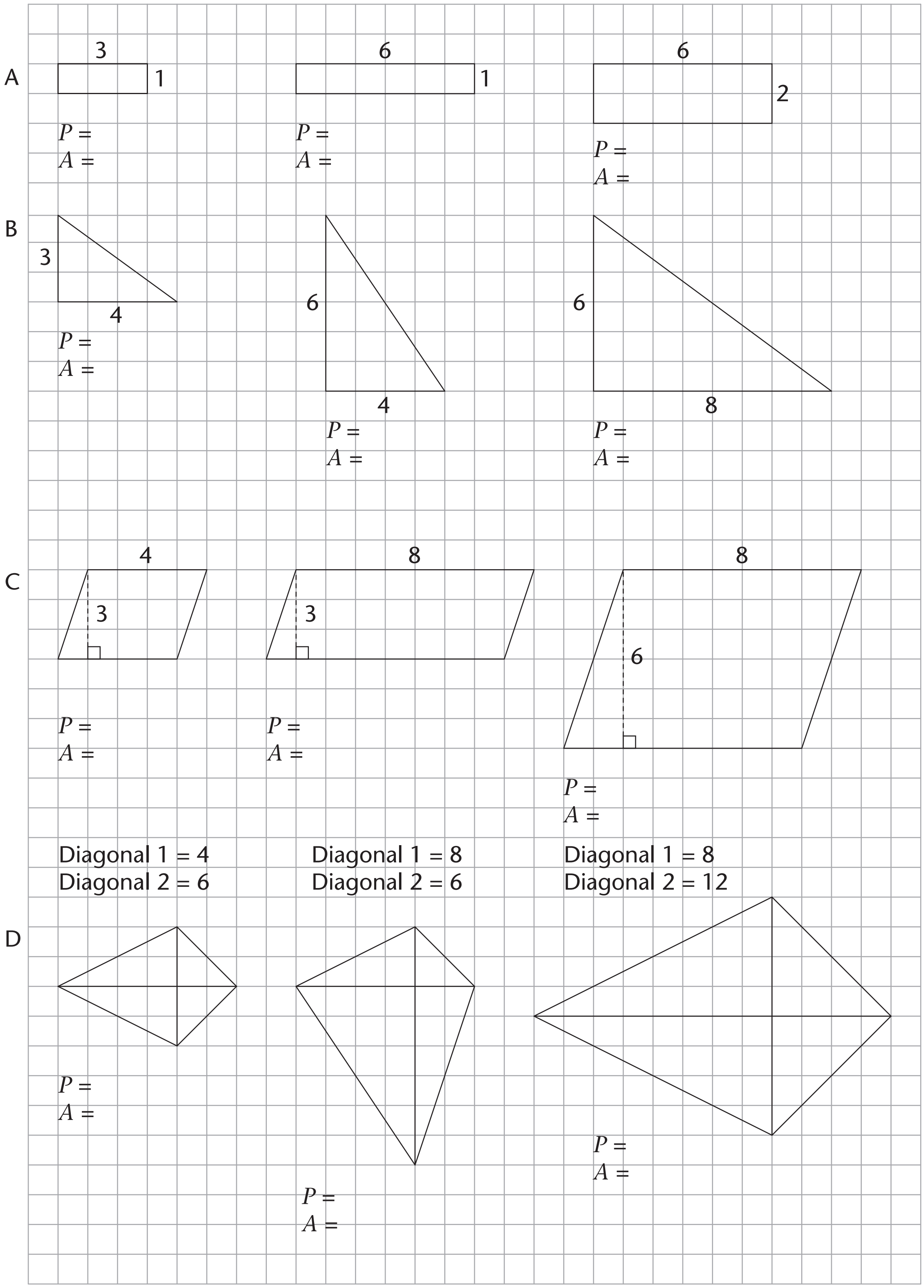
Original figure	One dimension doubled	Both dimensions doubled

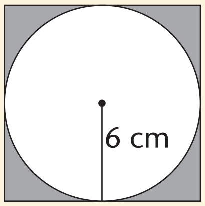


WORKSHEET

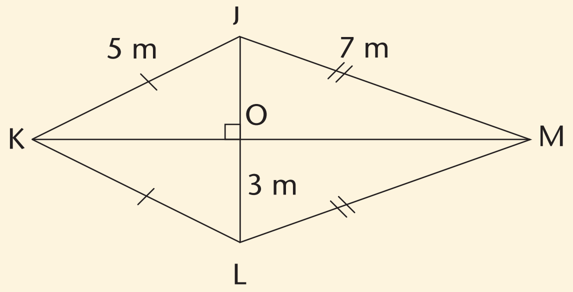
1. Write down the formulae for the following:

Perimeter of a square	
Perimeter of a rectangle	
Area of a square	
Area of a rectangle	
Area of a triangle	
Area of a rhombus	
Area of a kite	
Area of a parallelogram	
Area of a trapezium	
Diameter of a circle	
Circumference of a circle	
Area of a circle	

2. (a) Calculate the perimeter of the square and the area of the shaded parts of the square.



(b) Calculate the area of the kite.



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TERM 2
Revision and assessment

Revision

Remember to show all your steps of your working. Note that the diagrams drawn in this revision and assessment are not drawn to scale.

CONSTRUCTION OF GEOMETRIC FIGURES

Do not erase any construction arcs in these questions.

1. Construct and label the following triangles and quadrilaterals:
   (a) Triangle FGH where GH = 6,2 cm; $\hat{G} = 36°$ and $\hat{H} = 63°$

   (b) Parallelogram PQRS where PQ = 5,7 cm, PS = 7,8 cm and $\hat{R} = 112°$

2. (a) Construct ΔKLM where KL = 9,4 cm; LM = 7 cm and MK = 7,8 cm.

(b) Construct the perpendicular bisectors of all three sides of the triangle drawn in part (a). You should find that they all go through the same point.

(c) Use the point of intersection as the midpoint of a circle that passes through all three vertices of the triangle. Use your compass to draw this circle.

3. Construct the following angles without using a protractor:

(a) 45°

(b) 210°

4. Construct a regular hexagon in your exercise books by following these instructions:
   - Construct a horizontal line, AB, which is 2 cm long.
   - Set your compass to 2 cm, and from each of A and B, draw an arc above line AB. Call the point that the arcs intersect O.
   - Draw a circle of radius 2 cm, centred on O. It should go through A and B.
   - Place the compass on point B, and draw an arc crossing the circle on the side opposite to A. Call this point C.
   - Repeat the above step to create points D to F.
   - Join B to C with a straight line. Repeat with C to D, and so on, until you get back to point A. You have now constructed a regular hexagon!

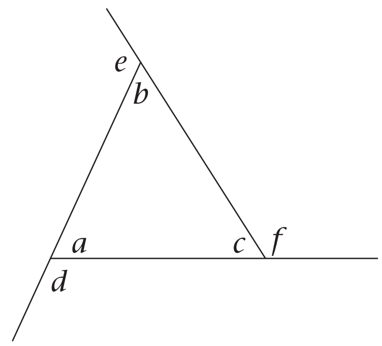
GEOMETRY OF 2D SHAPES

1. The following table summarises the properties of diagonals of quadrilaterals. Complete the table by placing ticks in the appropriate blocks.

	Parallelogram	**Rectangle**	**Square**	**Rhombus**	**Trapezium**	**Kite**
Diagonals bisect each other						
Diagonals cut at right angles						

2. Study the following figure.

   Note that *d*, *e* and *f* are the exterior angles of the triangle.



   (a) Write down an equation that shows the relationship between angle *d* and the sum of two other angles in the image.

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   (b) Determine the size of $d + e + f$. Give reasons for your answer.

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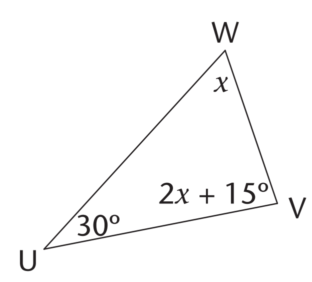
3. Determine the size of $\hat{V}$. Show all steps of your working and give reasons when using any geometrical theorem:



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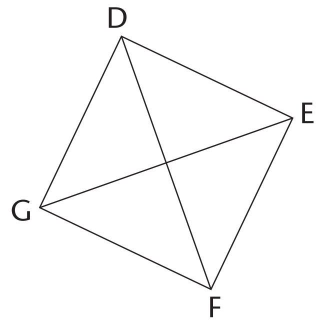
4. DEFG is a square with DF = 12 cm. Determine the length of the side of the square, correct to two decimal places. Show all steps of your working.
Give reasons when using any geometrical theorem:



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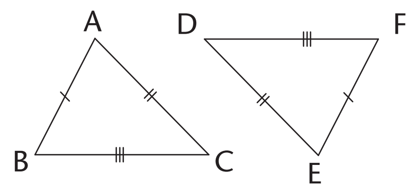
5. Are the following pairs of triangles congruent? If they are, write down the relationship in the form ΔXYZ ≡ ΔORQ, where X corresponds with O, Y with R, and so on. Also state the congruency condition (case) that proves the two triangles are congruent (for example, sss). If they are not congruent, explain why they are not congruent.
Note: the triangles are not drawn to scale.
(a)



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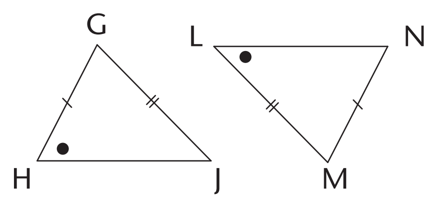
(b)



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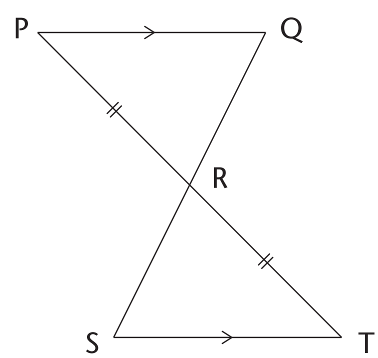
6. Provide a formal proof that the two triangles in the diagram below are congruent:



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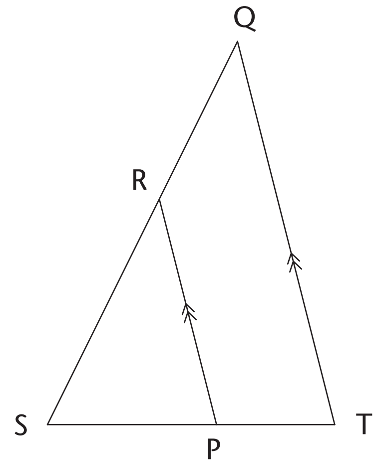
7. Study the diagram alongside:
   Prove that ΔSRP ||| ΔSQT.



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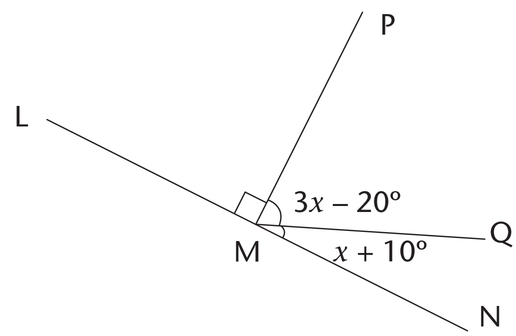
GEOMETRY OF STRAIGHT LINES

1. Determine the size of $P\hat{M}Q$.



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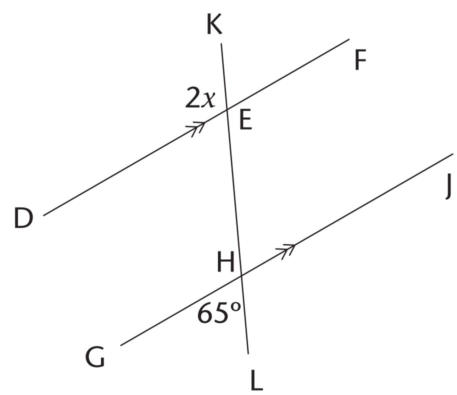
2. Determine the size of x in each case. Show all steps of your working and give reasons when using any geometrical theorem:

   (a)



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(b) Given: EH = EJ

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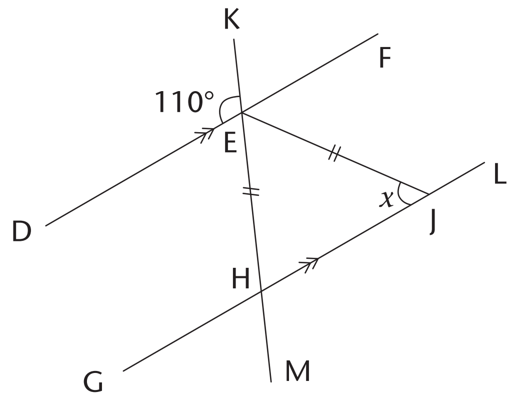

(c) $A\hat{B}G = x$; $B\hat{C}D = 130°$ and $C\hat{D}J = 72°$

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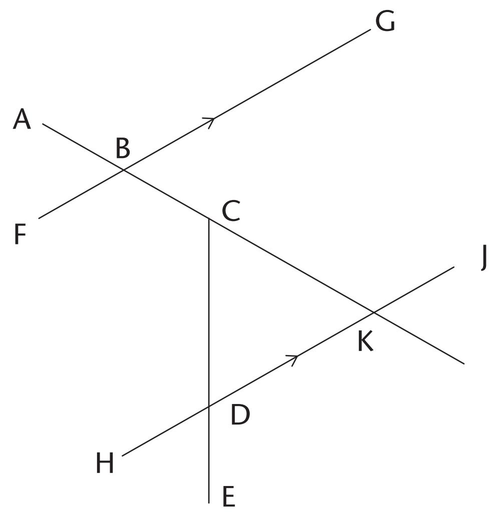

(d)

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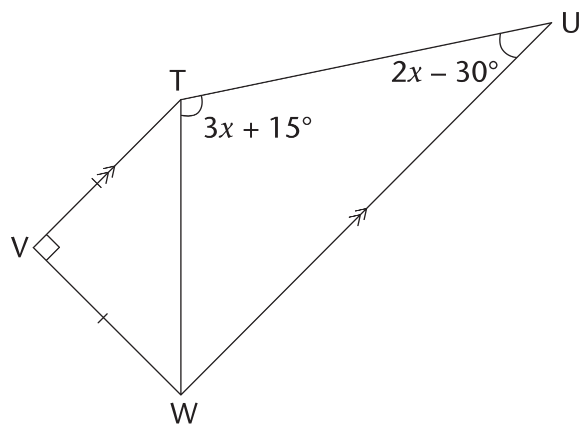
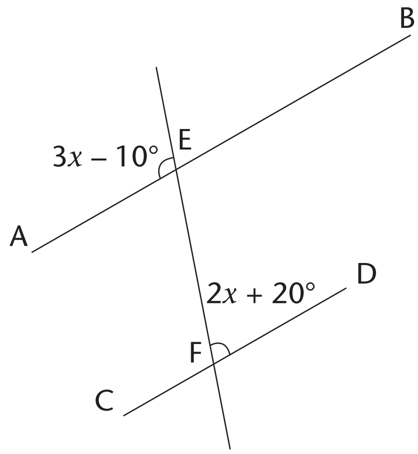


3. For which value of *x* are AB and CD parallel? Show all steps of your working and give reasons when using any geometrical theorem.



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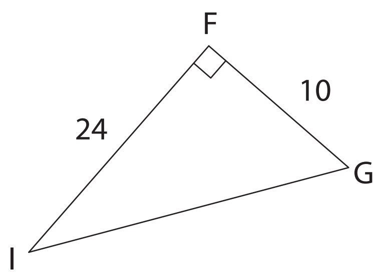
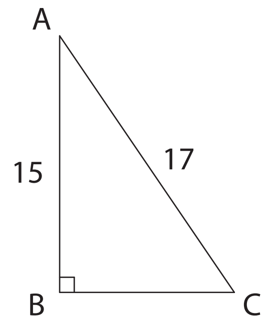
PYTHAGORAS' THEOREM

1. Calculate the missing side length in each of the following triangles:

(a)



(b)



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2. A river is 50 m wide. Assume that the river flows completely straight. If Camelia swims across the river to a point on the bank 12 m downstream from the point directly opposite her, how far will she swim? Give your answer correct to one decimal place.

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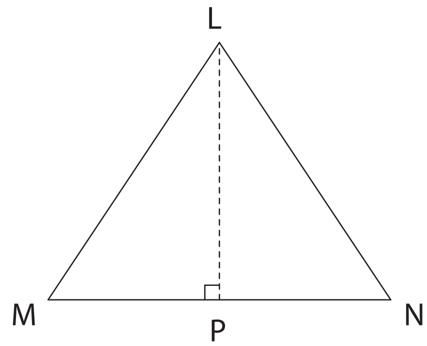
3. Triangle LMN is isosceles, with LM = LN. MN = 36 cm and LP = 24 cm. Determine the perimeter of the triangle.



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4. DEFG is a rectangle, with DE = 20 cm and diagonal EG = 101 cm. Determine the area of the rectangle.

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5. Is it possible to have a right-angled triangle with the following side lengths: 36; 76 and 84? Show all working necessary to support your answer.

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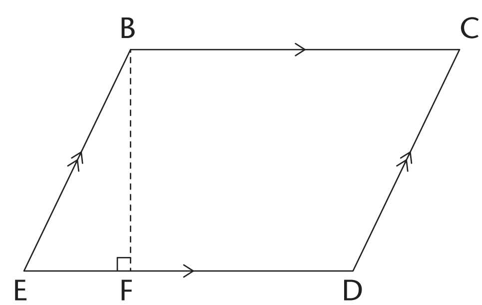
AREA AND PERIMETER OF 2D SHAPES

1. Determine (i) the perimeter, and (ii) the area, of each of the following shapes. If necessary, give your answers in centimetres or square centimetres, correct to one decimal place:

   (a) BF = 8 cm; BC = 10 cm; FD = 6 cm



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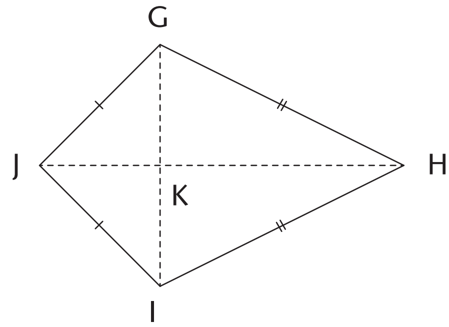
(b) GI = 12 cm; JK = 6 cm and JK : KH = 1 : 2



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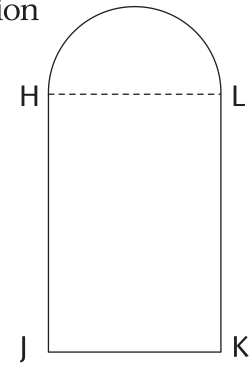
(c) The shape alongside is that of a window, consisting of a rectangular section HJKL, and a semi-circular top section. HJ = 0,5 m and JK = 0,2 m.



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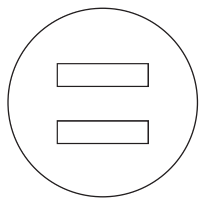
2. A button is made in the shape of a circle, with two congruent rectangles cut out, as shown in the diagram. The diameter of the button is 25 mm and the dimensions of each rectangle are 12 mm by 3 mm. Calculate the area of the top surface of the button in square centimetres.



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3. A rectangle has length $3d$ and width $7e$. Write simplified expressions for the:

(a) Area of the rectangle

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(b) Perimeter of the rectangle

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4. Complete the unshaded blocks in following table to show the impact on the perimeter and area of <u>doubling</u> <u>one dimension</u> of the shape (for a rectangle, the length; for a triangle, the base; and for a circle, the radius). Assume that the original perimeter was x, and the original area was y. One answer has already been added for you.

	Rectangle	Triangle	Circle
New perimeter/circumference			
New area	$2y$		

Assessment

In this section, the numbers in brackets at the end of a question indicate the number of marks the question is worth. Use this information to help you determine how much working is needed. The total number of marks allocated to the assessment is 60.

1. (a) Construct a triangle RST with RS = 7,3 cm, $\hat{R} = 42°$; and $\hat{S} = 67°$. (3)

   (b) Construct the bisectors of each of the angles of the triangle that you constructed in part (a). You should find that they have a common point of intersection. (4)

   (c) Use the common point of intersection of the bisectors of the angles that you constructed in part (b) as the midpoint of a circle touching all three sides of the triangle. Use your compass to draw this circle. (1)

   (d) Is it always possible to draw a triangle given the length of one of the lines and the sizes of the angles adjacent to that line (as was given in part (a), for example)? Explain your answer. (2)

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(e) Construct the following angle without using a protractor: 150°. (2)

(f) Mthunzi is thinking of a quadrilateral and provides the following clue to Sam: "Its diagonals cut perpendicularly, but not all the sides of the shape are equal in length." Help Sam by writing down the special name of the shape. (1)

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(g) Look at the figure below. Write down an equation, and use it to determine the size of x. (3)

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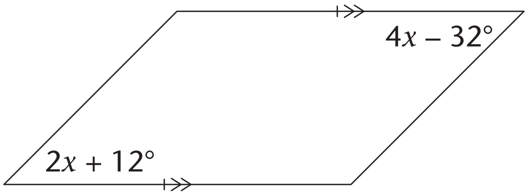
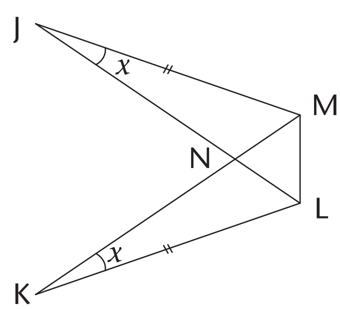


2. Study the diagram alongside:

(a) Prove that ΔJNM ≡ ΔKNL. (4)



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(b) Do you have enough information to prove that ΔJLM ≡ ΔKML? Explain your answer. (2)

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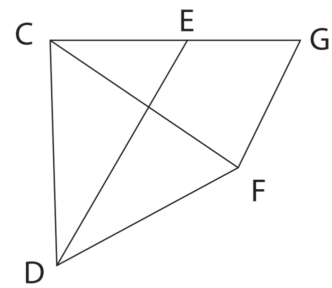
(c) Study the diagram alongside:
Given that ΔCDE ≡ ΔFCG, prove that ED || GF. Give reasons for all statements. (3)



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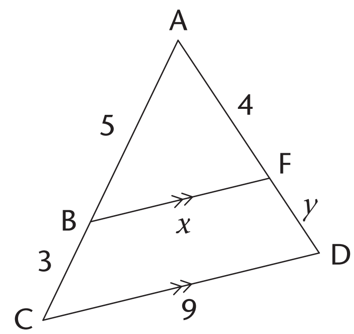
3. Study the diagram below. All numerical values represent lengths of sides:



(a) Briefly explain why ΔABF ||| ΔACD (a full proof is NOT required). (1)

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(b) Use the similarity of the triangles to determine the lengths of the line segments (correct to one decimal place). (6)

(i) x

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(ii) y

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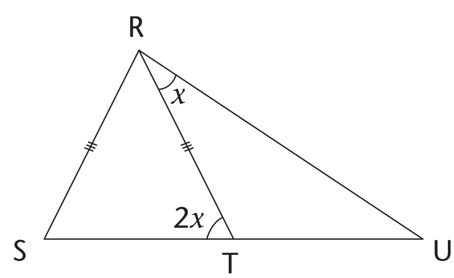
4. Study the diagram alongside. Determine, with reasons, the size of $\hat{U}$ in terms of x. (4)



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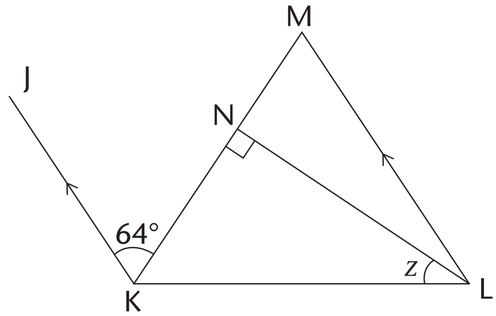
5. Study the diagram alongside. Given that MK = ML, determine, with reasons, the value of z. (5)



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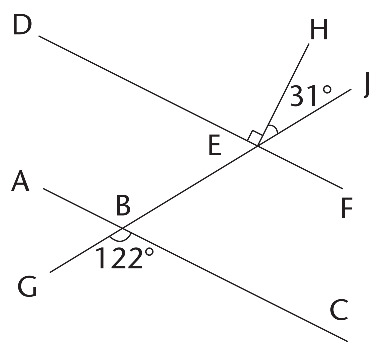
6. Is AC || DF? Explain your answer by means of a proof. (3)



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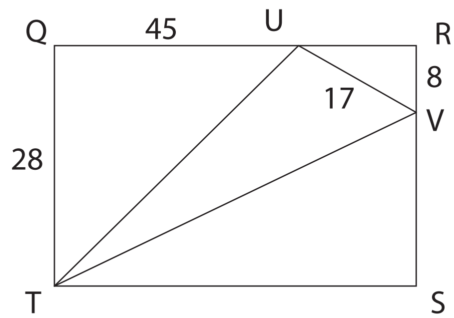
7. Study the diagram alongside. QRST is a rectangle. All numerical values represent lengths:

(a) Calculate the length of UT. (3)



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(b) Calculate the perimeter of triangle TUV, correct to one decimal place. (4)

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8. A rectangle has an area of $6a^2$ and a perimeter of $10a$. Determine, in terms of a, the dimensions of the rectangle. (2)

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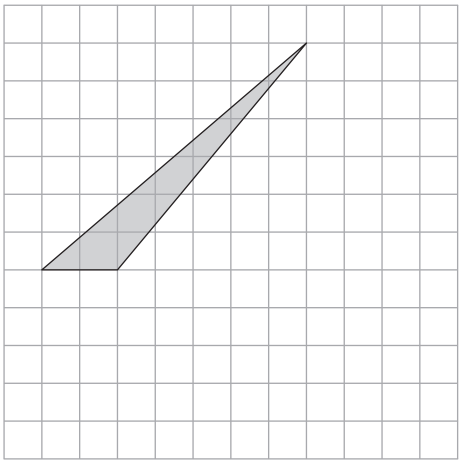
9. On the grid below, draw a parallelogram that has the same area as the triangle. (2)



10. The perimeter of a rhombus is 60 cm, and the length of one of its diagonals is 24 cm.

(a) Calculate the length of a side of the rhombus. (1)

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(b) Show that the area of the rhombus is 216 cm^2. (4)

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